# Chapter 4 Determining Change: Derivatives

## 4.1 Numerically Finding Slopes

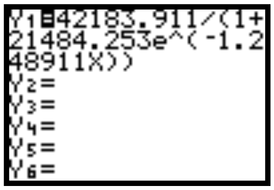
Using your calculator to find slopes of tangent lines does not involve a new procedure. However, the techniques in this section allow you to repeatedly apply a method of finding slopes that gives quick and accurate results.

### 4.1.1 NUMERICALLY INVESTIGATING SLOPES ON THE HOME SCREEN

Finding slopes of secant lines joining the point at which the tangent line is drawn to increasingly close points on a function to the left and right of the point of tangency is easily done using your calculator. Suppose we want to find the slope of the tangent line at $t = 8$ to the graph of the function giving the number of polio cases in 1949: $y = \frac{42183.911}{1 + 21484.253e^{-1.248911t}}$ where $t = 1$ on January 31, 1949, $t = 2$ on February 28, 1949, and so forth.

Enter the equation in the Y1 location of the Y= list. (Carefully check the entry of your equation, especially the location of the parentheses.)

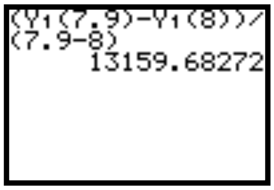
We now evaluate the slopes joining nearby points to the *left* of $x = 8$.



```
Y1■42183.911/(1+
21484.253e^(-1.2
48911X))
Y2=
Y3=
Y4=
Y5=
Y6=
```

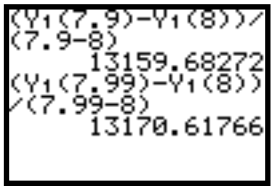
Type in the expression shown to the right to compute the slope of the secant line joining $x = 7.9$ and $x = 8$. You must use parentheses around both the numerator and the denominator of the slope formula.

Record each slope on your paper as it is computed. You are trying to find what these slopes are approaching.



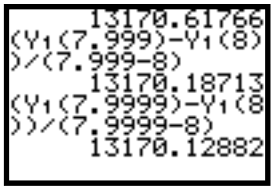
```
(Y1(7.9)-Y1(8))/
(7.9-8)
     13159.68272
```

Press 2nd ENTER (ENTRY) to recall the last entry, and then use the cursor keys to move the cursor over the 9 in the "7.9". Press 2nd DEL (INS) and press 9 to insert another 9 in <u>both</u> positions where 7.9 appears. Press ENTER to find the slope of the secant line joining $x = 7.99$ and $x = 8$.



```
(Y1(7.9)-Y1(8))/
(7.9-8)
     13159.68272
(Y1(7.99)-Y1(8))
/(7.99-8)
     13170.61766
```

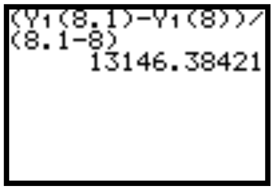
Continue in this manner, recording each result, until you can determine to which value the slopes from the left seem to be getting closer and closer.



```
     13170.61766
(Y1(7.999)-Y1(8)
)/(7.999-8)
     13170.18713
(Y1(7.9999)-Y1(8
))/(7.9999-8)
     13170.12882
```

We now evaluate the slopes joining nearby close points to the *right* of $x = 8$.

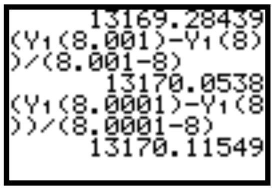
Clear the screen with CLEAR , recall the last expression with 2nd ENTER (ENTRY) , and edit it with DEL so that the nearby point is $x = 8.1$. Press ENTER .



```
(Y1(8.1)-Y1(8))/
(8.1-8)
      13146.38421
```

Continue in this manner as before, recording each result on paper, until you can determine the value the slopes from the right seem to be approaching.

When the slopes from the left and the slopes from the right approach the same value, that value is the slope of the tangent line at $x = 8$.



```
      13169.28439
(Y1(8.001)-Y1(8)
)/(8.001-8)
       13170.0538
(Y1(8.0001)-Y1(8
))/(8.0001-8)
      13170.11549
```

The slopes from the left and from the right appear to be getting closer and closer to 13,170. (The number of polio cases makes sense only as a whole number.)

## 4.1.2 NUMERICALLY INVESTIGATING SLOPES USING THE TABLE

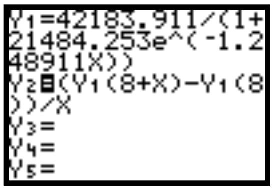
The process shown in Section 4.1.1 can be done in fewer steps when you use the TABLE. Recall that we are evaluating the slope formula

$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(8+h)-f(8)}{h}$$

for various values of $h$ where $h$ is the distance from 8 to the input of the close point. This process is illustrated using the logistic function given in Section 4.1.1 of this *Guide*.

Remember that when in the graphing list, you must use $x$ as the input variable. Since $h$ is what is varying in the slope formula, replace $h$ by $X$ and enter the slope formula in Y2.

