

# Chapter 9 Ingredients of Multivariable Change: Models, Graphs, Rates



## 9.1 Cross-Sectional Models and Multivariable Functions

For a multivariable function with two input variables described by data given in a table with the values of the *first* input variable listed *horizontally* across the top of the table and the values of the *second* input variable listed *vertically* down the left side of the table, obtain cross-sectional models by entering the appropriate data in the columns of  $\Sigma$ DAT and then proceeding as indicated in Chapter 1 of this *Guide*.

### 9.1.1 FINDING CROSS-SECTIONAL MODELS (holding the first input variable constant)

Using the elevation data in Table 9.1 of the text, find the cross-sectional model  $E(0.8, n)$  as described below. When viewing a table of data, remember that “rows” go from left to right horizontally and “columns” go from top to bottom vertically.

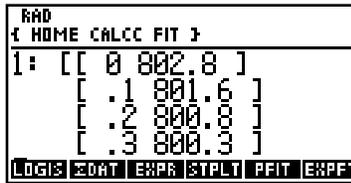
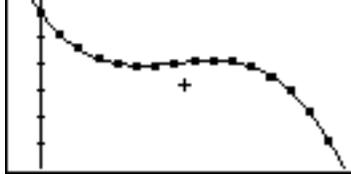
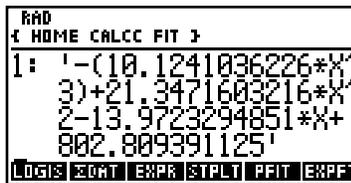
When considering  $E(0.8, n)$ ,  $e$  is constant at 0.8 and  $n$  varies.

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| <p>When considering <math>E(0.8, n)</math>, <math>e</math> is constant at 0.8 and <math>n</math> varies. Thus, enter the values for <math>n</math> appearing on the left side of the table (vertically) as the first column of <math>\Sigma</math>DAT and the elevations <math>E</math> obtained from the table in the <math>e = 0.8</math> column as the second column of <math>\Sigma</math>DAT.</p>                       |  |
| <p>Press <b>FIT</b> to enter the FIT directory and be sure <math>\Sigma</math>DAT is in level 1 of the stack.</p> <p>Draw a scatter plot of the <math>E(0.8, n)</math> data with <b>STPLT</b>.</p>   |  |
| <p>The data appears to be quadratic. Press <b>ΣDAT</b> to place the data on the stack, enter 2 for the quadratic fit, and press <b>PFIT</b>.</p> <p>(Remember that you can return the graph to the screen when the program finishes by pressing <b>◀</b>.)</p>   |  |
| <p>The quadratic model is returned to the stack. (We later use this model, so store it in the HP to the name NEQ.)</p> <p>Because we will find several different cross-sectional models, calling different variables by the same names <math>x</math> and <math>y</math> would be very confusing. It is very important that you call the variables by the names they have been assigned in the application. (See below.)</p> |  |

- Remember that when finding or graphing a model, your calculator always calls the input variable  $X$  and the output variable  $y$ . When working with multivariable functions, you

must translate the calculator's model  $y \approx -2.5x^2 + 2.497x + 799.490$  to the symbols used in this problem to obtain  $E(0.8, n) = -2.5n^2 + 2.497n + 799.490$ .

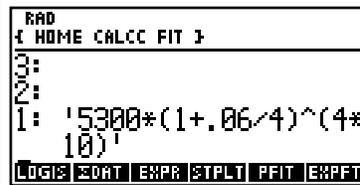
**9.1.2 FINDING CROSS-SECTIONAL MODELS (holding the second input variable constant)** Using the elevation data in Table 9.1 of the text, find the cross-sectional model  $E(e, 0.6)$  as discussed below. When considering  $E(e, 0.6)$ ,  $n$  is constant at 0.6 and  $e$  varies.

