Chapter 8 Repetitive Change: Cycles and Trigonometry

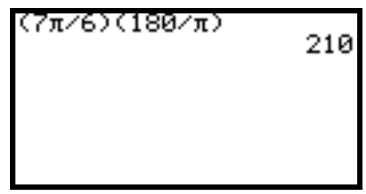
8.1 Functions on Angles: Sines and Cosines

Before you begin this chapter, go back to the first page of this *Guide* and check the basic setup, the statistical setup, and the window setup. If these are not set as specified in Figures 1, 2, and 3, you will have trouble using your calculator in this chapter. Pay careful attention to the third line in the MODE screen in the basic setup. The Radian/Degree mode setting affects the calculator's interpretation of the ANGLE menu choices. Your calculator's MODE menu should always be set to Radian unless otherwise specified.

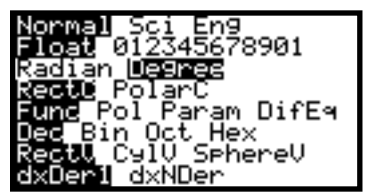
8.1.1 CONVERTING ANGLES FROM RADIANS TO DEGREES Even though the process of converting angles from one measure to another is a simple arithmetic task, we cover it here to show you more of your calculator's functionality. Suppose you want to convert $7\pi/6$ to degree measure. (Remember that when no units are specified, the angle is assumed to be expressed in radian measure.)

As shown in the text example, convert $7\pi/6$ radians to degrees with multiplication by the proper factor. (Let the units be your guide and carefully use parentheses.)

$$\left(\frac{7\pi}{6}\text{ radians}\right)\left(\frac{180^{o}}{\pi\text{ radians}}\right)=210^{o}$$



If you want to use your calculator for this conversion *to* degrees, first set the calculator to degree mode. Press 2nd MORE (MODE) , choose Degree in the third line and press ENTER . Return to the home screen.



Tell the calculator the angle and that it is measured in radians with (7 2nd ^ (π) ÷ 6) 2nd × (MATH) F3 (ANGLE) F2 (r) . Press ENTER to conve the angle to degrees.

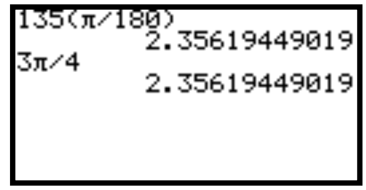
While it is not necessary in this examp[redacted] if you have an angle that you want expressed in degree[redacted]inutes, and seconds, first convert the angle to degree[redacted]nd then press 2nd × (MATH) F3 (ANGLE) F4 (DMS) .

(Don't forget to set your calculator mode back to radian measure when you finish the conversions!)

8.1.2 CONVERTING ANGLES FROM DEGREES TO RADIANS Converting angles from degree measure to radian measure is similar to process described above. However, since you wish to convert to radian measure, the mode setting should be Radian. Suppose that you need to convert 135^{o} to radian measure.

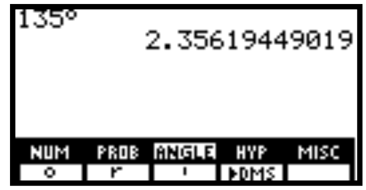
As shown in the text example, do this numerically by multiplication of the proper factor. (Let the units be your guide and carefully use parentheses.)

$$\left(135^{\circ}\right)\left(\frac{\pi \text{ radians}}{180^{\circ}}\right) = \frac{3\pi}{4} \text{ radians} = \frac{3\pi}{4}$$



Because you want to convert *to* radians, press [MODE], choose Radian in the third line and press [ENTER].

Enter the degree measure of the angle and tell the calculator the angle is in degrees with [2nd] [X] (MATH) [F3] (ANGLE) [F1] (°). [ENTER] converts to radians.

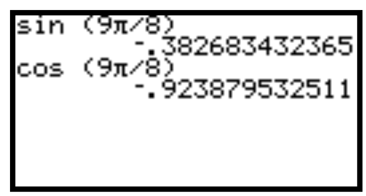


8.1.3 EVALUATING TRIGONOMETRIC FUNCTIONS

Just as it was important to have the correct mode set when changing from degrees to radians or vice versa, it is essential that you have the correct mode set when evaluating trigonometric function values. Unless you see the degree symbol, the mode setting should be Radian.