Turn Y1 off since we are looking only at the output from Y2.



```
Y1=42183.911/(1+
21484.253e^(-1.2
48911X))
Y2=(Y1(8+X)-Y1(8
))/X
Y3=
Y4=
Y5=
```

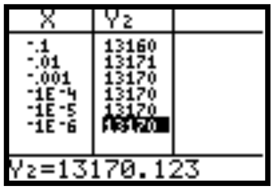
Press 2nd WINDOW (TblSet) and choose the settings on the right.

(Since we are using the ASK feature, the settings for TblMin and ΔTbl do not matter.)

```
TABLE SETUP
 TblMin=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Access the table with 2nd GRAPH (TABLE) and either delete or type over any previous entries in the X column.

Let X (really $h$) take on values that move the nearby point on the left closer and closer to 8.

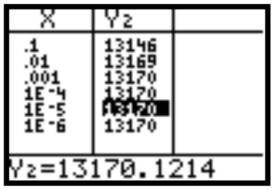


```
   X    | Y2    |
--------+-------+------
 -.1    | 13160 |
 -.01   | 13171 |
 -.001  | 13170 |
 -1E-4  | 13170 |
 -1E-5  | 13170 |
 -1E-6  | 13170 |
Y2=13170.123
```

- Notice that after a certain point, the calculator switches your input values to scientific notation and displays rounded output values so that the numbers can fit on the screen in the space allotted for outputs of the table. You should position the cursor over each output value and record on paper as many decimal places as necessary in order to determine the limit from the left to the desired degree of accuracy.

Repeat the process, letting X (really *h*) take on values that move the nearby point on the right closer and closer to 8.

View the entire decimal value for each output and determine the limit from the right.



```
   X     Y2
 .1      13146
 .01     13169
 .001    13170
 1E-4    13170
 1E-5    13170
 1E-6    13170

Y2=13170.1214
```

### 4.1.3 VISUALIZING THE LIMITING PROCESS (optional)

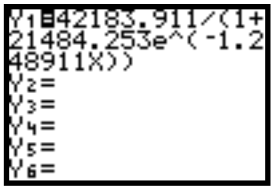
Program SECTAN can be used to view secant lines between a point *(a, f(a))* and some close points on a curve $y = f(x)$ and the tangent line at the point *(a, f(a))*. Using this program either before or after numerically finding the limit of the slopes can help you understand the numerical process.

We use the function giving the number of polio cases in 1949: $y = \dfrac{42183.911}{1 + 21484.253e^{-1.248911t}}$ where $t = 1$ on January 31, 1949, $t = 2$ on February 28, 1949, and so forth.

(Program SECTAN is given in the TI-82/83 Appendix and should be in your calculator before you work through the following illustration.)

Before using program SECTAN, the function must be entered in the Y1 location of the Y= list, and you *must* draw a graph of the function.

Enter the function, using *x* as the input variable, in Y1.

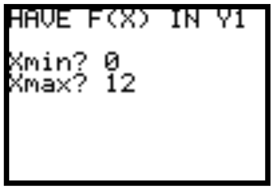


```
Y1=42183.911/(1+
21484.253e^(-1.2
48911X))
Y2=
Y3=
Y4=
Y5=
Y6=
```

Since $t = 1$ represents January 31, 1949, 0 represents the beginning of 1949. The function gives the number of polio cases for the entire year, so we view the graph through December 31, 1949 ($t = 12$).

When you need to draw the graph of a function, you usually are given the input values in the statement of the problem in your text. *Always carefully read the problem before starting the solution process.*

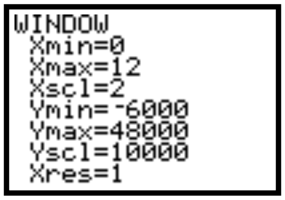
**TI-82** We use program AUTOSCL to draw the graph.



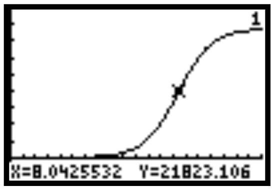
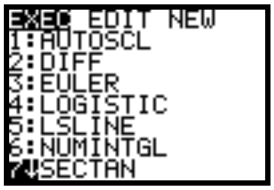
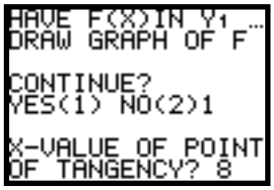
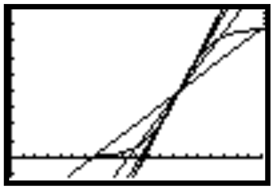
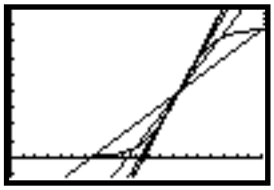
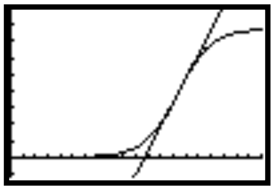
```
HAVE F(X) IN Y1

Xmin? 0
Xmax? 12
```

**TI-83** Press WINDOW , set Xmin to 0, and set Xmax to 12.

Press ZOOM ▲ (0: ZoomFit) ENTER . Press WINDOW . Reset Ymin to ⁻6000 to allow room to see the trace cursor and Ymax to 48,000 so that the equation does not cover the top of the graph.