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| <p>Enter the values for <math>e</math> appearing across the top of the table (horizontally) as the first column of <math>\Sigma</math>DAT and the elevations <math>E</math> obtained from the table in the <math>n = 0.6</math> column as the second column of <math>\Sigma</math>DAT.</p>   |    |
| <p>Be sure <math>\Sigma</math>DAT is in level 1 of the stack.</p> <p>Draw a scatter plot of the <math>E(e, 0.6)</math> data with <b>STPLT</b>.</p>   |    |
| <p>The data appears to be cubic. Press <b>ΣDAT</b> to place the data on the stack, enter 3 for the cubic fit, and press <b>PFIT</b>.</p>   |   |
| <p>The cubic model is returned to the stack. Be certain that you translate the calculator's model to the symbols used in the example. (See the note at the end of Section 9.1.1.)</p> <p>Write this cross-sectional model as</p> $E(e, 0.6) = -10.124e^3 + 21.347e^2 - 13.972e + 802.809$ <p>(We later use this model, so store it in the HP to the name EEQ.)</p> |  |

**9.1.3 EVALUATING OUTPUTS OF MULTIVARIABLE FUNCTIONS** Outputs of multivariable functions are found by evaluating the function at the given values of the input variables. One way to find multivariable function outputs is to evaluate them on the home screen. For instance, the compound interest formula tells us that

$$A(P, t) = P \left( 1 + \frac{0.06}{4} \right)^{4t} \text{ dollars}$$

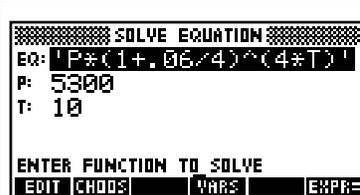
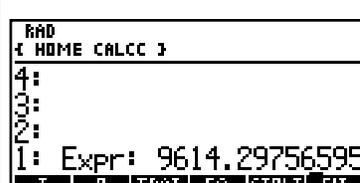
is the amount to which  $P$  dollars will grow over  $t$  years in an account paying 6% interest compounded quarterly. Let's review how to find  $A(5300, 10)$ .

|  |   |
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| <p>Type, on the stack, the formula for <math>A(P, t)</math>, substituting in <math>P = 5300</math> and <math>t = 10</math>. Press <b>ENTER</b> and then <b>EQVAL</b>.</p> <p>Again, be warned that you must carefully use the correct placement of parentheses. Check your result with the values in Table 9.4 in the text to see if \$9614.30 is a reasonable amount.</p> |  |
|--|---|

Even though it is not necessary in this example, you may encounter activities in this section in which you need to evaluate a multivariable function at several different inputs. Instead of individually typing each one on the stack, there are easier methods. You will also use the techniques shown below in later sections of this chapter.

When evaluating a multivariable function at several different input values, you may find it more convenient to enter the multivariable function in the SOLVE application.

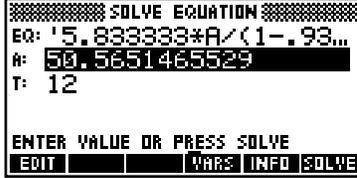
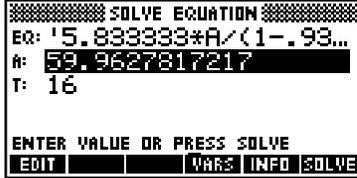
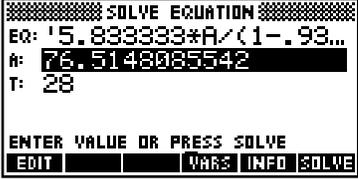
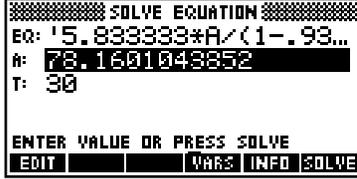
It is very important at this point to note that while we have previously used X as the input variable when entering functions in EQ, we do *not* follow this rule when *evaluating* functions with more than one input variable.

|   |   |
|---|---|
| <p>Clear any previously-entered equations. Enter</p> $A(P, t) = P \left( 1 + \frac{0.06}{4} \right)^{4t}$ <p>on the stack by pressing <b> </b> <b>α</b> <b>P</b> <b>X</b> <b>←</b> <b>÷</b> <b>( )</b> <b>1</b> <b>+</b> <b>.</b> <b>06</b> <b>÷</b> <b>4</b> <b>▶</b> <b>y<sup>x</sup></b> <b>←</b> <b>÷</b> <b>( )</b> <b>4</b> <b>X</b> <b>α</b> <b>T</b> <b>ENTER</b>. Go up one directory with <b>←</b> <b> </b>. Store this equation in EQ with <b>←</b> <b>EQ</b>.</p> |   |
| <p>Press <b>→</b> <b>7</b> (SOLVE), press <b>OK</b> to choose SOLVE EQUATION, and enter the values 5300 in the P location and 10 in the T location.</p>   |  |
| <p>Have the EQ field highlighted and press <b>EXPR=</b>.</p> <p>Use <b>NXT</b> <b>OK</b> to return to the stack and view the value of <math>A(5300, 10)</math>.</p>   |  |

