PART C

HEWLETT-PACKARD HP-48G SERIES CALCULATORS

Setup

The HP-48G series (48G/48GX) offers a choice of two methods of accessing calculator applications: *screen interface* or *stack interface*. The screen interface, available for use when the green right-shift key [→] is pressed before an application printed in green type, provides access through dialog boxes on the screen. The stack interface uses the standard HP-48 softkeys and the stack to easily approach all commands related to a particular topic when the application key (printed in purple type) is preceded by the purple left-shift [←] key. Both operational methods will be illustrated in this *Guide*.

Turn your HP-48 on. At the top left of the display screen is information telling you where you are in the calculator's directory system. Directories are like file folders that organize information and applications. If only {HOME} is printed in the top left corner, you are in your HOME directory. The menus at the bottom of the screen change depending on the directory you have chosen. Anything you store in the calculator is in the VAR directory. Many directories have a lot of menu items. To see all of these, press [NXT] to scroll thorough the menu items when you enter a particular directory.

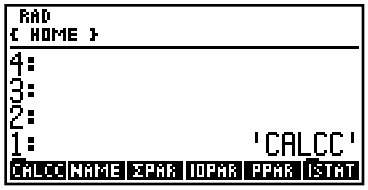
It is helpful to create a calculus directory to hold the programs given in this *Guide*. Before creating your directory, press [→] ['] (HOME) to return to your home directory. To create your calculus concepts directory, first press [→] [VAR] (MEMORY) [NEW] and [▽]. You should now be in the NAME: location. Although this directory can be given any name, we suggest CALCC. Enter CALCC on the stack.

- All alphabetic characters must be preceded by the alphabetic key [α]. Whenever you are typing several alphabetic characters, you will find it convenient to hold down the [α] key with one hand and continue holding it while you type in the letters with your other hand. When you finish typing, release the [α] key.

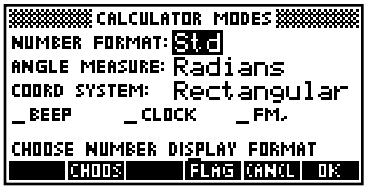
Press [OK] and [√CHK] [OK]. [OK] returns you to the stack, and you should see the name of the new directory in your variables ([VAR]) menu. See Figure 1. (The other menu items will vary according to what you have entered in your HP.)

Check the HP's basic setup by pressing [→] [CST] (MODES) . Choose Std for number format, Radians as the angle measure, and Rectangular for the coordinate system. See Figure 2.

- If a particular setting is not chosen, press [CHOOS] , use the cursor keys to move to the desired setting, and press [OK] . Press [ON] to return to the stack when done.



Creating a Directory
FIGURE 1

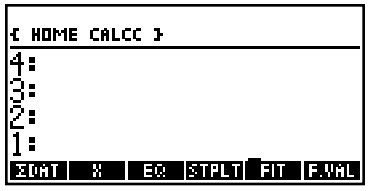


HP-48G/GX Mode Setup
FIGURE 2

Basic Operation

You should be familiar with the basic operation of your calculator. With calculator in hand, go through each of the following.

1. CALCULATING HP calculators offer two ways of working with numbers and expressions. When using either method, you need to remember that the times sign [×] must be used to indicate a product, and you must press the [α] key before typing any letter, even X. Now, press [CALCC] to enter your calculus directory.

Levels 1-4 of the *stack* are on your home screen. Like lines on a piece of paper, the stack is a sequence of temporary storage locations for numbers and the other kinds of objects used by the calculator.	

Use the stack to combine numbers and expressions. The idea is this -- first put inputs on the stack and then execute commands that use the inputs.