```
WINDOW
  Xmin=0
  Xmax=12
  Xscl=2
  Ymin=-6000
  Ymax=48000
  Yscl=10000
  Xres=1
```

| | |
|---|---|
| **Both** The graph of the function is displayed. If the point of tangency is not such that you can easily see points to the left and right of it, adjust the window settings. | <br>X=8.0425532 Y=21823.106 |
| Press PRGM and the number or letter corresponding to the location of program SECTAN .<br>(Your program list may not look like the one on the right.)<br>Press ENTER . | <br> |
| If you start this program and have forgotten to enter the function in Y1 or to draw a graph of the function, enter 2. Otherwise, type 1 and press ENTER .<br>At the prompt, enter the input value of the tangent point. (For this example, $x = 8$.) | <br>HAVE F(X)IN Y1 ...<br>DRAW GRAPH OF F<br>CONTINUE?<br>YES(1) NO(2)1<br>X-VALUE OF POINT<br>OF TANGENCY? 8 |
| The next message that appears tells you to press enter to see secant lines drawn between the point of tangency and close points to the *left*.<br>Press ENTER . (Five secant lines will draw ; they may take a little time to draw.) |  |
| When you finish looking at the graph of the secant lines, press ENTER to continue.<br>The next message that appears tells you to press enter to see secant lines drawn between the point of tangency and close points to the *right*. Press ENTER . |  |
| Press ENTER to continue the program.<br>You are next instructed to press ENTER to see a graph of the tangent line. (Note that the tangent line cuts through the graph because an inflection point occurs at $x \approx 8$.) |  |

- *Caution:* In order to properly view the secant lines and the tangent line, it is essential that you first draw a graph of the function clearly showing the function, the point of tangency, and enough space so that the close points on either side can be seen.

## 4.3 Slope Formulas

Your calculator can draw slope formulas. However, to do so, you must first enter a formula for the function whose slope formula you want the calculator to draw. Because you will probably be asked to draw slope formulas for functions whose equations you are not given, you must not rely on your calculator to do this for you. You should instead use technology to check your hand-drawn graphs and to examine the relationships between a function graph and its slope graph. It is very important in both this chapter and several later chapters that you know these relationships.

### 4.3.1 UNDERSTANDING YOUR CALCULATOR'S SLOPE FUNCTION

Both the TI-82 and the TI-83 use the slope of a secant line to approximate the slope of the tangent line at a point on the graph of a function. However, instead of using a secant line through the point of tangency and a close point, these calculators use the slope of a secant line through two close points that are equally spaced from the point of tangency.

Figure 7 illustrates the secant line joining the points $(a\text{-}k,\ f(a\text{-}k))$ and $(a+k,\ f(a+k))$. Notice that the slopes appear to be close to the same value even though the secant line is not the same line as the tangent line.

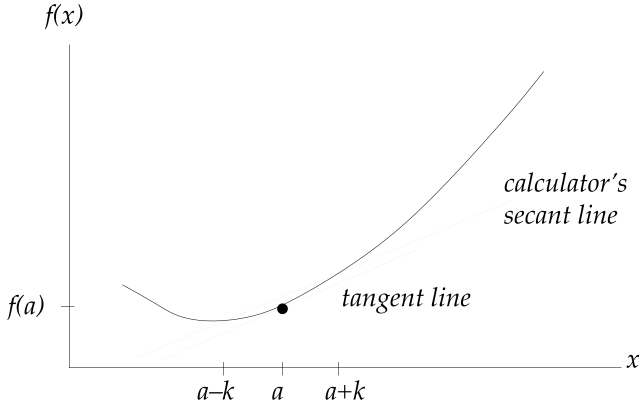


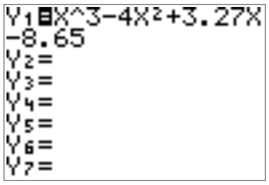
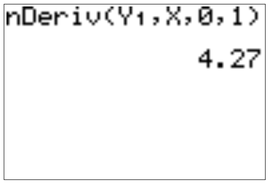
Figure 7

As $k$ gets closer and closer to 0, the two points move closer and closer to $a$. Provided the slope of the tangent line exists, the limiting position of the secant line will be the tangent line.

The calculator's notation for the slope of the secant line shown in Figure 7 is

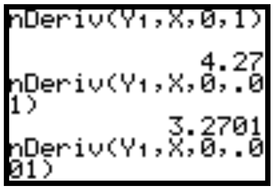
nDeriv(function, symbol for input variable, *a*, *k*)

Specifying the value of $k$ is optional. If it is not given, the calculator automatically uses $k = 0.001$. Any smooth, continuous function will do, so let's investigate these ideas with the function $f(x) = x^3 - 4x^2 + 3.27x - 8.65$.

| | |
|---|---|
| Enter $f(x) = x^3 - 4x^2 + 3.27x - 8.65$ in one of the locations of the Y= list, say Y1.<br>Return to the home screen with 2nd MODE (QUIT) . | <br>Y1=X^3-4X²+3.27X<br>-8.65<br>Y2=<br>Y3=<br>Y4=<br>Y5=<br>Y6=<br>Y7= |
| Suppose you want to find the slope of the secant line between the points $(^{-}1, f(^{-}1))$ and $(1, f(1))$. That is, you are finding the slope of the secant line between $(a\text{-}k,\ f(a\text{-}k))$ and $(a+k,\ f(a+k))$ for $a = 0$ and $k = 1$.<br>Type the expression on the right. Access nDeriv( with<br>**Error!** | <br>nDeriv(Y1,X,0,1)<br>4.27 |

Press 2nd ENTER (ENTRY) , and edit the expression so that *k* changes from 1 to 0.1. Press ENTER .

Again press 2nd ENTER (ENTRY) , and edit the expression so that *k* changes from 0.1 to 0.01. Press ENTER . Repeat the process for *k* = 0.001 and *k* = 0.0001. Did you get the values 3.28, 3.2701, 3.27001, and 3.27000001?