- If you need to evaluate several output values, keep changing the values in P and T and pressing **EXPR=** before returning to the stack to view all the outputs.

When you store values or equations, those names appear on your variables menu. Any time you find your menu cluttered with names that are no longer needed, recall that you can clean-up the menu with the following procedure. It is advisable to press the menu key corresponding to each variable before beginning this procedure to be certain that you no longer are using the stored equation or value.



|   |   |   |
|---|---|---|
| <p>Choose a value for <math>t</math>, say <math>t = 12</math> years.</p> <p>Highlight the <math>A</math> location and press <b>SOLVE</b>.</p> <p>You should find <math>A \approx 50.565</math> thousand dollars.</p>              |  |   |
| <p>Choose another value for <math>t</math>, say <math>t = 16</math> years.</p> <p>Highlight the <math>A</math> location and press <b>SOLVE</b>.</p>   |  |   |
| <p>Repeat this process for <math>t = 20, t = 24, t = 28</math>, and <math>t = 30</math>.</p>  |   |  |
| <p>Make a table of the values of <math>t</math> and <math>A</math> as you find them. You need to find as many points as it takes for you to see the pattern the points are indicating when you plot them on a piece of paper.</p> |   |   |

Step 3: You need a model to draw a contour graph using your calculator. Even though there are several models that seem to fit the data points obtained in Step 2, their use would be misleading since the real best-fit model can only be determined by substituting the appropriate values in a multivariable function. The focus of this section is to use a graph to study the relationships between input variables, not to find the equation of a model for a contour curve. Thus, we always sketch the contours on paper, not with your calculator.



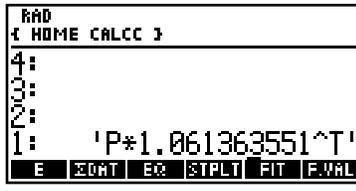
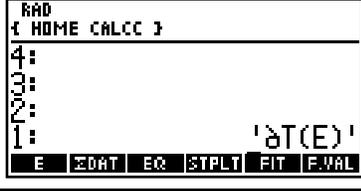
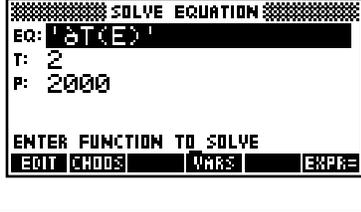
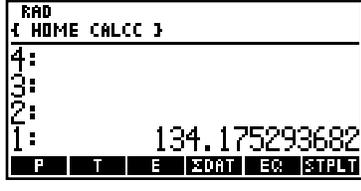
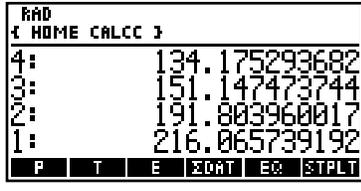
## 9.3 Partial Rates of Change

When you hold all but one of the input variables constant, you are actually looking at a function of one input variable. Therefore, all the techniques we previously discussed can be used. In particular, the calculator's numerical and sometimes, symbolic, derivatives can be used to find partial rates of change at specific values of the varying input variable. You can use your calculator as discussed in Sections 4.4.1 and/or 4.4.2 of this *Guide* to check your algebraic formula for a partial derivative.

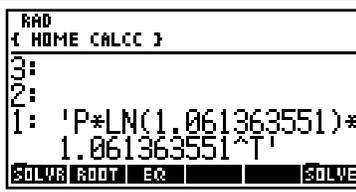
### 9.3.1 NUMERICALLY CHECKING PARTIAL DERIVATIVE FORMULAS

The basic concept behind checking your partial derivative formula is that your formula and the calculator's formula should have the same outputs when each is evaluated at several different randomly-chosen inputs. You can use the methods in Section 9.1.3 of this *Guide* to evaluate each formula at several different inputs and determine if the same numerical values are obtained from each formula. If so, your formula is *probably* correct.

