Chapter 4 Determining Change: Derivatives
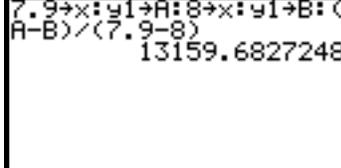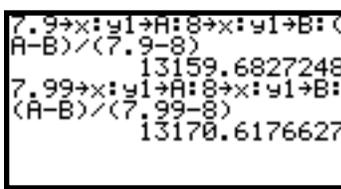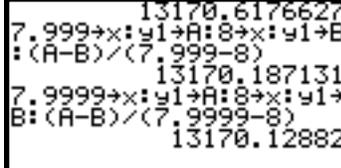
📖 4.1 Numerically Finding Slopes

Using your calculator to find slopes of tangent lines does not involve a new procedure. However, the techniques in this section allow you to repeatedly apply a method of finding slopes that gives quick and accurate results.
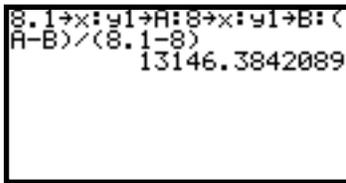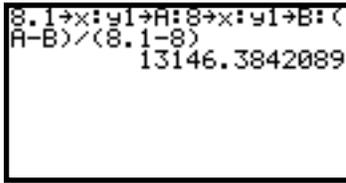
4.1.1 NUMERICALLY INVESTIGATING SLOPES ON THE HOME SCREEN

Finding slopes of secant lines joining the point at which the tangent line is drawn to increasingly close points on a function to the left and right of the point of tangency is easily done using your calculator. Suppose we want to find the slope of the tangent line at $t = 8$ to the graph of the function giving the number of polio cases in 1949: $y = \dfrac{42183.911}{1 + 21484.253e^{-1.248911t}}$

where $t = 1$ on January 31, 1949, $t = 2$ on February 28, 1949, and so forth.

Enter the equation in the y1 location of the y(x)= list. (Carefully check the entry of your equation, especially the location of the parentheses. Parentheses are needed around the denominator and around the exponent.)
 	
 We now evaluate the slopes joining nearby points to the *left* of $x = 8$.
	`y1█…53e^(-1.248911x))`
	
	 `y(x)= RANGE ZOOM TRACE GRAPH`
	 `  x  │  y  │ INSf │ DELf │SELCT▸`

TI-85 Type in the expressions shown to the right to compute the slope of the secant line joining $x = 7.9$ and $x = 8$. Carefully watch the parentheses!
 	
 Record each slope on your paper as it is computed. You are trying to find what these slopes are approaching.
	`7.9→x:y1→A:8→x:y1→B:(`
	 `A-B)/(7.9-8)`
	 `          13159.6827248`

Press ⌷2nd⌷ ⌷ENTER⌷ (ENTRY) to recall the last entry, and then use the cursor keys to move the cursor over the 9 in the "7.9". Press ⌷2nd⌷ ⌷DEL⌷ (INS) and press ⌷9⌷ to insert another 9 in <u>both</u> positions where 7.9 appears.
 Press ⌷ENTER⌋ to find the slope of the secant line joining $x = 7.99$ and $x = 8$.)
	`7.9→x:y1→A:8→x:y1→B:(`
	 `A-B)/(7.9-8)`
	 `          13159.6827248`
	 `7.99→x:y1→A:8→x:y1→B:`
	 `(A-B)/(7.99-8)`
	 `          13170.6176627`

Continue in this manner, recording each result, until you can determine to which value the slopes from the left seem to be getting closer and closer.	`          13170.6176627`
	 `7.999→x:y1→A:8→x:y1→B`
	 `:(A-B)/(7.999-8)`
	 `          13170.187131`
	 `7.9999→x:y1→A:8→x:y1→`
	 `B:(A-B)/(7.9999-8)`
	 `          13170.12882`