```
nDeriv(Y₁,X,0,1)
            4.27
nDeriv(Y₁,X,0,.0
1)
          3.2701
nDeriv(Y₁,X,0,.0
01)
```

In the table on the next page, the first row lists some values of *a*, the input of the point of tangency, and the second row gives the slope of the tangent line at those values. (You will later learn how to find these exact values of the slope of the tangent line to *f(x)* at various input values.)

Use your calculator to verify the values in the third through sixth rows that give the slope of the secant line between the points *(a-k, f(a-k))* and *(a+k, f(a+k))* for the indicated values of *k*. Find each secant line slope by calculating the value of nDeriv(Y1, X, *a*, *k*).

| *a* = input of point of tangency | -1 | 2.3 | 12.82 | 62.7 |
|---|---|---|---|---|
| slope of tangent line | 14.27 | 0.74 | 393.7672 | 11295.54 |
| slope of secant line, *k* = 0.1 | 14.28 | 0.75 | 393.7772 | 11295.55 |
| slope of secant line, *k* = 0.01 | 14.2701 | 0.7401 | 393.7673 | 11295.5401 |
| slope of secant line, *k* = 0.001 | 14.270001 | 0.740001 | 393.767201 | 11295.54 |
| slope of secant line, *k* = 0.0001 | 14.27000001 | 0.74000001 | 393.7672 | 11295.54 |

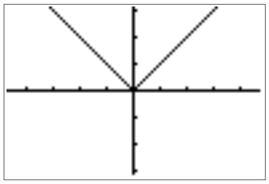
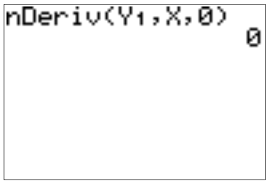
You can see that the slope of the secant line is very close to the slope of the tangent line for small values of *k*. The slope of this secant line does a good job of approximating the slope of the tangent line when *k* is very small.

Now repeat the process, but do not include *k* in the instruction. That is, find the secant line slope by calculating nDeriv(Y1, X, *a*). Did you obtain the following?

|  |  |  |  |  |
|---|---|---|---|---|
| slope of secant line | 14.270001 | 0.740001 | 393.767201 | 11295.54 |

These values are those in the fifth row of the above table -- the values for *k* = 0.001. From this point forward, we use *k* = 0.001 and therefore do not specify *k* when evaluating nDeriv using the calculator.

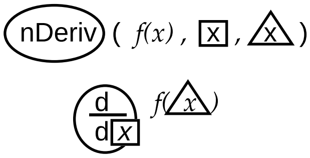
Will the slope of this secant line always do a good job of approximating the slope of the tangent line when *k* is very small? Yes, it does, as long as the instantaneous rate of change exists at the input value (*a*) at which you evaluate nDeriv. When the instantaneous rate of change does not exist at a point, nDeriv should *not* be used to approximate something that does not have a value!

| | |
|---|---|
| **TI-82** Clear all functions from the Y= list, and enter the function $y = \lvert x \rvert$ in Y1 with the keystrokes 2nd x-1 (ABS) X-T-θ .<br>**TI-83** Clear all functions from the Y= list, and enter the function $y = \lvert x \rvert$ in Y1 with the keystrokes MATH ▷ (NUM) 1 (abs( ) X-T-θ-n ) .<br>**Both** Draw the graph with ZOOM 4 (ZDecimal) . | <br>Notice that this function has a “sharp point” at $x = 0$. The instantaneous rate of change does not exist at $x = 0$. |
| Return to the home screen and find the slope of the secant line joining two points 0.001 unit on either side of $x = 0$.<br>Notice that the slope of the secant line is 0, but the instantaneous rate of change at $x = 0$ does not exist! | <br>nDeriv(Y1,X,0)<br>0 |

- Be certain the instantaneous rate of change exists at a point before using nDeriv. Two places where nDeriv usually does *not* give correct results *for the instantaneous rate of change* are at sharp points and the joining point(s) of piecewise continuous functions.
- Provided the instantaneous rate of change exists at a point, we use the secant line slope nDeriv to provide a good approximation to the slope of the tangent line at that point. Since the slope of the tangent line is the slope of the curve which is the derivative of the function, we call nDeriv the calculator’s *numerical derivative*.

## 4.3.2 DERIVATIVE NOTATION AND CALCULATOR NOTATION

You can often see a pattern in a table of values for the slopes of a function at indicated values of the input variable and discover a formula for the slope (derivative). The process of calculating the slopes uses the calculator's numerical derivative, nDeriv($f$(X), X, X). The correspondence between the derivative notation $\frac{d\,f(x)}{dx}$ and the calculator's notation nDeriv($f$(X), X, X) is

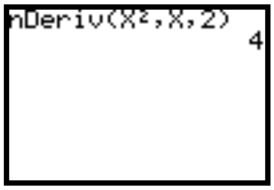
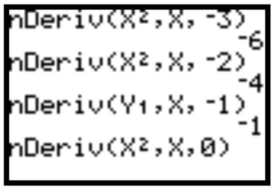
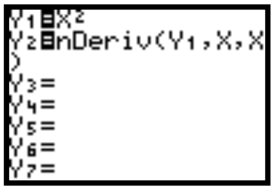
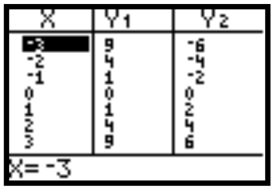


○ indicates we are taking a derivative or slope.

□ indicates the letter corresponding to the name of the input variable.