Suppose you have a multivariable function  $A(P, t) = P(1.061363551)^t$  and find, using derivative formulas, that  $\frac{\partial A}{\partial t} = P(\ln 1.061363551)(1.061363551)^t$ . We now illustrate how to check to see if you obtained the correct formula for  $\frac{\partial A}{\partial t}$ .

|  |   |
|--|---|
| <p>Store <math>P(1.061363551)^t</math> in some location, say E.</p>  |    |
| <p>Enter <math>\frac{\partial A}{\partial t}</math>, as computed by the calculator, with <math>\boxed{\text{'}}</math> <math>\boxed{\rightarrow}</math> <math>\boxed{\text{SIN}}</math> <math>\boxed{(\partial)}</math> <math>\boxed{\alpha}</math> <math>\boxed{\text{T}}</math> <math>\boxed{\leftarrow}</math> <math>\boxed{\div}</math> <math>\boxed{()}</math> <math>\boxed{\text{E}}</math> <math>\boxed{\text{ENTER}}</math> and store this in the EQ location with <math>\boxed{\leftarrow}</math> <math>\boxed{\text{EQ}}</math>.</p> |    |
| <p>Press <math>\boxed{\rightarrow}</math> <math>\boxed{7}</math> (SOLVE). Since <math>P</math> is constant, store <i>any</i> reasonable value except 0 in <math>P</math>.</p> <p>Enter a value for <math>t</math>, say 2. Use <math>\boxed{\triangle}</math> to move the cursor so that the EQ field is highlighted and press <math>\boxed{\text{EXPR=}}</math>.</p>   |   |
| <p>Press <math>\boxed{\text{NXT}}</math> <math>\boxed{\text{OK}}</math> to return to the stack and press <math>\boxed{\text{EVAL}}</math>.</p>   |  |
| <p>Then, repeat the process for several other values for <math>t</math>, say 4, 8, and 10.</p> <p>Make a table, on your paper, of the values of <math>t</math> and the values of <math>\frac{\partial A}{\partial t}</math>.</p>   |  |

Remember that you are checking to see if the values of the calculator's derivative are the same as the values of your derivative. You could now repeat the above process with your derivative formula, instead of the calculator's derivative formula, stored in EQ. However, we take this opportunity to present another method of evaluating the outputs so that you can choose the one you prefer to use.

|  |   |
|--|---|
| <p>Press <math>\boxed{\leftarrow}</math> <math>\boxed{7}</math> (SOLVE) <math>\boxed{\text{ROOT}}</math> and store <i>your</i> derivative formula in EQ.</p> |  |
|--|---|

Press **SOLVR** . Again, since  $P$  is constant, store the same value as you did when finding the calculator's derivative outputs in  $P$  by placing the value, 2000, on the stack and pressing **P** .

Enter the same input values for  $t$ : 2, 4, 8, and 10. Press **T** and **EXPR=** after each entry to obtain the values shown on the right.

| RAD            |                       |
|----------------|-----------------------|
| { HOME CALCC } |                       |
| 4:             | Expr: 134.175293682   |
| 3:             | Expr: 151.147473745   |
| 2:             | Expr: 191.803960017   |
| 1:             | Expr: 216.065739192   |
| <b>P</b>       | <b>T</b> <b>EXPR=</b> |

- Since the values shown for the calculator's derivative and your derivative are approximately the same, the formula you computed for  $\frac{\partial A}{\partial t}$  is probably correct.

Note that if you have rounded a model fit by the calculator before computing your partial derivative, the two sets of values will very likely be *slightly* different.

- To check your formula for  $\frac{\partial A}{\partial P}$  , repeat the above procedure, but this time enter '  $\partial P(E)$  ' as the calculator's partial derivative and store any reasonable value (except 0) in  $T$  before computing the output values. Remember that the variable that is changing always appears in the denominator of the partial derivative symbol.