Evaluate $\frac{1}{4 * 15 + \frac{895}{7}}$. The result 5.32319391635Ê-3 means $5.32319391635*10^{-3}$, the scientific notation expression for 0.00532319391635.	Type 4 and press [ENTER] Type 15 and press [×] Type 895 and press [ENTER] Type 7 and press [÷] Press [+] and [1/x] to see 5.32319391635Ê-3

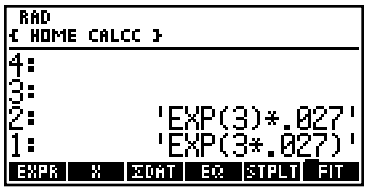
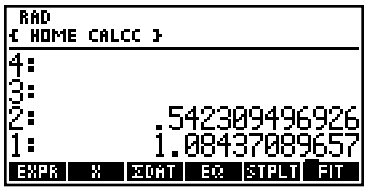
- To recapture the stack after you perform an operation, press [→] [EVAL] (UNDO) .
- To change object positions in the bottom two levels of the stack, press [▷] (SWAP).

- Pressing ◀ drops what is in level 1 of the stack.

Evaluate $\frac{(-3)^4 - 5}{8 + 1.456}$. (Use +/– for the negative symbol and – for the subtraction sign.)	Type 3; press +/- ENTER Type 4 and press y^x Type 5 and press – Type 8 and press ENTER Type 1.456 and press + ÷ to see 8.0372250423

- You can build complicated expressions using stack operations. Just remember that whatever operation key you press will take as its inputs what is in level 1 and level 2 of the stack.
- Any time you want to erase or clear the entire stack, press ↶ DEL (CLEAR) .

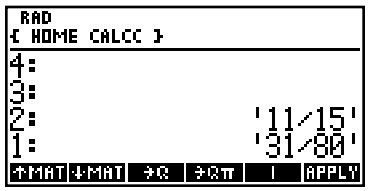
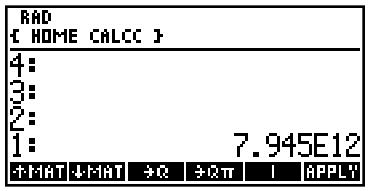
Another method of working with numbers or expressions on the HP is to use algebraic entry form. When you enclose an expression inside of the "tic" symbols ' ', it becomes an algebraic object and is not evaluated until you press EVAL .

Evaluate e^3*0.027 and $e^{3*0.027}$. (After keying in the 3 in the first expression, use ▶ to move outside the right parenthesis before continuing to type. The calculator will assume you mean the first expression unless you use parentheses around the two values in the exponent.	
Press EVAL ▶ to evaluate e^3*0.027 and place its value in level 2 of the stack. Press EVAL to evaluate $e^{3*0.027}$. Press ▶ to swap the values back to their original positions.	

- You can type in lengthy expressions using this format; just make sure that you use parentheses when you are not sure of the calculator's order of operations. As a general rule, numerators and denominators of fractions and powers consisting of more than one term should be enclosed in parentheses.

2. USING THE *ANS* MEMORY This feature is not available on the HP-48G/GX. The result of your last calculation is always in level 1 of the stack, so it is not a needed feature.

3. ANSWER DISPLAY The HP-48 can provide the calculated answer as a fraction. The calculator often uses scientific display as the answer format.

The "to a fraction" key is obtained by pressing ◁ 9 (SYMBOLIC) NXT →ℚ . Perform each calculation below and then press this key. Find the sum $\frac{2}{5}+\frac{1}{3}$ and convert to a fraction. Convert 0.3875 to a fraction.	
The calculator's symbol for "times 10^{12}" is È12. Thus, 7.945È12 means 7,945,000,000,000. Type in (without the commas) 5,600,000,000,000, press ENTER ; type 2,345,000,000,000 and press + .	

4. STORING VALUES Sometimes it is beneficial to store numbers or expressions for later recall. To store a number or expression, type it on the display, press ' , enter the name in which you wish to store the variable, and press STO .

Store 5 in *A* and 3 in *B*. Press VAR . Notice that when you pressed STO , the variable name appeared on your variable menu at the bottom of the display screen.	
To recall the value of a variable you have stored, press the white key on the main keyboard directly under the name of the variable. Calculate 4 *A* – 2*B* by pressing 4 ENTER A ENTER × 2 ENTER B ENTER × – **or** by typing, within the ' ' symbols, the expression shown to the right. Press EVAL .	