△ indicates the value of the input variable at which the slope is calcualated.

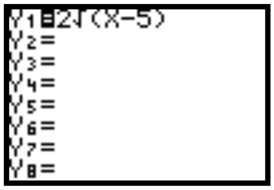
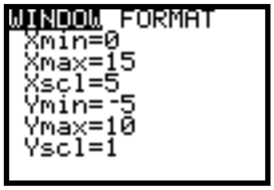
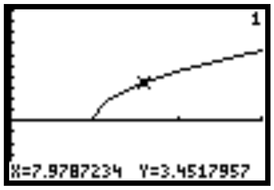
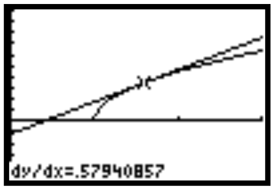
Suppose you are asked to construct a table of values of $f'(x)$ where $f(x) = x^2$ evaluated at different values of $x$. Two methods of doing this are illustrated below:

| | |
|---|---|
| Return to the home screen and type the expression on the right. Access nDeriv( with $\boxed{\text{MATH}}$ $\boxed{8}$ (nDeriv().<br>nDeriv( X$^2$, X, 2) ≈ $\frac{dy}{dx}$ for $y = x^2$ evaluated at $x = 2$. | <br>nDeriv(X²,X,2)<br>4 |
| Recall the last entry, and edit the expression nDeriv( X$^2$, X, 2) by changing the 2 to a $^-3$.<br>Press $\boxed{\text{ENTER}}$ . Continue with this process until you have found all the slope values. | <br>nDeriv(X²,X,-3)<br>-6<br>nDeriv(X²,X,-2)<br>-4<br>nDeriv(Y₁,X,-1)<br>-1<br>nDeriv(X²,X,0) |
| You might prefer to use the TABLE. If so, recall that you must have the expression being evaluated in the Y= list.<br>If you enter the slope formula as indicated on the right, you will only have to change Y1 when you work with a different function. | <br>Y₁=X²<br>Y₂=nDeriv(Y₁,X,X<br>)<br>Y₃=<br>Y₄=<br>Y₅=<br>Y₆=<br>Y₇= |
| You can either type in the $x$-values using the ASK feature of the table or you can set TblMin = $^-3$ and ΔTbl = 1 with the AUTO setting chosen. | <br>TABLE SETUP<br>TblMin=-3<br>ΔTbl=1<br>Indpnt: Auto Ask<br>Depend: Auto Ask |
| Use the table and evaluate the function and the numerical derivative at $x = {}^-3, {}^-2, {}^-1, 0, 1, 2,$ and 3.<br>Determine a relationship (pattern) between the slopes and the values of $x$.<br>Function values appear in the Y1 column and slopes appear in the Y2 column. | <br>X Y₁ Y₂<br>-3 9 -6<br>-2 4 -4<br>-1 1 -2<br>0 0 0<br>1 1 2<br>2 4 4<br>3 9 6<br>X=-3 |

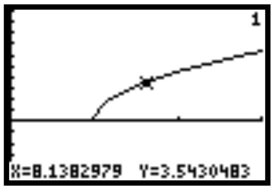
- If you have difficulty determining a pattern, enter the $x$-values at which you are evaluating the slope in list L1 and the values of nDeriv in list L2. Draw a scatter plot of the $x$-values and the slope values. The shape of the scatter plot should give you a clue as to the equation of the slope formula. If not, try drawing another scatter plot where L1 contains the values of $y = f(x)$ and L2 contains the calculated slope formula values. Note that this method might help only if you consider a variety of values for $x$ in list L1.
- The TI-82 and the TI-83 only calculate approximate numerical values of slopes -- they do not give the slope in formula form.

**4.3.3 DRAWING TANGENT LINES FROM THE GRAPHICS SCREEN** Chapter 3 of this *Guide* (specifically, Section 3.3.2) presented a method of drawing tangent lines from the home screen. We now examine another method for drawing tangent lines, this time using the graphics screen. You may not find this method as useful as the previous one, however, because the point at which the tangent line is drawn depends on the horizontal settings in the window.

We illustrate this method of drawing tangent lines with $f(x) = 2\sqrt{x-5}$. Without the context of a real-world situation, how do you know what input values to consider? The answer is that you need to call upon your knowledge of functions. Remember that we graph only real numbers. If the quantity under the square root symbol is negative, the output of $f(x)$ is not a real number. We therefore know that $x$ must be greater than or equal to 5. Many different horizontal views will do, but we choose to use $0 \le x \le 15$. You can use previously-discussed methods to set height of the window, or you can use the one given below.

| | |
|---|---|
| Enter $f(x) = 2\sqrt{x-5}$ in Y1.<br>The parentheses around the $x - 5$ are necessary to include the entire quantity under the square root. | <br>Y1■2√(X-5)<br>Y2=<br>Y3=<br>Y4=<br>Y5=<br>Y6=<br>Y7=<br>Y8= |
| Set the window to that shown on the right.<br>Press GRAPH . | <br>WINDOW FORMAT<br>Xmin=0<br>Xmax=15<br>Xscl=5<br>Ymin=-5<br>Ymax=10<br>Yscl=1 |
| **TI-82** With the graph on the screen, press 2nd<br>**Error!**<br>Press ENTER . | <br>X=7.9787234 Y=3.4517957 |
| The tangent line is drawn at the position of the cursor.<br>At the bottom of the screen, the TI-82 also gives the slope of the tangent line at that point. | <br>dy/dx=.57940857 |

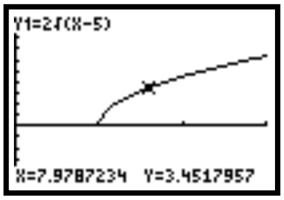
Suppose you want the tangent line drawn at $x = 8$.
Press
**Error!**