### 9.3.2 SYMBOLICALLY CHECKING PARTIAL DERIVATIVE FORMULAS

One advantage of the HP-48 is that it computes derivative formulas. However, you may or may not be able to understand the calculator's notation. When the multivariable function is not too complicated, you can have the calculator compute the symbolic derivative and compare it with your derivative formula.

- You must remember that when evaluating derivatives symbolically, the changing input *cannot* appear on the menu or on the menu of any directory above the one you are in. If it does, the calculator's derivative formula will automatically be evaluated at the value that is stored in that input variable.

Press **←** **+** ({} ) and press the menu keys that correspond to the names you want to delete.

To delete the items in your list, press **ENTER** **←** **EEX** (PURGE) .

| RAD            |   |
|----------------|---|
| { HOME CALCC } |   |
| 4:             |   |
| 3:             |   |
| 2:             |   |
| 1:             | { T P }   |
| <b>E</b>       | <b>EQAT</b> <b>EQ</b> <b>STPLT</b> <b>FIT</b> <b>EWAL</b> |

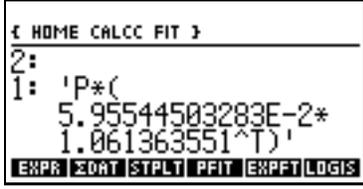
- You must also have symbolic mode chosen in order to have the calculator find an algebraic formula for partial derivatives.

Press **←** **CST** (MODES) and press **MISC** .

If **SYM** has a dot to the right of the "M", your calculator is in symbolic mode. If not press **SYM** and **SYM** should appear. Press **VAR** .

| RAD            |  |
|----------------|--|
| { HOME CALCC } |  |
| 4:             |  |
| 3:             |  |
| 2:             |  |
| 1:             |  |
| <b>BEEP</b>    | <b>CLK</b> <b>SYM</b> <b>STR</b> <b>ARG</b> <b>CMD</b> |

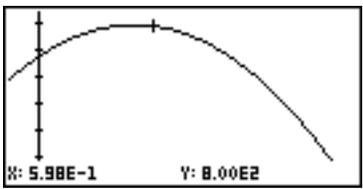
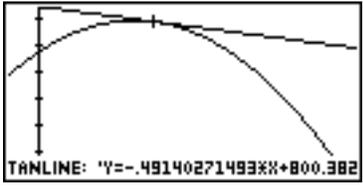
Then, use the following procedure to have the HP-48 find the algebraic formula for the partial derivative of a multivariable function. Be certain that you also know how to do this without your calculator because the final formula reached by the calculator may not be in a recognizable form.

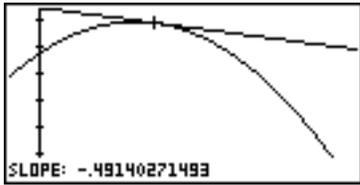
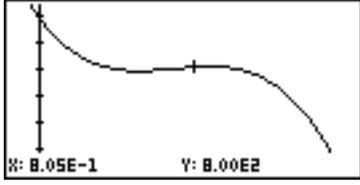
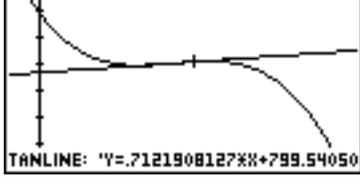
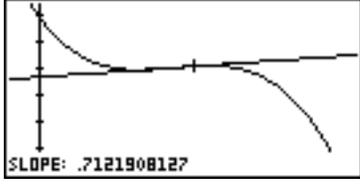
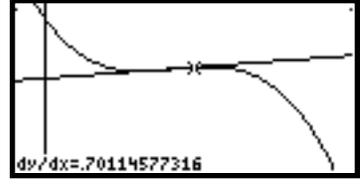
|   |   |
|---|---|
| <p>Press <math>\rightarrow</math> <b>9</b> (SYMBOLIC), and choose Differentiate. You should still have <math>P(1.061363551)^t</math> stored in E. If so, store this expression in EXPR with <b>CHOOS</b>, highlight E, and press <b>OK</b>. Enter T as the changing input, VAR, and choose Symbolic.</p>                                      |  |
| <p>With Symbolic highlighted, find <math>\frac{\partial A}{\partial T}</math> with <b>OK</b>. Evaluate the logarithm in your derivative, <math>\frac{\partial A}{\partial t} = P (\ln 1.061363551) (1.061363551)^t</math>, to see that these two formulas are equivalent. If they are not, your derivative formula is probably incorrect.</p> |  |

- If the expression that appears on the stack is not in a recognizable form, press  $\leftarrow$  **9** and **COLCT** until the expression on the stack no longer changes.