We now evaluate the slopes joining nearby close points to the *right* of $x = 8$. Clear the screen with $\boxed{\text{CLEAR}}$, recall the last expression with $\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ (ENTRY) , and edit it with $\boxed{\text{DEL}}$ so that the nearby point is $x = 8.1$. Press $\boxed{\text{ENTER}}$.	```8.1→x:y1→A:8→x:y1→B:(A-B)/(8.1-8) 13146.3842089```
Continue in this manner as before, recording each result on paper, until you can determine the value the slopes from the right seem to be approaching. When the slopes from the left and the slopes from the right approach the same value, that value is the slope of the tangent line at $x = 8$.	```8.1→x:y1→A:8→x:y1→B:(A-B)/(8.1-8) 13146.3842089```
TI-86 Type in the expression shown to the right to compute the slope of the secant line joining $x = 7.9$ and $x = 8$. You must use parentheses around both the num-erator and the denominator of the slope formula. Record each slope on your paper as it is computed. You are trying to find what these slopes are approaching.	
Press $\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ (ENTRY) to recall the last entry, and then use the cursor keys to move the cursor over the 9 in the "7.9". Press $\boxed{\text{2nd}}$ $\boxed{\text{DEL}}$ (INS) and press $\boxed{9}$ to insert another 9 in <u>both</u> positions where 7.9 appears. Press $\boxed{\text{ENTER}}$ to find the slope of the secant line joining $x = 7.99$ and $x = 8$.	
Continue in this manner, recording each result, until you can determine to which value the slopes from the left seem to be getting closer and closer.	
We now evaluate the slopes joining nearby close points to the *right* of $x = 8$. Clear the screen with $\boxed{\text{CLEAR}}$, recall the last expression with $\boxed{\text{2nd}}$ $\boxed{\text{ENTER}}$ (ENTRY) , and edit it with $\boxed{\text{DEL}}$ so that the nearby point is $x = 8.1$. Press $\boxed{\text{ENTER}}$.	

Continue in this manner as before, recording each result on paper, until you can determine the value the slopes from the right seem to be approaching. When the slopes from the left and the slopes from the right approach the same value, that value is the slope of the tangent line at $x = 8$.	

Both   The slopes from the left and from the right appear to be getting closer and closer to 13,170. (The number of polio cases makes sense only as a whole number.)

4.1.2 NUMERICALLY INVESTIGATING SLOPES USING THE TI-86 TABLE

The process shown in Section 4.1.1 can be done in fewer steps when you use the TABLE. Recall that we are evaluating the slope formula

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(8+h) - f(8)}{h}$$

for various values of h where h is the distance from 8 to the input of the close point. This process is illustrated using the logistic function given in Section 4.1.1 of this *Guide*.

TI-86 Remember that when in the graphing list, you must use x as the input variable. Since h is what is varying in the slope formula, replace h by x and enter the slope formula in y2. Turn y1 off since we are looking only at the output from y2.	
Press TABLE F2 (TBLST) and choose the ASK setting. (Since we are using the ASK feature, the settings for TblStart and ΔTbl do not matter.)	
Access the table with F1 (TABLE) , and either delete or type over any previous entries in the x column. Let x (really h) take on values that move the nearby point on the left closer and closer to 8.	

- Notice that after a certain point, the calculator switches your input values to scientific notation and displays rounded output values so that the numbers can fit on the screen in the space allotted for outputs of the table. You should position the cursor over each output value and record on paper as many decimal places as necessary in order to determine the limit from the left to the desired degree of accuracy.

Repeat the process, letting x (really *h*) take on values that move the nearby point on the right closer and closer to 8. View the entire decimal value for each output and determine the limit from the right.	

4.1.3 VISUALIZING THE LIMITING PROCESS (optional)

Program SECTAN can be used to view secant lines between a point *(a, f(a))* and some close points on a curve *y = f(x)* and the tangent line at the point *(a, f(a))*. Using this program either before or after numerically finding the limit of the slopes can help you understand the numerical process.

We use the function giving the number of polio cases in 1949: $y = \dfrac{42183.911}{1 + 21484.253e^{-1.248911t}}$

where $t = 1$ on January 31, 1949, $t = 2$ on February 28, 1949, and so forth.

(Program SECTAN is given in the TI-85/86 Appendix and should be in your calculator before you work through the following illustration.)

Before using program SECTAN, the function must be in the y1 location of the y(x)= list, and you *must* draw a graph of the function. Enter the function, using *x* as the input variable, in y1. If the function is already entered, be certain it is turned on.	y1⊟…53e^(-1.248911x)) y(x)= RANGE ZOOM TRACE GRAPH x y INSf DELf SELCT▸

Since $t = 1$ represents January 31, 1949, 0 represents the beginning of 1949. The function gives the number of polio cases for the entire year, so we view the graph through December 31, 1949 ($t = 12$).
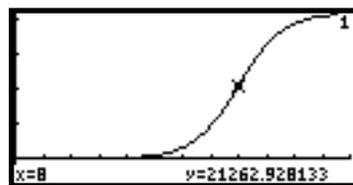
When you need to draw the graph of a function, you usually are given the input values in the statement of the problem in your text. *Always carefully read the problem before starting the solution process.*