```
1
X=8.1382979  Y=3.5430483
```

**TI-83** With the graph on the screen, press 2nd
**Error!**

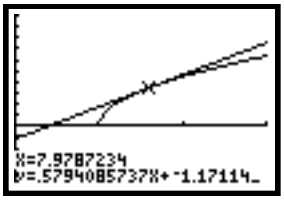
Press ENTER .



```
Y1=2√(X-5)
X=7.9787234  Y=3.4517957
```

The tangent line is drawn at the position of the cursor.

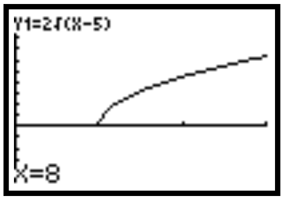
The input value of the point of tangency and the equation of the tangent line are displayed at the bottom of the screen.



```
X=7.9787234
y=.5794085737X+ ⁻1.17114_
```

Suppose you want the tangent line drawn at $x = 8$.
Press
**Error!**

However, on the TI-83, if at this point you simply press 8 , you obtain the screen to the right. Pressing ENTER causes the tangent line to be drawn at the point on the curve where $x = 8$.



```
Y1=2√(X-5)
X=8
```

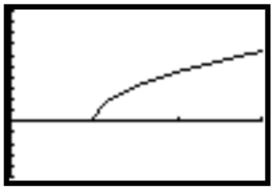
**Both** If you want to draw the tangent line at a certain value of the input variable that is not a possible trace value, return to the home screen and use the method given in Section 3.3.2 of this *Guide*. (On the TI-83, either method can be used.) It is very important to remember the situations discussed in that section in which the instantaneous rate of change does not exist, but yet the calculator's tangent line draws on the screen.

### 4.3.4 CALCULATING $\frac{dy}{dx}$ AT SPECIFIC INPUT VALUES

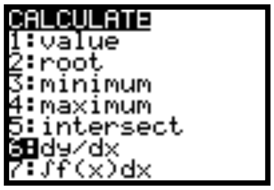
Section 4.3.1 of this *Guide* examined the calculator's numerical derivative nDeriv(Y1, X, *a*) and illustrated that it gives a good approximation of the slope of the tangent line at points where the instantaneous rate of change exists. You can also evaluate the calculator's numerical derivative from the graphics screen using the CALC menu. However, instead of being called nDeriv in that menu, it is called $\frac{dy}{dx}$. We illustrate its use with the function $f(x) = 2\sqrt{x-5}$.

Enter $f(x) = 2\sqrt{x-5}$ in Y1, and draw a graph of $f(x)$.

(Refer to Section 4.3.3 of this *Guide*. If you have the graph on the screen with the tangent line from the previous section, retype the 2 in $f(x)$ and the graph will draw as on the right.)



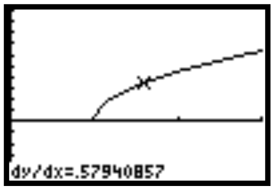
Press 2nd TRACE (CALC) 6 (dy/dx) .



Use ▷ or ◁ to move to some point on the graph.

Press ENTER .

Record on paper the value at the bottom of the screen.

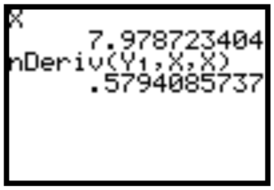