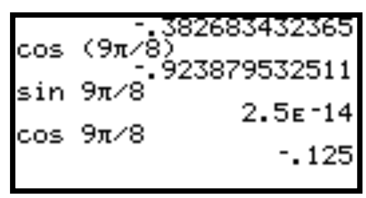
Find $\sin \frac{9\pi}{8}$ and $\cos \frac{9\pi}{8}$.

Because these angles are in radians, be certain that Radian is chosen in the third line of the Mode screen. The [SIN] and [COS] keys are above the [,] and [(] keys on your calculator's keyboard.



It is also essential that you use parentheses -- the calculator's order of operations are such that the sine function takes precedence over division.

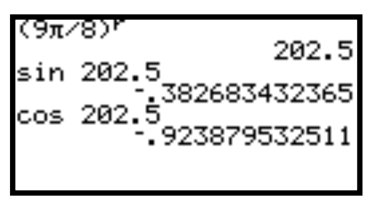
Without telling your calculator to first divide, it evaluates $\frac{\sin(9\pi)}{8}$ and $\frac{\cos(9\pi)}{8}$.



The values of the trigonometric functions are the same regardless of whether the angles are expressed in radian measure or degree measure. To verify this, first convert the angle in radians to degree measure, and then re-evaluate the sine and cosine of the angle.

Change the mode setting to Degree and then convert $\frac{9\pi}{8}$ to degrees. Evaluate the sine and cosine functions.

(When you finish, don't forget to change the mode setting back to Radian!)



8.2 Cyclic Functions as Models

We now introduce another model -- the sine model. As you might expect, this model is fit to data that repeatedly varies between alternate extremes. The form of the model is $f(x) = a \sin(bx + c) + d$ where a is the amplitude, $2\pi/|b|$ is the period, $^-c/b$ is the horizontal shift, and d is the vertical shift.

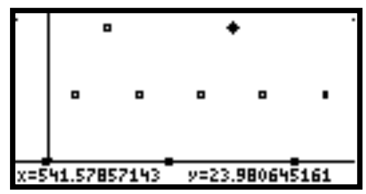
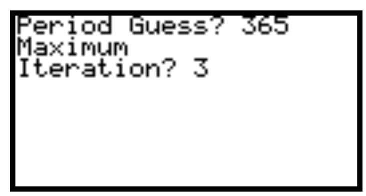
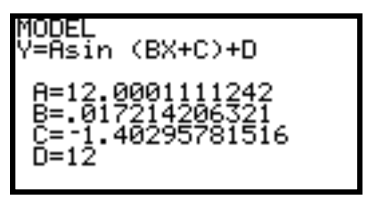
The sine model is available in the TI-86, but on the TI-85, we use program[1] SINREG to fit the sine model. (SINREG is given in the TI-85 /86 Appendix.)

8.2.1 FITTING A SINE MODEL TO DATA

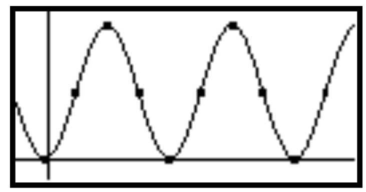
Before fitting any model to data, remember that you should construct a scatter plot of the data and observe what pattern the data appears to follow. We illustrate finding a sine model for cyclic data with the hours of daylight on the Arctic Circle as a function of the day of the year on which the hours of daylight are measured. (January 1 is day 1.)

Day of the year	⁻10	81.5	173	264	355	446.5	538	629	720	811.5
Hours of daylight	0	12	24	12	0	12	24	12	0	12

Enter these data with the day of the year in L1 and the hours of daylight in L2.

Construct a scatter plot of the data. It appears to be cyclic. Either look at the data, move around the screen with the arrow keys on the TI-85, or use TRACE on the TI-86 to measure the horizontal distance between one high point and the next (or between two successive low points, etc.). One cycle of the data appears to be about 365 days.	 period ≈ 542– 177
TI-85 Run program SINREG. The program's initial message reminds you that the input data should be in list L1 and the output data in list L2. *The input data should be in order, from smallest to largest, when using this program.* You must supply the program with the period of the model and the maximum iterations -- the number of times the program should cycle thorough the process of determining the best-fit model.	 The more iterations you use, the longer it takes the program to run. Try a value between 3 and 10 for best results.
If the program finds a good fit before it completes the maximum number of iterations, it stops. For instance, these data are so close to a sine curve that it took only 1 iteration. (You need to look at the program to know this, however.) The program stores the model in y1, so all you need do is run program STPLT to graph the model on the scatter plot of the data.	 (If the graph of the model looks like a line, you have forgotten to put your calculator back in radian mode!)