### 9.3.3 VISUALIZING AND ESTIMATING PARTIAL RATES OF CHANGE

A partial rate of change of a multivariable function (evaluated at a specific point) is the slope of the line tangent to a cross-sectional model at a given location. We illustrate this concept with the two elevation cross-sectional models  $E(0.8, n)$  and  $E(e, 0.6)$ .

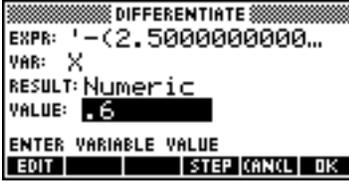
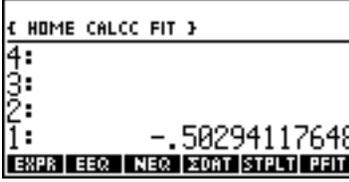
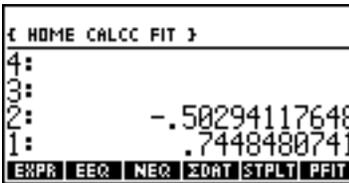
|  |   |
|--|---|
| <p>Have <math>E(0.8, n)</math> stored in NEQ and <math>E(e, 0.6)</math> in EEQ. (If not, reenter these models when they are required.)</p> <p>Press <math>\rightarrow</math> <b>1</b> (PLOT), choose NEQ in the EQ location, and set appropriate horizontal and vertical views.</p>          |  |
| <p><b>ERASE</b> <b>DRAW</b> graphs the <math>E(0.8, n)</math> model.</p> <p>Press <b>TRACE</b> <b>+</b> and use <math>\leftarrow</math> to trace to 0.6. (Your horizontal view will probably be set in such a way that you will not be able to locate <math>n = X = 0.6</math> exactly.)</p> |  |
| <p>Press any white key to return the menu to the bottom of the screen. Press <b>FCN</b> <b>NXT</b>, and draw the tangent line to <math>E(0.8, n)</math> at <math>n = 0.6</math> with <b>TANL</b>.</p>  |  |

|   |   |
|---|---|
| <p>Estimate the partial rate of change <math>\frac{\partial E(0.8, n)}{\partial n}</math> at <math>n = 0.6</math> by first pressing any white key to return the menu to the bottom of the graphics screen and then pressing <b>NXT</b> <b>SLOPE</b> .</p> <p>Remember that this is only an <i>estimate</i> of the true value of <math>\frac{\partial E(0.8, n)}{\partial n}</math> at <math>n = 0.6</math>.</p> |    |
| <p>Press <b>ON</b> <b>→</b> <b>1</b> (PLOT) , and choose EEQ in the EQ location</p>   |    |
| <p><b>ERASE</b> <b>DRAW</b> graphs the <math>E(e, 0.6)</math> model.</p> <p>Press <b>TRACE</b> <b>+</b> and use <b>◀</b> to trace to 0.8. (Your horizontal view will probably be set in such a way that you will not be able to locate <math>e = X = 0.8</math> exactly.)</p>   |    |
| <p>Press any white key to return the menu to the bottom of the screen. Press <b>FCN</b> <b>NXT</b> , and draw the tangent line to <math>E(e, 0.6)</math> at <math>e = 0.8</math> with <b>TANL</b> .</p>   |   |
| <p>Estimate the partial rate of change <math>\frac{\partial E(e, 0.6)}{\partial e}</math> at <math>e = 0.8</math> by first pressing any white key to return the menu to the bottom of the graphics screen and then pressing <b>NXT</b> <b>SLOPE</b> .</p> <p>Remember that this is only an <i>estimate</i> of the true value of <math>\frac{\partial E(e, 0.6)}{\partial e}</math> at <math>e = 0.8</math>.</p> |  |
| <p>Next, determine the slope of the tangent line at the point whose coordinates are shown at the bottom of the graphics screen in the previous step with <b>ENTER</b> .</p> <p>Remember that this is only an <i>estimate</i> of the true value of <math>\frac{\partial E(e, 0.6)}{\partial e}</math> at <math>e = 0.8</math>.</p>   |  |