- To see the what value is stored in the variable, press ▷ followed by the white key at the bottom of the display screen that corresponds to the variable .
- If your variable menu gets cluttered, delete (*purge*) all unwanted variables (*e.g. A* and *B*) by first pressing ' , then pressing the white key corresponding to the unwanted variable on the menu at the bottom of the display screen, and finally pressing ◁ EEX (PURGE) . Be *very careful* that you only purge unwanted variables in your VAR menu and not a built-in routine or a program you wish to keep. Things that are purged may not be able to be recovered.

5. ERROR MESSAGES When your input is incorrect, an error message is displayed. Pressing ON or ENTER should return the calculator to its normal operation mode. If things are not going well on the command line (where you enter expressions), remember that the ⌫ key will backspace and delete. If you get an invalid syntax message, the cursor will appear at the

location of the error. In that case, use ◁ or ▷ to move the cursor, delete any incorrect symbol or insert a needed symbol, and press ENTER .

Chapter 1 Ingredients of Change: Functions and Linear Models

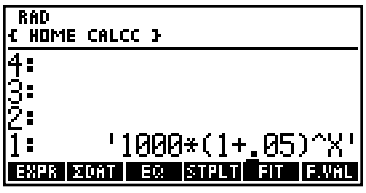
1.1 Fundamentals of Modeling

There are many uses for a function that is entered in the graphing list. Graphing the function in an appropriate viewing window is one of these. Because you must enter all functions on one line (that is, you cannot write fractions and exponents the same way you do on paper) it is very important to have a good understanding of the calculator's order of operations and to use parentheses whenever they are needed.

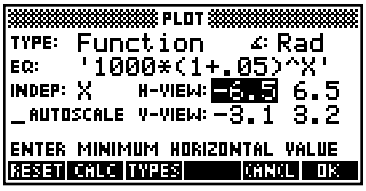
1.1.1 ENTERING AN EQUATION TO BE GRAPHED You can use either the screen interface or the stack interface to enter an equation to be graphed.

SCREEN INTERFACE METHOD: Press [↰] [8] (PLOT) . If the Ptype is not FUNCTION, press [PTYPE] and [FUNC] . Press ['] and enter your *expression* in level 1 of the stack. Press [↰] [EQ] to store $y =$ *expression* as the current equation.

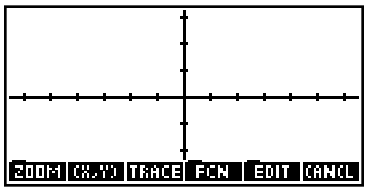
STACK INTERFACE METHOD: Press [↱] [8] (PLOT) . If the TYPE is not Function, press [△] [CHOOS] and select Function with [OK] . Press [▽], enter the expression, and press [OK] . You can use any letter you wish for the input variable. However, you must tell your HP-48 the name of the input variable in the INDEP location by first moving to that location, entering the name of the variable, and then pressing [OK] . Then, [OK] exits.

Choose the method you prefer, and enter as your current EQ the right-hand side of $A = 1000(1 + 0.05)^t$. (The expression to the right is entered as it appears in either the screen or stack interface method.)	

1.1.2 DRAWING A GRAPH Follow the basic procedures shown next to draw a graph with your calculator. Always begin by storing the *expression* part of $y =$ *expression* in EQ. We now draw the graph of $A = 1000(1 + 0.05)^t$. (You can call the input either t or x. We choose x.)

Press [↱] [8] (PLOT) . EQ should contain the expression entered in Section 1.1.1 of this *Guide*. Use [▽] and [▷] to move to the H-VIEW values. Press [NXT] [RESET] . Use [▽] to move to the Reset plot option and press [OK] .	

- Remember that whenever you want to change a setting on the PLOT screen, you must first use the cursor keys to move to the position containing the setting you want to change. That position should then be highlighted and ready for changes.