Press [RANGE] (WIND on the TI-86) , set xMin to 0, and set xMax to 12. Press [ZOOM] [MORE] [F1] (ZFIT) . Reset yMin to ⁻6000 to allow room to see the trace cursor. Press [F5] (GRAPH) .	RANGE xMin=0 xMax=12 xScl=1 yMin=⁻6000 yMax=41904.8860405 yScl=10000 y(x)= RANGE ZOOM TRACE GRAPH▸
The graph of the function is displayed. If the point of tangency is not such that you can easily see points to the left and right of it, adjust the window settings. Return to the home screen.	 x=8 y=21262.928133
Press [PRGM] (NAMES) and the F-key corresponding to the location of program SECTAN . (Your program list may not look like the one on the right.) Press [ENTER] .	SECTAN NAMES EDIT NUINT SECTA SINRE STPLT TABLE▸

If you start this program and have forgotten to enter the function in y1 or to draw a graph of the function, press F2 (No) . Otherwise, press F1 (Yes) . At the prompt, enter the input value of the tangent point. (For this example, $x = 8$.)	``` Have f(x) in y1 and draw graph of f Continue? x-value of point of tangency? 8 ```
The next message that appears tells you to press enter to see secant lines drawn between the point of tangency and close points to the *left*. Press ENTER . (Five secant lines will draw ; they may take a little time to draw.)	
When you finish looking at the graph of the secant lines, press ENTER to continue. The next message that appears tells you to press enter to see secant lines drawn between the point of tangency and close points to the *right*. Press ENTER .	
Press ENTER to continue the program. You are next instructed to press ENTER to see a graph of the tangent line. (Note that the tangent line cuts through the graph because an inflection point occurs at $x \approx 8$.)	

- *Caution:* In order to properly view the secant lines and the tangent line, it is essential that you first draw a graph of the function clearly showing the function, the point of tangency, and enough space so that the close points on either side can be seen.

📖 4.3 Slope Formulas

Your calculator can draw slope formulas. However, to do so, you must first enter a formula for the function whose slope formula you want the calculator to draw. Because you will probably be asked to draw slope formulas for functions whose equations you are not given, you must not rely on your calculator to do this for you. You should instead use technology to check your hand-drawn graphs and to examine the relationships between a function graph and its slope graph. It is very important in both this chapter and several later chapters that you know these relationships.

4.3.1 UNDERSTANDING YOUR CALCULATOR'S SLOPE FUNCTION   Both the TI-85 and the TI-86 use the slope of a secant line to approximate the slope of the tangent line at a point on the graph of a function. However, instead of using a secant line through the point of tangency and a close point, these calculators use the slope of a secant line through two close points that are equally spaced from the point of tangency.

Figure 7 illustrates the secant line joining the points *(a-k, f(a-k))* and *(a+k, f(a+k))*. Notice that the slopes appear to be close to the same value even though the secant line is not the same line as the tangent line.
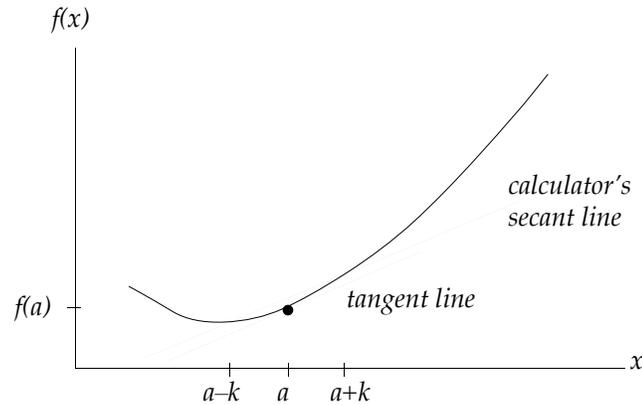
Figure 7

As *k* gets closer and closer to 0, the two points move closer and closer to *a*. Provided the slope of the tangent line exists, the limiting position of the secant line will be the tangent line.

The calculator's notation for the slope of the secant line shown in Figure 7 is

nDer(function, symbol for input variable, *a*)

The *k* that determines the location of the points *(a-k, f(a-k))* and *(a+k, f(a+k))* can be set to different values, but specifying this value is optional. If it is not given, the calculator automatically uses *k* = 0.001 (or whatever the current setting happens to be).
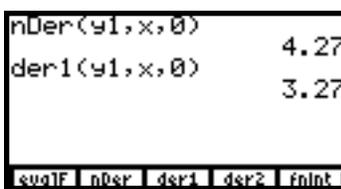
Your calculator also computes for many functions[1] the slope of the tangent line. This tangent line slope, the calculator's first derivative der1, uses the syntax

der1(function, symbol for input variable, *a*)

We now investigate these ideas and compare nDer and der1. We begin our discussion with the smooth, continuous function $f(x) = x^3 - 4x^2 + 3.27x - 8.65$.

Enter $f(x) = x^3 - 4x^2 + 3.27x - 8.65$ in one of the locations of the y(x)= list, say y1.
 	
 Return to the home screen with 2nd EXIT (QUIT) .
 	