```
dy/dx=.57940857
```

**TI-82** Return to the home screen; press X-T-θ ENTER .

**TI-83** Return to the home screen; press X-T-θ-n ENTER .

**Both** From the home screen, evaluate the calculator's numerical derivative at X.



```
X
          7.978723404
nDeriv(Y1,X,X)
          .5794085737
```

The value at which you evaluated the calculator's numerical derivative is stored in X. (The values you see probably will not be the same as those displayed on the above screens.)

As you would expect, the two values of the calculator's numerical derivative are the same. Other than fewer digits being printed on the graphics screen with the value of *dy/dx*, nDeriv on the home screen and dy/dx on the CALC screen are the same.

## 4.4 The Sum Rule, 4.5 The Chain Rule, and 4.6 The Product Rule

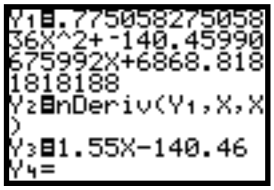
If you have time, it is always a good idea to check your answer. Although your calculator cannot give you a general rule for the derivative of a function, you can use graphical and numerical techniques to check your derivative formula answers. These same procedures apply when you check your results after applying the Sum Rule, the Chain Rule, or the Product Rule.

**4.4.1 NUMERICALLY CHECKING SLOPE FORMULAS** When you use a formula to find the derivative of a function, it is possible to check your answer using the calculator's numerical derivative nDeriv. The basic idea of the checking process is that if you evaluate your derivative and the calculator's numerical derivative at several randomly chosen values of the input variable and the output values are very close to the same values, your derivative is *probably* correct.

The average yearly fuel consumption per car in the United States from 1980 through 1990 can be modeled by $g(t) = 0.775t^2 - 140.460t + 6868.818$ gallons per car where $t$ is the number of years since 1900. Applying the sum, power, and constant multiplier rules for derivatives, suppose you determine $g'(t) = 1.55t - 140.460$ gallons per year per car. We now numerically check this answer.

(As we have mentioned several times, if you have found a model from data, you should have the complete model, not the rounded one given by $g(t)$, in the Y= list of the calculator.)

Enter the function you are taking the derivative of in Y1, the calculator's derivative in Y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in Y3.



```
Y1=.775058275058
36X^2+-140.45990
675992X+6868.818
1818188
Y2=nDeriv(Y1,X,X
)
Y3=1.55X-140.46
Y4=
```

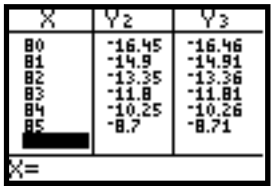
Since the *g(t)* model represents average fuel consumption per car where *t* = 80 in 1980, it makes sense to check using only whole number values of *t* greater than or equal to 80 (that are now denoted by *x* since we are using the Y= list).

In TABLE SETUP, choose ASK in the independent variable location.

Turn off Y1 since you are checking to see if Y2 ≈Y3.

Press 2nd GRAPH (TABLE) and check to see that Y2 and Y3 are very close to the same values for at least four values of *x*.

(Recall that we are using a rounded derivative in Y3. This accounts for most of the differences in the two columns of output values.)



| X | Y2 | Y3 |
|---|---|---|
| 80 | ⁻16.45 | ⁻16.46 |
| 81 | ⁻14.9 | ⁻14.91 |
| 82 | ⁻13.35 | ⁻13.36 |
| 83 | ⁻11.8 | ⁻11.81 |
| 84 | ⁻10.25 | ⁻10.26 |
| 85 | ⁻8.7 | ⁻8.71 |

X=

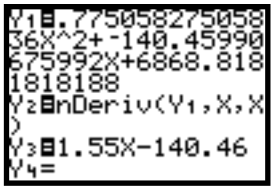
If the two columns of output values are *not* very close to the same, you have either incorrectly entered a function in the Y= list or you have made a mistake in your derivative formula.

## 4.4.2 GRAPHICALLY CHECKING SLOPE FORMULAS

Another method of checking your answer for a slope formula (derivative) is to draw the graph of the calculator's numerical derivative and draw the graph of your derivative. If the graphs appear identical *in the same viewing window*, your derivative is probably correct.

We again use the fuel consumption functions from Section 4.4.1 of this *Guide*.

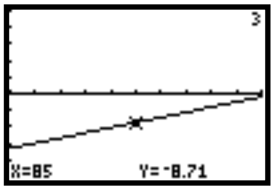
**TI-82** Have the function you are taking the derivative of in Y1, the calculator's numerical derivative in Y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in Y3.