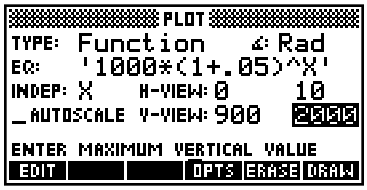
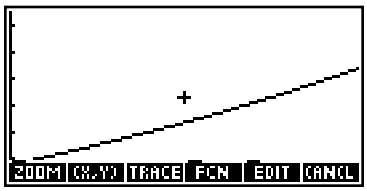
Press NXT ERASE DRAW . Notice that the graphics screen is blank. Press ON to return to the PLOT menu.	

- The settings of the left and right edges of the viewing screen are the values in H-VIEW, and the settings for the lower and upper edges of the viewing screen are the values in V-VIEW. If you want to set the spacing between the tick marks on the x- and y-axes, press OPTS on the PLOT menu and choose appropriate settings for H-TICK and V-TICK.

Follow the procedures shown in either 1.1.3 or 1.1.4 to draw a graph with your calculator. Whenever you draw a graph, you have the option of manually changing the view or having the calculator automatically find a view of the graph.

1.1.3 MANUALLY CHANGING THE VIEW OF A GRAPH If you do not have a good view of the graph or if you do not see the graph, change the view with one of the ZOOM options or manually set the view. (We later discuss the ZOOM options.)

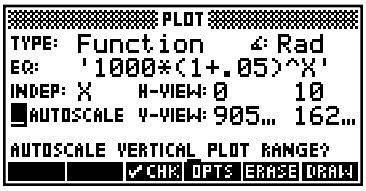
Again, be sure the function you are graphing, $y = 1000(1 + 0.05)^x$, is entered in EQ. To check, simply press EQ in your VAR menu.

Access the H-VIEW and V-VIEW settings as discussed in 1.1.2 of this *Guide*. Set H-VIEW to 0 10, and set V-VIEW to 900 2000 by pressing OK after you enter each of the four values.	
Draw the graph with ERASE DRAW . Remember, to return to the stack or to the previous application when there is a graph on the screen, just press ON .	

- Note that when a graph is on the screen, − removes the menu from the bottom of the screen. + removes the menu and shows the coordinates of the cursor. After using either of these, pressing a white key returns the menu to the graphics screen.

1.1.4 AUTOMATICALLY CHANGING THE VIEW OF THE GRAPH If your view of the graph is not good or if you do not see the graph, change the view using the built-in autoscaling feature of your calculator. This option will automatically find a view to see the equation you have entered in EQ.

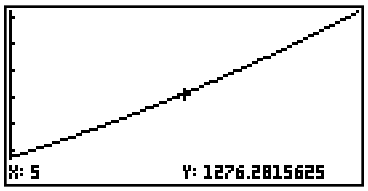
Use the cursor keys to move to the AUTOSCALE location on the PLOT menu. Choose AUTOSCALE with √CHK. Draw the graph with ERASE DRAW. Press ON and notice the V-VIEW.



1.1.5 TRACING You can display the coordinates of certain points on the graph by tracing. The x-values shown when you trace depend on the horizontal view that you choose, and the y-values are calculated by substituting the x-values into the equation that is being graphed.

Press OK to return to the stack, and then press ◁ to return the last-drawn graph to the screen. Now, press TRACE, and use ▷ to move the trace cursor to the right and ◁ to move the trace cursor to the left.

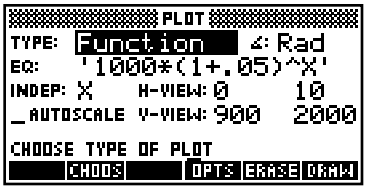
(x,y) removes the menu from the display screen and shows the coordinates as you trace with ▷ or ◁.



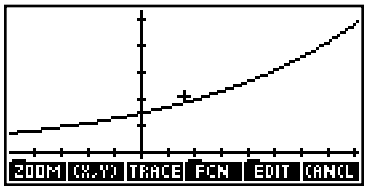
1.1.6 ESTIMATING OUTPUTS You can estimate outputs from the graph using TRACE. It is important to realize that such outputs are *never* exact values unless the displayed x-value is *identically* the same as the value of the input variable.

Estimate the value of A where $A = 1000(1 + 0.05)^x$ when x = 5, x = 7, and x = 10.