 Suppose you want to find the slope of the secant line between the points $(^-1, f(^-1))$ and $(1, f(1))$. That is, you are finding the slope of the ⬛ tween *(a-k, f(a-k))* and *(a+k, f(a+k))* fo⬛ 0 and *k* = 1.
	`y1∎..-4 x²+3.27 x-8.65`
	
	 `⬛⬛⬛ RANGE ZOOM TRACE GRAPH`
	 `x │ y │ INSf │ DELf │SELCT▶`

Your calculator uses the symbol ⬛ he "tol" sets the calculator's tolerance -- a q⬛ty that can affect the speed and accuracy of some c⬛utations. For our purposes, have tol = $10^{\wedge-6} = 1$ ⬛, ⬛ = 1.
 	
 TI-85 Set the value of *k* with 2nd CLEAR (TOLER)
 	
 TI-86 Set the value of *k* with 2nd 3 (MEM) F4 (TOL)
	`TOLERANCE`
	 `  tol=1ɛ⁻6`
	 `  δ=1`
	
	 δ = *k*

---

[1]The first derivative, der1, and the second derivative, der2, are valid for single-argument functions. For more information, consult your TI-85 or TI-86 owner's *Guidebook*.

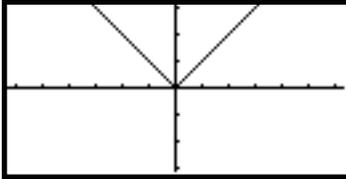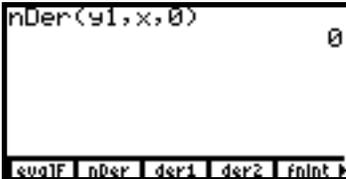|	

Both Return to the home screen. Type the expressions on the right. Access nDer(with [2nd] [÷] (CALC) [F2] and with der1 with [2nd] [÷] (CALC) [F3] .
 	
Press [ENTER] and see that the slope of the secant line between ($^-$1, $f(^-1)$) and (1, $f(1)$) is 4.27 and the slope of the tangent line at (0, $f(0)$) is 3.27.
	nDer(y1,x,0)
	                    4.27
	der1(y1,x,0)
	                    3.27
	
	evalF nDer der1 der2 fnInt

Go to the TOLERANCE screen and change the value of k from 1 to 0.1. Find the slope of the secant line and the slope of the tangent line.
 	
Repeat the process for $k = 0.001$ and $k = 0.0001$. Did you get the values 3.28, 3.2701, 3.270001, and 3.27000001 for the secant line slopes? Note that the value of der1 has remained constant at 3.27.
	nDer(y1,x,0) 3.27
	                  3.2701
	nDer(y1,x,0)
	               3.270001
	nDer(y1,x,0)
	             3.27000001
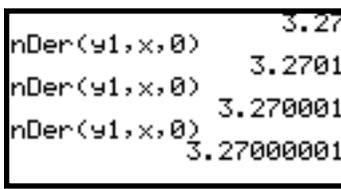
In the table on the next page, the first row lists some values of a, the input of the point of tangency, and the second row gives the slope of the tangent line at those values. (You will later learn how to find these exact values of the slope of the tangent line to $f(x)$ at various input values.)

Use your calculator to verify the values in the third through sixth rows that give the values of der1 and nder, the slope of the secant line between the points $(a-k, f(a-k))$ and $(a+k, f(a+k))$ for the indicated values of k. Find each secant line slope by changing the value of k on the tolerance screen and then calculating the value of nDer(y1, X, a).

a = input of point of tangency	-1	2.3	12.82	62.7
slope of tangent line	14.27	0.74	393.7672	11295.54
value of der1(y1,x,a)	14.27	0.74	393.7672	11295.54
slope of secant line, $k = 0.1$	14.28	0.75	393.7772	11295.55
slope of secant line, $k = 0.01$	14.2701	0.7401	393.7673	11295.5401
slope of secant line, $k = 0.001$	14.270001	0.740001	393.767201	11295.54
slope of secant line, $k = 0.0001$	14.27000001	0.74000001	393.7672	11295.54

You can see that the value of der1(y1,x,a) equals the slope of the tangent line and that the slope of the secant line is very close to the slope of the tangent line for small values of k. The slope of this secant line does a good job of approximating the slope of the tangent line when k is very small. For consistency, we choose $k = 0.001$. Set k to this value and do not change it.
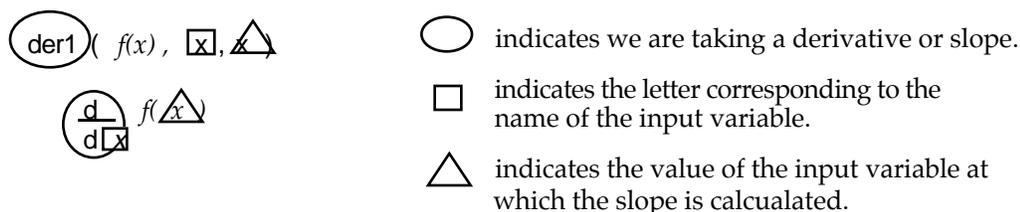
Will the slope of this secant line always do a good job of approximating the slope of the tangent line when k is very small? Yes, it does, as long as the instantaneous rate of change exists at the input value (a) at which you evaluate nDer. When the instantaneous rate of change does not exist at a point, neither nDer nor der1 should be used to approximate or find something that does not have a value! Consider the following.