### 9.3.4 FINDING PARTIAL RATES OF CHANGE USING CROSS-SECTIONAL MODELS

The procedure described above gave estimates of the partial rates of change. You will obtain a more accurate answer if you use your calculator's numerical derivative on the home screen to evaluate partial rates of change at specific input values.

You should still have  $E(0.8, n)$  stored in NEQ and  $E(e, 0.6)$  in EEQ. (If not, re-enter these models when they are required.)

|  |   |
|--|---|
| <p>Enter <math>\frac{\partial E(0.8, n)}{\partial n}</math> evaluated at <math>n = 0.6</math> with </p> <p> (SYMBOLIC), choose Differentiate, choose NEQ in the EXPR location, and have the other fields set as shown to the right.</p> <p>(Recall that  switches the result field from SYMBOLIC to NUMERIC.)</p> |    |
| <p>Press  to find <math>\frac{\partial E(0.8, n)}{\partial n}</math> evaluated at <math>n = 0.6</math>.</p>   |    |
| <p>Enter <math>\frac{\partial E(e, 0.6)}{\partial e}</math> evaluated at <math>e = 0.8</math> with </p> <p> (SYMBOLIC), choose Differentiate, choose EEQ in the EXPR location, and have the other fields set as shown to the right.</p>  |   |
| <p>Press  to find <math>\frac{\partial E(e, 0.6)}{\partial e}</math> evaluated at <math>e = 0.8</math>.</p>   |  |

- Note that if you use the above methods to check your partial derivative formulas evaluated at specific input values, the calculator's "answers" may differ slightly from your answers since the HP-48's Differentiate function is using the full models NEQ and EEQ (with more decimal places).



## 9.4 Compensating for Change

As you have just seen, your calculator can find numerical values of partial derivatives using several different methods. This technique can also be very beneficial and help you eliminate many potential calculation mistakes when you find the rate of change of one input variable with respect to another input variable (that is, the slope of the tangent line) at a point on a contour curve.

### 9.4.1 EVALUATING PARTIAL DERIVATIVES OF MULTIVARIABLE FUNCTIONS

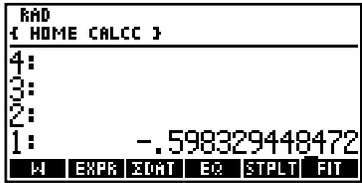
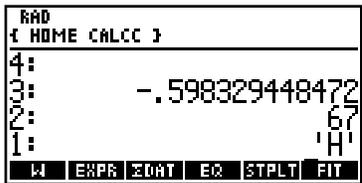
The previous section of this *Guide* indicated how to estimate and evaluate partial derivatives using cross-sectional models. Your calculator can be very helpful when evaluating partial derivatives calculated directly from multivariable function formulas. The most important thing to remember is that you must supply the name of the input variable that is changing and the values at which the partial derivative is evaluated.

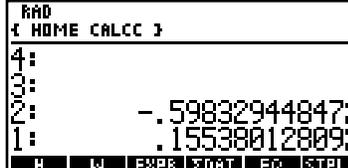
Consider, for instance, the formula for a person's body-mass index:

$$B(h, w) = \frac{0.45w}{0.00064516h^2}$$

where  $h$  is the person's height in inches and  $w$  is the person's weight in pounds. We first find the partial derivatives  $\frac{\partial B}{\partial h}$  and  $\frac{\partial B}{\partial w}$  at a specific height and weight and then use those values in the next section of this *Guide* to find the value of the derivative  $\frac{dw}{dh}$  at that particular height and weight.

Suppose the person in this example is 5 feet 7 inches tall and weighs 129 pounds.