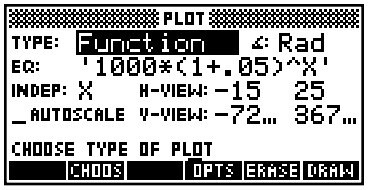
If you do not have the settings shown to the right on your PLOT menu, reset those values.



Draw the graph, and press ZOOM ZOUT.

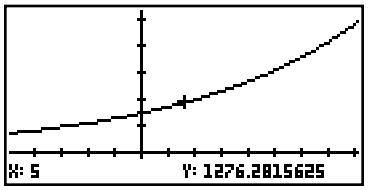


Press ON and observe the values now defining the graphics screen.



Press NXT OK, and use ◁ to return the graph to the screen. Press TRACE (x,y). (Your screen may look slightly different than the one shown to the right.)

Continue pressing ▷ and notice that the values 7 and 10 cannot be obtained by tracing in this view. Therefore, choose values close to these numbers to obtain *estimates* such as A is approximately 1.40E3 or \$1400 when $x = 7$ and A is about 1.65E3 or \$1650 when $x = 10$.

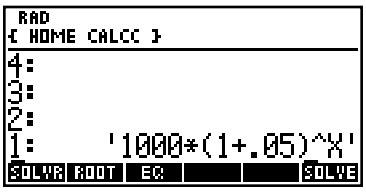
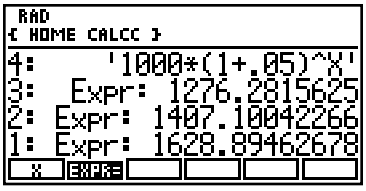


Because the number 5, *not* a value close to 5 is shown, A = \$1276.28 when $x = 5$.

- If your H-VIEW has an upper value of 10, you should obtain from tracing the *exact* value $A = 1500$ when x = 10 because 10, not a value "close to" 10, is the displayed x-value.

- If you want "nice, friendly" values displayed for x when tracing, set the H-VIEW so that the difference in the upper and lower values is a multiple of 13, the width of the RESET viewing screen. For instance, if you set the H-VIEW: 0 13 in the example above, the *exact* values when x = 5, x = 7, and x = 10 are displayed when you trace. Another view that gives friendly values is H-VIEW: -5 21 since 26 = 2(13). Try it!

1.1.7 EVALUATING OUTPUTS The values obtained by this evaluation process are *actual* output values of the equation, not estimated values such as those generally obtained by tracing. The HP-48 evaluates outputs from an expression entered in EQ using either the stack interface or the screen interface. We use the stack interface below and the screen interface in Section 1.2.2 of this *Guide*. Explore both and choose the one you prefer.

Press ◀ 7 (SOLVE) ROOT . Press EQ . If you do not have what is shown to the right, enter '1000(1 + 0.05)^X' and press ◀ EQ .	
Press SOLVR . Type the input at which you want the expression evaluated and press X . Press EXPR= and the output is returned to the stack. Evaluate EQ at x = 5, 7, and 10.	

1.2 Functions and Graphs

When you are asked to *estimate* or *approximate* an output or an input value, you can use your calculator in the following ways:

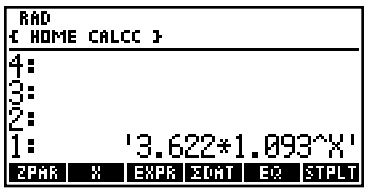
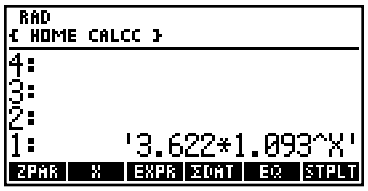
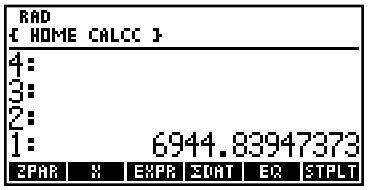
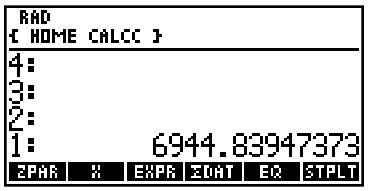
- trace a graph (Sections 1.1.5, 1.1.6)