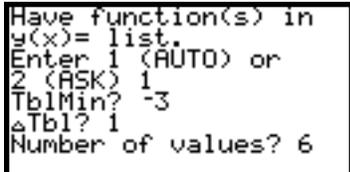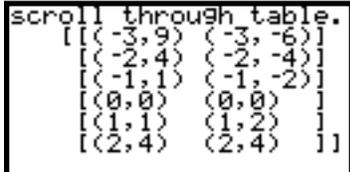|	
Clear all functions from the y(x)= list, and enter the function $y = \lvert x \rvert$ in y1 with the keystrokes [2nd] [X] (MATH) [F1] (NUM) [F5] (ABS) [x-VAR] Draw the graph with [ZOOM] [MORE] [F4] (ZDECM) . Notice that this function has a "sharp point" at $x = 0$. The instantaneous rate of change does not exist at $x = 0$.	
Return to the home screen and find the slope of the secant line joining two points 0.001 unit on either side of $x = 0$. Notice that the slope of the secant line is 0, but the instantaneous rate of change at $x = 0$ does not exist!	nDer(y1,x,0) 0 evalF nDer der1 der2 fnInt ▶
Now evaluate der1 at x = 0. When you enter the expression shown on the right, you receive an ERROR 4 DOMAIN message. This message tells you that something is wrong! Look again at the function and see that the instantaneous rate of change at $x = 0$ does not exist.	nDer(y1,x,0) 0 der1(y1,x,0)
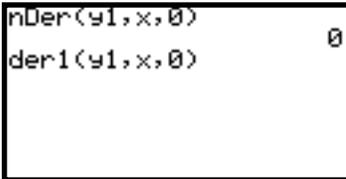
- Be certain the instantaneous rate of change exists at a point before using nDer. Two places where nDer usually does *not* give correct results *for the instantaneous rate of change* are at sharp points and the joining point(s) of piecewise continuous functions.

- Provided the instantaneous rate of change exists at a point, we can use the secant line slope nDer to provide a good *approximation* to the slope of the tangent line at that point or we can use der1 to give the *value* of the slope of the tangent line at that point. Since the slope of the tangent line is the slope of the curve which is the derivative of the function, we choose to use der1 rather than nDer, and we call der1 the calculator's *numerical derivative*.

4.3.2 DERIVATIVE NOTATION AND CALCULATOR NOTATION   You can often see a pattern in a table of values for the slopes of a function at indicated values of the input variable and discover a formula for the slope (derivative). The process of calculating the slopes uses the calculator's numerical derivative, der1(f(X), X, X). The correspondence between our notation $\dfrac{d\,f(x)}{dx}$ and the calculator's notation der1($f(x)$, x, x) is shown below:

der1($f(x)$, [x], △)   ◯ indicates we are taking a derivative or slope.

$\dfrac{d}{d\,x} f(△)$ ☐ indicates the letter corresponding to the name of the input variable.

△ indicates the value of the input variable at which the slope is calcualated.

Suppose you are asked to construct a table of values of $f'(x)$ where $f(x) = x^2$ evaluated at different values of x. Two methods of doing this are illustrated below:
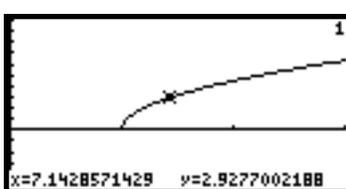
Return to the home screen and type the expression on the right. Access der1 with 2nd ÷ (CALC) F3 . der1(X^2, X, 2) = $\frac{dy}{dx}$ for $y = x^2$ evaluated at $x = 2$.	``` der1(x²,x,2) 4 ```
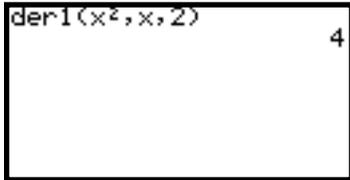
Recall the last entry, and edit the expression der1(X^2, X, 2) by changing the 2 to a ⁻3.
 	