```
Y1=.775058275058
36X^2+-140.45990
675992X+6868.818
1818188
Y2=nDeriv(Y1,X,X
)
Y3=1.55X-140.46
Y4=
```

Turn off Y1 and Y2 so that only the graph of Y3 will draw.

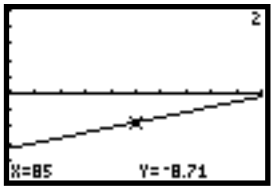
Set an appropriate viewing window such as *x* between 80 and 90 and *y* between ⁻25 and 25.



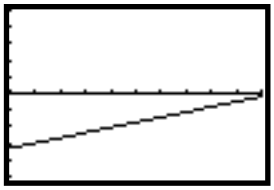
With the functions in their current locations, you cannot use program AUTOSCL since it only draws the graph of the function in Y1. You could change the location of the functions and use the program, or you could use ZOOM 4 (ZDecimal). You probably won't see much, but press TRACE and ▽ until you see a "3" in the upper-right-hand corner of the screen. Press ▷ and ◁ several times and watch the *y*-coordinates. Reset Ymin and Ymax in the window based on the y-value observations, and press GRAPH . If necessary, repeat this process until you see a good graph of Y3.

Now, turn off Y3, turn on Y2, and draw the graph of Y2 *in the same viewing window.*

(The graph of the calculator's numerical derivative takes slightly longer to draw because your calculator computes the output before plotting each point.)

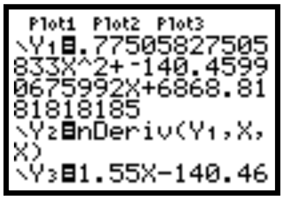


To be certain the graphs appear identical, turn on Y3 and draw the graphs of both Y2 and Y3 in this same viewing window. If you see only *one* graph on the screen after both graphs finish drawing, your derivative is very likely correct.



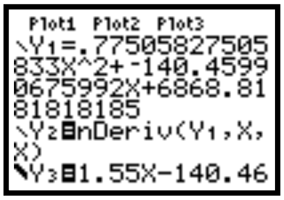
It is tempting to try to shorten the above process for graphically checking your derivative formula by drawing only the graphs of your derivative and the calculator's derivative at the same time without first graphing each separately. If your derivative is such that it cannot be seen in the viewing window in which you see the calculator's derivative or vice versa, you will see only one graph and think that your slope formula is correct. It is better to perform a numerical check on the derivatives than to incorrectly use the graphical checking process.

**TI-83** Have the function you are taking the derivative of in Y1, the calculator's numerical derivative in Y2, and your derivative formula, $\frac{dg}{dt} = g'(t)$, in Y3.



```
 Plot1 Plot2 Plot3
\Y1■.77505827505
833X^2+-140.4599
0675992X+6868.81
81818185
\Y2■nDeriv(Y1,X,
X)
\Y3■1.55X-140.46
```

Turn off Y1 so that only the graphs of Y2 and Y3 will draw. Use ▽ until you reach the Y3 location, and press ◁ ◁ to move the cursor over the "\" mark to the left of Y3. Press ENTER and notice the blinking cursor changes to a heavier slanted line.

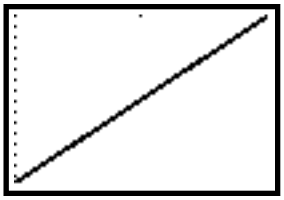


```
 Plot1 Plot2 Plot3
\Y1=.77505827505
833X^2+-140.4599
0675992X+6868.81
81818185
\Y2■nDeriv(Y1,X,
X)
\Y3■1.55X-140.46
```

Press WINDOW , set Xmin to 80, and set Xmax to 90.

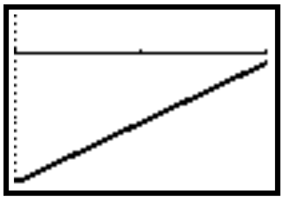
Press ZOOM △ (0: ZoomFit) ENTER .

*Carefully watch the screen.* If you see the first graph (Y2) draw and then see the darker graph (Y3) draw *on top of* the first graph, your derivative is probably correct. You must see <u>both</u> graphs draw for this to method to work as a check of your answer!



If you want to see the graphs draw again, just change a value in the window and press GRAPH .

It is not necessary to reset the darker line to the left of Y3 for future graphing. When you clear the equation in Y3, the setting returns to the normal slanted line.



**Both** When trying to determine an appropriate viewing window, read the problem again; it will likely indicate the values for Xmin and Xmax. Also use your knowledge of the general shape of the function being graphed to know what you should see.