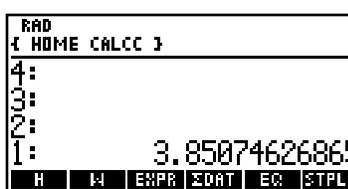
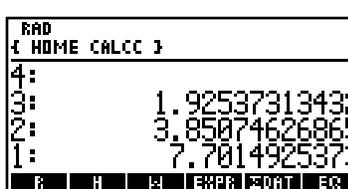
|  |   |
|--|---|
| <p>Enter <math>B(h, w)</math> on the stack now, using the same letters that appear in the formula above, and store it in <math>EXPR</math>, or enter it in <math>EXPR</math> after you enter the symbolic mode.</p>  |    |
| <p>Let's first find <math>\frac{\partial B}{\partial h}</math> at <math>(h, w) = (67, 129)</math>. Because <math>h</math> is changing, <math>w</math> is constant. Enter the value of the input that is held constant by storing 129 in <math>W</math>.</p>        |   |
| <p>Press <math>\rightarrow</math> <math>\boxed{9}</math> (SYMBOLIC), and choose Differentiate.</p> <p>Enter <math>H</math> as the changing input, and set the other values shown on the right. (VALUE is the value of the VAR at which it is being evaluated.)</p> |  |
| <p>Find the value of <math>\frac{\partial B}{\partial h}</math> at <math>h = 67</math> and <math>w = 129</math> by pressing <math>\boxed{OK}</math>.</p> <p>(Leave this value on the stack.)</p>   |  |
| <p>We now find <math>\frac{\partial B}{\partial w}</math> at <math>(h, w) = (67, 129)</math>. Because <math>w</math> is changing, <math>h</math> is constant. On the stack, store the value of the input that is held constant.</p>                                |  |

|   |   |
|---|---|
| <p>Press <math>\rightarrow</math> <math>\boxed{9}</math> (SYMBOLIC), and choose Differentiate.</p> <p>Enter W as the changing input, and set the other values shown on the right.</p>                   |  |
| <p>Find the value of <math>\frac{\partial B}{\partial h}</math> at <math>h = 67</math> and <math>w = 129</math> by pressing <math>\boxed{\text{OK}}</math>.</p> <p>(Leave this value on the stack.)</p> |  |

**9.4.2 COMPENSATING FOR CHANGE** When one input of a two-variable multivariable function changes by a small amount, the value of the function is no longer the same as it was before the change. The methods illustrated below show how to determine the amount by which the other input must change so that the output of the function remains at the value it was before any changes were made.

We use the body-mass index formula of the previous section of this *Guide* to illustrate the procedure for calculating the compensating amount.

We first determine  $\frac{dw}{dh}$  at the point (67, 129) on the contour curve corresponding to the person's current body-mass index. The formula is  $\frac{dw}{dh} = \frac{-\partial B}{\partial h} \div \frac{\partial B}{\partial w}$ . An easy way to remember this formula is that whatever variable is in the numerator of the derivative (in this case,  $w$ ) is the same variable that appears in the denominator of the partial derivative in the denominator of the quotient. The partial derivative with respect to the other variable (here,  $h$ ) appears in the numerator of the quotient. Don't forget to put a minus sign in front of the numerator.

|   |   |
|---|---|
| <p>In the previous section, we first found <math>\frac{\partial B}{\partial h}</math> and then found <math>\frac{\partial B}{\partial w}</math>. So, <math>\frac{\partial B}{\partial h}</math> is now on level 2 of the stack and <math>\frac{\partial B}{\partial w}</math> is on level 1 of the stack. So, to find <math>\frac{dw}{dh} = \frac{-\partial B}{\partial h} \div \frac{\partial B}{\partial w}</math>, simply press <math>\boxed{\div}</math> and <math>\boxed{+/-}</math>.</p> <p>The rate of change is about 3.85 pounds per inch.</p> |  |
| <p>To estimate the weight change needed to compensate for growths of 0.5 inch, 1 inch, and 2 inches if the person's body-mass index is to remain constant, you need to find</p> $\Delta w \approx \frac{dw}{dh} (\Delta h)$ <p>at the various values of <math>\Delta h</math>. (Hint: Store the value of <math>\frac{dw}{dh}</math> to some name, say R, and multiply it by each value of <math>\Delta h</math>.)</p>   |  |