Press ENTER . Continue with this process until you have found all the slope values.
	``
	                   4
	der1(x²,x,-3)
	                  -6
	der1(x²,x,-1)
	                  -2
	der1(x²,x,0)
	                   0
	``
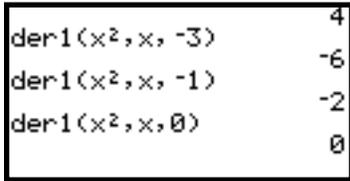
You might prefer to use the TABLE. If so, recall that you must have the expression being evaluated in the y(x)= list. (On the TI-85, it must be in y1.) If you enter the slope formula as indicated on the right, you will only have to change y1 when you work with a different function.	``` y1=x² y2=der1(y1,x,x) x y INSƒ DELƒ SELCT evalF nDer der1 der2 fnInt ▶ ```
TI-85 Run program TABLE, and choose the AUTO option. Set TblMin = ⁻3 and ΔTbl = 1. Choose to generate 6 table values.	``` Have function(s) in y(x)= list. Enter 1 (AUTO) or 2 (ASK) 1 TblMin? -3 ΔTbl? 1 Number of values? 6 ```
Press ENTER when the message appears to see the points (x, y1(x)) in the first column and the points (x, der1(y1, x, a) in the second column for a = ⁻3, ⁻2, ⁻1, 0, 1, and 2.	``` scroll through table. [[(-3,9) (-3,-6)] [(-2,4) (-2,-4)] [(-1,1) (-1,-2)] [(0,0) (0,0)] [(1,1) (1,2)] [(2,4) (2,4)]] ```
TI-86 You can either type in the x-values using the ASK feature of the table or you can set TblMin = ⁻3 and ΔTbl = 1 with the AUTO setting chosen.	
Evaluate the function and the numerical derivative at $x = ⁻3, ⁻2, ⁻1, 0, 1, 2$, and 3. Determine a relationship (pattern) between the slopes and the values of x. Function values appear in the y1 column and slopes appear in the y2 column.	

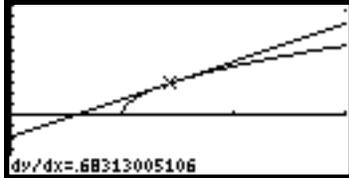| Both Determine a relationship (pattern) between the slopes and the values of x. Function values appear in the y1 column and slopes appear in the y2 column.	The TI-85 and the TI-86 only calculate or approximate numerical values of slopes -- they do not give the slope in formula form.

- If you have difficulty determining a pattern, enter the x-values at which you are evaluating the slope in list L1 and the values of der1 in list L2. Draw a scatter plot of the x-values and the slope values. The shape of the scatter plot should give you a clue as to the equation of the slope formula. If not, try drawing another scatter plot where L1 contains the values of $y = f(x)$ and L2 contains the calculated slope formula values. Note that this method might help only if you consider a variety of values for x in list L1.

4.3.3 DRAWING TANGENT LINES FROM THE GRAPHICS SCREEN   Chapter 3 of this *Guide* (specifically, Section 3.3.2) presented a method of drawing tangent lines from the home screen. We now examine another method for drawing tangent lines, this time using the graphics screen. You may not find this method as useful as the previous one, however, because the point at which the tangent line is drawn depends on the horizontal settings in the viewing window.

We illustrate this method of drawing tangent lines with $f(x) = 2\sqrt{x-5}$. Without the context of a real-world situation, how do you know what input values to consider? The answer is that you need to call upon your knowledge of functions. Remember that we graph only real numbers. If the quantity under the square root symbol is negative, the output of $f(x)$ is not a real number. We therefore know that x must be greater than or equal to 5. Many different horizontal views will do, but we choose to use $0 \le x \le 15$. You can use previously-discussed methods to set height of the window, or you can use the one given below.

Either clear or turn off any previously-entered functions. Enter $f(x) = 2\sqrt{x-5}$ in y1. The parentheses around the $x-5$ are necessary to include the entire quantity under the square root.	y1■2√(x−5) ◄y(x)= RANGE ZOOM TRACE GRAPH► x y INSf DELf SELCT►

Set the range (window) to that shown on the right.
 	
Press F5 (GRAPH) to graph the function.
	RANGE
	  xMin=0
	  xMax=15
	  xScl=5
	  yMin=-5
	  yMax=10
	  yScl=1
	◄y(x)= RANGE ZOOM TRACE GRAPH►
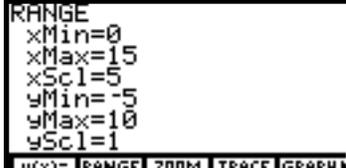
TI-85 With the graph on the screen, press MORE F1 (MATH) MORE MORE F3 (TANLN) **TI-86** With the graph on the screen, press MORE F1 (MATH) MORE MORE F1 (TANLN) **Both** Use ▶ or ◀ to move to some point on the curve. Press ENTER .	x=7.1428571429 y=2.9277002188

