Chapter 11 Dynamics of Change: Differential Equations and Proportionality

11.1 Differential Equations and Accumulation Functions

Many of the differential equations we encounter have solutions that can be found by determining an antiderivative of the given rate-of-change function. So, many of the techniques we learned with the calculator's numerical integration function apply to this chapter. (See Chapters 6 and 7 of this *Guide*.)

11.1.1 EULER'S METHOD FOR *dy*/*dx* = *g*(*x*) You may encounter some differential equations that cannot be solved by standard methods. You may want to draw an accumulation graph for a differential equation without first finding an antiderivative. In either of these cases, Euler's method is helpful. Euler's method relies on the use of the derivative of a function to approximate the change in the function. Recall from Section 5.3 of *Calculus Concepts* that the approximate change in f is the rate of change of f times a small change in x. That is,

$$f(x + h) - f(x) \approx f'(x)\cdot h \text{ where represents the small change in } x.$$

Now, if we let $b = x + h$, and $x = a$, the above expression becomes

$$f(b) - f(a) \approx + f'(a)\cdot(b - a) \quad \text{or} \quad f(b) \approx f(a) + (b - a)\cdot f'(a)$$

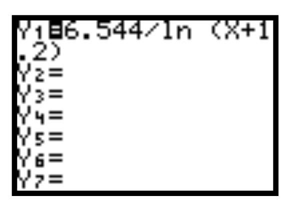
The first values used for the coordinates of the point (a, b) will be given to you and are often called the initial condition. Then use the formula given above to involve the slope of the tangent line at a to approximate the change in the function between a and b. When h, the distance between a and b, is fairly small, Euler's method will often give close numerical estimates of points on the solution to the differential equation containing $f'(x)$.

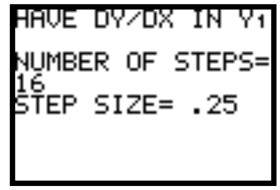
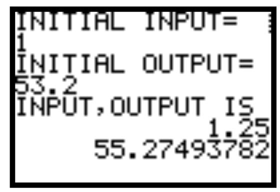
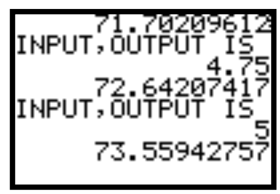
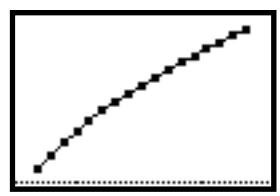
Be wary of the fact that there is some error involved in each step of the approximation process that is compounded when each result is used to obtain the next result.

We illustrate Euler's method with the differential equation giving the total sales, in billions of dollars, of a computer product:

$$\frac{dS}{dt} = \frac{6.544}{\ln(t + 1.2)} \text{ billion dollars per year}$$

where t is the number of years after the product is introduced. Because Euler's method involves a repetitive process, a program that performs the calculations used to find the approximate change in the function can save you time and eliminate computational errors.

Before using this program, you must have the differential equation in location Y1 of the Y= list with X as the input variable. **Note:** If the differential equation is a function of two variables, those variables must be called x and y when using program EULER.	
Add program EULER to your list of programs. (The program is found in the TI-82/83 Appendix.)	

Run the program. Each time the program stops for input or for you to view a result, press ENTER to continue. We are told in this example to use 16 steps. Enter this value. We are also told to use steps of size 0.25. Enter this value.	
The initial condition is given as the point (1, 52.3). Enter these values when prompted for them.	
The first application of the formula gives an us an estimate for the value of the quantity function at $x = 1.25$: $S(1.25) \approx 55.275$.	
Continue pressing ENTER to obtain more estimates of points on the quantity function *S*. Record the input values and the output estimates on paper as they are displayed.	
When the number of steps has been completed, the program draws a graph of the points *(input, output estimate)* connected with straight line segments. This is an approximation to the graph of the solution of the differential equation.	

11.1.2 EULER'S METHOD FOR *dy/dx = h(x, y)*

Program EULER can be used when the differential equation is a function of x and y with $y = f(x)$. Follow the same process as illustrated in Section 11.1.1 of this *Guide*, but enter $\frac{dy}{dx}$ in Y1 in terms of both X and Y.

If the differential equation is written in terms of variables other than x and y, let the derivative symbol be your guide as to which variable is the input and which is the output. For instance, if the rate of change of a quantity is given by $\frac{dP}{dn} = 1.346P \cdot (1 - n^2)$, you would enter Y1= 1.346Y(1 – X)^2, using ALPHA 1 (Y) to type Y.

When the differential equation is given in terms of x and y, such as $\frac{dy}{dx} = 5.9x - 3.2y$, enter Y1= 5.9X – 3.2Y. The differential equation may be given in terms of y only. For instance, if $\frac{dy}{dx} = k(30 - y)$ where k is a constant, enter Y1= K(30 – Y). Of course, you need to store a value for k or substitute a value for k in the equation before using the program. It is always better to store the exact value for the constant instead of using a rounded value.