When you are asked to *find* or *determine* an output or an input value, you should use your calculator in the following ways:

- find the EXPR= value using SOLVE (Section 1.1.7)
- evaluate an output on the stack with EVAL (Section 1.2.1)
- determine an input using SOLVE (Section 1.2.2)

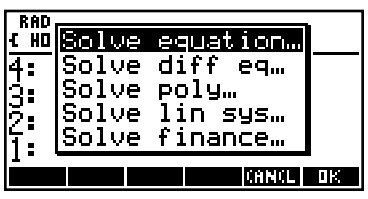
1.2.1 DETERMINING OUTPUTS Function outputs can be determined using the SOLVE application as discussed in 1.1.7 of this *Guide*. You can also evaluate functions from your VAR menu or automate the process using program F.val found in the HP-48 Appendix.

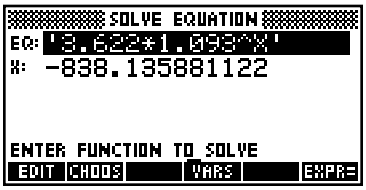
First we use EQ in the VAR menu to determine the output of the function $v(t) = 3.622(1.093)^t$ when t =85. Press VAR to return to your variables menu.

Store '3.622*(1.093)^X ' in EQ by entering it on the stack and pressing [◁] [EQ] . Store the input value 85 in X by typing it and then pressing [◁] [X] . If there is no X on your variables menu, store the value with 85 [ENTER] ['] [α] X [STO] .	
Store '3.622*(1.093)^X ' in EQ by entering it on the stack and pressing [◁] [EQ] . Store the input value 85 in X by typing it and then pressing [◁] [X] .	
Press [EQ] [EVAL] and the output for this input is returned to the stack. (Repeat this process for other input values.) Note that you could use any letter as the input variable symbol when determining outputs with the EVAL key.	
Program F.val automates this process. *Whenever you use this program, you must call the input variable X.* Locate the program on your VAR menu. (If necessary, return to the home directory with [◁] ['] . Then, press [CALCC] , and maybe [NXT] , until you see F-VAL.) Before running the program, enter the input value, 85, on the stack. Execute the program by pressing [F.VAL] .	

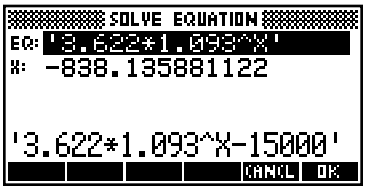
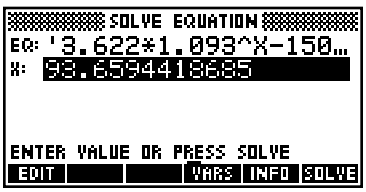
1.2.2 SOLVING FOR INPUT VALUES Your calculator will solve for input values of an equation that you enter in the SOLVE application. We now explore the screen interface with the SOLVE. You can use any letter you wish for the input variable when using the SOLVE. You can even enter an equation consisting of several variables! You should find your calculator's solver a very valuable tool.

Suppose we want to solve $v(t) = 3.622(1.093)^t$ for t when $v = \$15{,}000$.

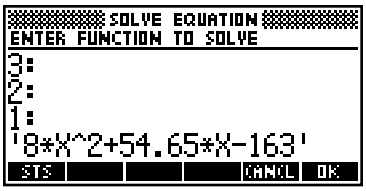
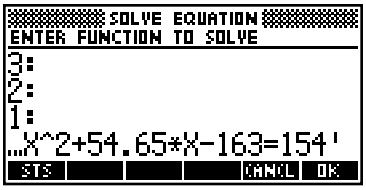
Press [▷] [7] (SOLVE) . Choose Solve Equation from the menu that appears by pressing [OK] .	