The tangent line is drawn at the position of the cursor. At the bottom of the screen, the calculator also gives the slope of the tangent line at that point.	 dy/dx=.68313005106
TI-85 Suppose you want the tangent line drawn at x = 8. Press MORE F1 (MATH) MORE MORE F3 (TANLN) and ▷ or ◁ to move to $x = 8$. You can't do it! With the horizontal settings for this screen, it is not possible to obtain $x = 8$ by tracing.	1 x=8.095230952 y=3.5186577527
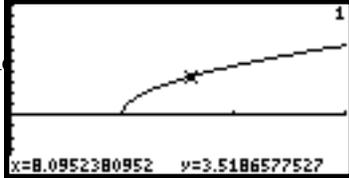
TI-86 With the graph on the screen, press MORE F1 (MATH) MORE MORE F1 (TANLN) . The TI-86 allows you to input the exact value of the point at which you want the tangent drawn, even if that value can not be obtained by tracing. Press 8 and ENTER . The line and the slope at $x = 8$ are displayed.	
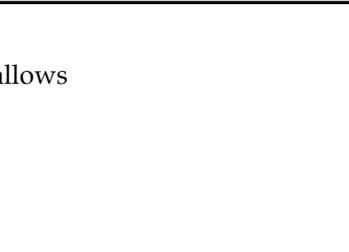
Both If you want to draw the tangent line at a certain value of the input variable that is not a possible trace value, return to the home screen and use the method given in Section 3.3.2 of this *Guide*. (On the TI-86, either method can be used.) It is very important to remember the situations discussed in that section in which the instantaneous rate of change does not exist, but for which the calculator's tangent line incorrectly draws on the screen and the situations in which it doesn't draw but should.

4.3.4 CALCULATING $\dfrac{dy}{dx}$ AT SPECIFIC INPUT VALUES   Section 4.3.1 of this *Guide*

examined two forms of the calculator's numerical derivative and illustrated that one gives a good approximation of the slope of the tangent line and the other gives the slope of the tangent line for most functions at points where the instantaneous rate of change exists. You can also evaluate the calculator's numerical derivative from the graphics screen using the GRAPH MATH menu. However, instead of being called der1 in that menu, it is called $\dfrac{dy}{dx}$. We illustrate its use with the function $f(x) = 2\sqrt{x-5}$.

Enter $f(x) = 2\sqrt{x-5}$ in y1, and draw a graph of *f(x)*. (Refer to Section 4.3.3 of this *Guide*. If you have the graph on the screen with the tangent line from the previous section, retype the 2 in *f(x)* and the graph will draw as on the right.)	 y(x)= RANGE ZOOM TRACE GRAPH▶

TI-85 Press MORE F1 (MATH) F4 (dy/dx) **TI-86** Press MORE F1 (MATH) F2 (dy/dx) **Both** Use ▷ or ◁ to move to some point on the graph and press ENTER . Record on paper the value at the bottom of the screen.	dy/dx=.63245553203
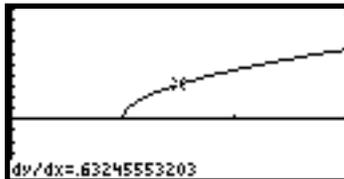
Return to the home screen; press x-VAR ENTER .
 	
From the home screen, evaluate the calculator's numerical derivative der1 at x.
 	
Note that the approximating secant line slope, nDer, is close to, but not the same, value.
	x
	                          7.5
	der1(y1,x,x)
	              .632455532034
	nDer(y1,x,x)
	                .6324555447
	evalF nDer der1 der2 fnInt

- The value at which you evaluated the calculator's numerical derivative is stored in x. (The values you see probably will not be the same as those displayed on the above screens.)

📖 4.4 The Sum Rule, 4.5 The Chain Rule, and 4.6 The Product Rule

If you have time, it is always a good idea to check your answer. Although your calculator cannot give you a general rule for the derivative of a function, you can use graphical and numerical techniques to check your derivative formula answers. These same procedures apply when you check your results after applying the Sum Rule, the Chain Rule, or the Product Rule.

4.4.1 NUMERICALLY CHECKING SLOPE FORMULAS
When you use a formula to find the derivative of a function, it is possible to check your answer using the calculator's numerical derivative der1. The basic idea of the checking process is that if you evaluate your derivative and the calculator's numerical derivative at several randomly chosen values of the input variable and the output values are very close to the same values, your derivative is *probably* correct.

The average yearly fuel consumption per car in the United States from 1980 through 1990 can be modeled by $g(t) = 0.775t^2 – 140.460t + 6868.818$ gallons per car where t is the number of years since 1900. Applying the sum, power, and constant multiplier rules for derivatives, suppose you determine $g'(t) = 1.55t – 140.460$ gallons per year per car. We now numerically check this answer. (As we have mentioned several times, if you have found a model from data, you should have the complete model, not the rounded one given by $g(t)$, in the y(x)= list of the calculator.)