[1]This program is based on a program that was modified by John Kenelly from material by Charles Scarborough of Texas Instruments. The authors sincerely thank Robert Simms for this help with this version of the program.

TI-86 The sine model SinR on the TI-86 is in the list with the other models in the STAT (CALC) menu. Enter the list containing the input data, the list containing the output data, and the location of the y(x)= list that you wish the model to appear. Press ENTER .	
The sine model of best fit appears on the screen and is pasted into the y1 location of the y(x)= list. (If you wish, you can tell the calculator how many times to go through the routine that fits the model. This number of iterations is 3 if not specified. The number should be typed before L1 when initially finding the model.)	
Press GRAPH F5 (GRAPH) to graph the model on the scatter plot of the data. (If the graph of the model looks like a line, you have forgotten to put your calculator back in radian mode!)	
Even though it did not occur in this example, you may get a SINGULAR MATRIX error when trying to fit a sine model to certain data. If so, try specifying an estimate for the period of the model. Recall that our estimate of the period is 365 days.	
Notice how much faster the TI-86 finds the model! (If you do not think the original model the calculator finds fits the data very well, try specifying a period and see if a better-fitting model results. It didn't here, but it might with a different set of data.)	

8.3 Rates of Change and Derivatives

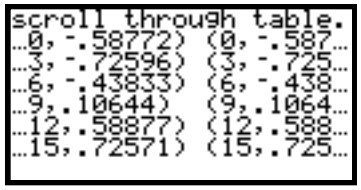
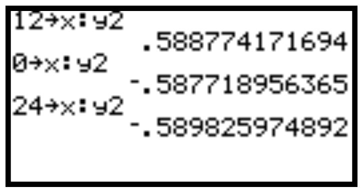
All the previous techniques given for other models also hold for the sine model. You can find intersections, maxima, minima, inflection points, derivatives, integrals, and so forth.

8.3.1 DERIVATIVES OF SINE AND COSINE MODELS Evaluate der1 at a particular input to find the value of the derivative of the sine model at that input. Suppose the calls for service made to a county sheriff's department in a certain rural/suburban county can be modeled as $c(h) = 2.8 \sin(0.262h + 2.5) + 5.38$ calls during the hth hour after midnight.

Enter the model in some location of the y(x)= list, say y1. In another location, say y2, enter the calculator's numerical derivative.	

Even though your calculator will not give you an equation for the derivative, you can use one of the methods illustrated in Sections 4.4.1 and/or 4.4.2 of this *Guide* to check the derivative you obtain using derivative rules.

Enter your derivative formula in y3, turn off y1, and use the table to compare several values of the calculator's derivative (y2) and your derivative (y3).

y2 = der1(y1, x, x) y3 = 0.7336 cos (0.262x + 2.5) (Turn y1 off.)	 **TI-85**	**TI-86**
How quickly is the number of calls received each hour changing at noon? at midnight?	 **TI-85**	**TI-86**

- The calculator screens shown above show that to answer these questions, simply evaluate y2 (or your derivative in y3) at 12 for noon and 0 (or 24) for midnight. Also, you weren't told if "midnight" refers to the initial time or 24 hours after that initial time.

8.5 Accumulation in Cycles

As with the other models we have studied, applications of accumulated change with the sine and cosine models involve the calculator's numerical integrator fnInt.

8.5.1 INTEGRALS OF SINE AND COSINE MODELS Suppose that the rate of change of the temperature in Philadelphia on August 27, 1993 can be modeled as

$$t(h) = 2.733 \cos(0.285h - 2.93) \text{ }^{o}\text{F per hour}$$

h hours after midnight. Find the accumulated change in the temperature between 9 a.m. and 3 p.m. on August 27, 1993.

You could enter the model in the y(x)= list. However, since we are not asked to draw a graph, we choose to find the result using the calculator's numerical integrator and the home screen. The temperature increased by approximately 13 °F.