If you have the expression '3.622*(1.093)^X ' stored in EQ, it will appear in the EQ: field here also. (Note that you can use T as the variable in EQ. If you do, the screen to the right would show a T instead of an X.)	

What x gives $15{,}000 = 3.622(1.093)^x$? To solve for the input variable using this method, *you should have an expression that equals 0 in the EQ: field*. In other words, you need to have $3.622(1.093)^x - 15{,}000$ in EQ.

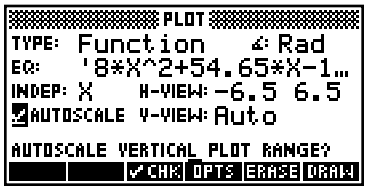
With the EQ: field highlighted, press EDIT and use ▶ to move the cursor just to the right of the last symbol inside the ' mark at the end of the expression. Then, type − 15000 and press OK.	
The X field should now be highlighted. (Pay no attention to the value that is currently there. It is just the last value stored in X.) To find the input, press SOLVE. Be careful that you do not use the current EQ to find an output of the original function because you now have $3.622(1.093)^x - 15{,}000$ stored in EQ, not $3.622(1.093)^x$.	

If there is more than one solution to an equation, you need to give the solver an approximate location for each answer. Suppose you are given $q(x) = 8x^2 + 54.65x - 163$ and asked to find what input(s) correspond to an output of $q(x) = 154$. (The procedure outlined below also applies to finding where two functions are equal.)

From within the SOLVE application with the EQ: field highlighted, press NXT and CALC to access the stack. Type in the $q(x)$, the expression shown to the right.	
Next, you can type − 154 to make this an expression that equals 0 (as explained in the previous section) or you can type the equation you are trying to solve. Your HP is a versatile machine! We choose to type the complete equation. Access the equal sign with ◀ 0. Press OK to store the equation.	

There are two solutions to this equation. To see this, we draw its graph. Exit the SOLVE application with OK and use ↱ 7 to access the PLOT application. Notice that your equation is the current EQ.

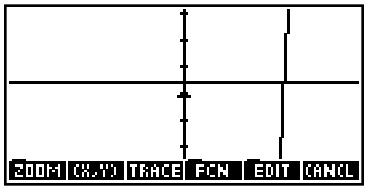
Reset the horizontal view and check AUTOSCALE. Draw the graph with ERASE DRAW .



The left- and right-hand sides of EQ will draw as two separate graphs.

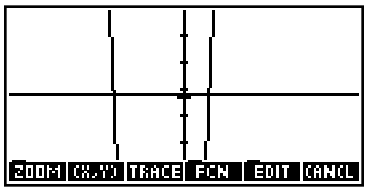
You want to see a "good" graph, that is, one that shows all the important features. In this case, the important features are where the line and the parabola intersect.

This is not a good graph for viewing the intersections because the horizontal view is not wide enough.

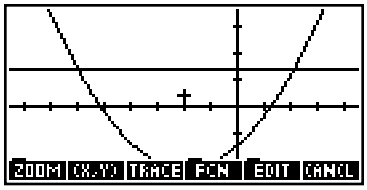


Press ZOOM ZOUT . The resulting picture shows both points of intersection.

(There are many other windows that work just as well as the one shown to the right.)

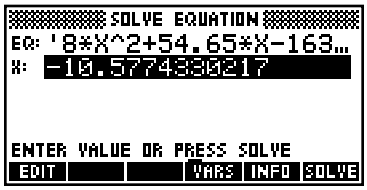


If you are not sure there are only two intersection points, change the view until you see the shape of the equations. A good horizontal view for this purpose is -15 8 and a vertical view of -300 400 works well. (Note: When entering the negative values, type the value and then press +/– OK .)



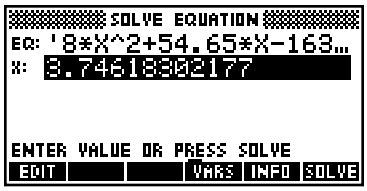
You can now trace the graphs to determine approximately where they intersect. Record the approximate *p*-location of each intersection point on paper. Two possible estimates are the points (-10.6, 154) and (3.75, 154).