Enter the function you are taking the derivative of in y1, the calculator's derivative in y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in y3. (Remember, since we are going to graph in the y(x)= list, the input variable must be called x.)	y1■.775058827505836 x… y2■der1(y1,x,x) y3■1.55 x–140.460 x y INSf DELf SELCT evalF nDer der1 der2 fnInt

Since the *g(t)* model represents average fuel consumption per car where *t* = 80 in 1980, i t makes sense to use only whole number values of *t* = *x* greater than or equal to 80 when checking the derivative formula. Turn off y1 by having the cursor on the y1 line and pressing F5 (SELCT) since you are checking to see if y2 ≈ y3.

TI-85 Run program TABLE and choose option 2 for ASK. Input *x* = 80 and see if the values of y2 and y3 are very close to the same. (Recall that we are using a rounded derivative in y3.) Repeat the process for at least 4 other values of *x*. If <u>all</u> the values of y2 and y3 are very close, your formula for the derivative is *probably* correct.	[(input, output)]. 1st column is for y1, 2nd column for y2,... Press right arrow to scroll through table. ...-16.45058) (80, -16....
TI-86 In TABLE SETUP, choose ASK in the Indpnt location. Press F1 (TABLE) and check to see that y2 and y3 are very close to the same values for at least four values of *x*. (Recall that we are using a rounded derivative in y3. This accounts for most of the differences in the two columns of output values.)	

• If the two columns of output values are *not* very close to the same, you have either incorrectly entered a function in the y(x)= list or made a mistake in your derivative formula.

4.4.2 GRAPHICALLY CHECKING SLOPE FORMULAS Another method of checking your answer for a slope formula (derivative) is to draw the graph of the calculator's numerical derivative and draw the graph of your derivative. If the graphs appear identical *in the same viewing window*, your derivative is probably correct.

We again use the fuel consumption functions from Section 4.4.1 of this *Guide*.

TI-85 Have the function you are taking the derivative of in y1, the calculator's numerical derivative in y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in y3.	y1=.77505827505836x²... y2=der1(y1,x,x) y3◼1.55x-140.460 y(x)= RANGE ZOOM TRACE GRAPH x y INSf DELf SELCT
Turn off y1 and y2 so that only the graph of y3 will draw. Set an appropriate viewing window such as *x* between 80 and 90 and *y* between ⁻25 and 25 or draw the graph with ZFIT in the ZOOM menu.	x=85 y=-8.71
Now, turn off y3, turn on y2, and draw the graph of y2 *in the same viewing window*. (The graph of the calculator's numerical derivative der1 takes slightly longer to draw because the calculator computes the output before plotting each point.)	x=85 y=-8.7

To be certain the graphs appear identical, turn on y3 and draw the graphs of both y2 and y3 in this same viewing window. If you see only *one* graph on the screen after both graphs finish drawing, your derivative is very likely correct.	
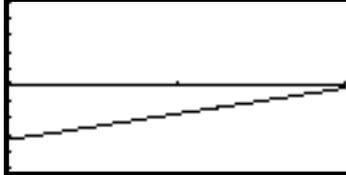
It is tempting to try to shorten the above process for graphically checking your derivative formula by drawing only the graphs of your derivative and the calculator's derivative a t the same time without first graphing each separately. If your derivative is such that i t cannot be seen in the viewing window in which you see the calculator's derivative or vice versa, you will see only one graph and think that your slope formula is correct. It is better to perform a numerical check on the derivatives than to incorrectly use the graphical checking process.

TI-86 Have the function you are taking the derivative of in y1, the calculator's numerical derivative in y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in y3.	
Turn off y1 so that only the graphs of y2 and y3 will draw. Use ▽ until you reach the y3 location, press MORE F3 (STYLE) , and notice that the "\" mark to the left of y3 has changed to a heavier slanted line.	
Press EXIT F2 (WIND) , set xMin to 80, and set xMax to 90. Press F3 (ZOOM) MORE F1 (ZFIT) . *Carefully watch the screen.* If you see the first graph (y2) draw and then see the darker graph (y3) draw *on top of* the first graph, your derivative is probably correct. You must see <u>both</u> graphs draw for this to method to work as a check of your answer!	
If you want to see the graphs draw again, just change a value in the window and press GRAPH . It is not necessary to reset the darker line to the left of y3 for future graphing. When you delete the equation in y3, the setting returns to the normal slanted line.	

Both   When trying to determine an appropriate viewing window, read the problem again; it will likely indicate the values for xMin and xMax. Also use your knowledge of the general shape of the function being graphed to know what you should see.