To finish solving this equation using SOLVE, press ON , ↱ 7, OK , and with the X: field highlighted, enter -10.6. Again highlight the and press SOLVE . Repeat the process with the other estimate.



The two solutions to the equation, reported to four decimal places, are $x = -10.5774$ and $x = 3.7462$.

Return to the stack with OK . Note that the two solutions appear there also.

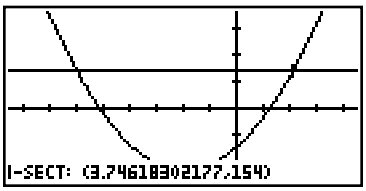


1.2.3 GRAPHICALLY FINDING INTERCEPTS Finding where a function graph crosses the vertical and horizontal axis can be done graphically as well as by the methods

indicated in 1.2.2 of this *Guide*. In fact, it is easier graphically. Remember the process by which we find intercepts:

- To find the y-intercept of a function $y = f(x)$, set x=0 and solve the resulting equation.
- To find the x-intercept of a function $y = f(x)$, set y=0 and solve the resulting equation.

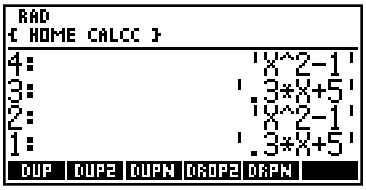
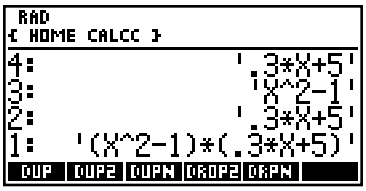
Also remember that an x-intercept of a function $y = f(x)$ has the same value as the root or solution of the equation $f(x) = 0$. Press ◀ to return the last-drawn graph to the screen.

Press FCN ISECT . One of the points of intersection is returned to the screen. Press one of the white keys to return the menu to the screen, NXT PICT TRACE . Use ◀ or ▶ to move near the other point of intersection and press FCN ISECT . Press ON and notice that the two points are on the stack.	

1.3 Constructed Functions

Your calculator can find output values of and graph combinations of functions in the same way that you do these things for a single function. The only additional information you need is how to enter constructed functions in the graphing list. Suppose that a function $f(x)$ has been entered in level 1 of the stack and a function $g(x)$ has been entered in level 2 of the stack.

- Press + to obtain the sum function $(f+g)(x) = f(x) + g(x)$.
- Press − to obtain the sum function $(f-g)(x) = f(x) - g(x)$.
- Press × to obtain the product function $(f \cdot g)(x) = f(x) * g(x)$.
- Press ÷ to obtain the quotient function $(f \div g)(x) = \dfrac{f(x)}{g(x)}$.

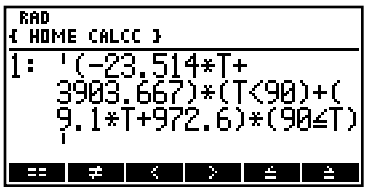
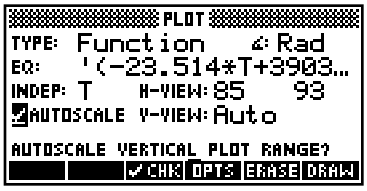
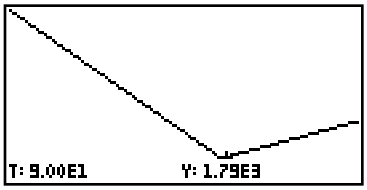
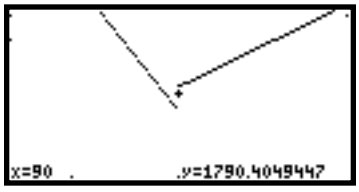
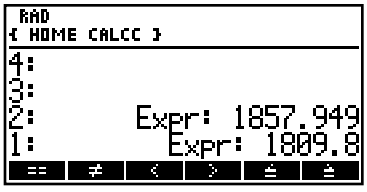
Enter $f(x) = x^2 - 1$ on the stack . Enter $g(x) = 0.3x + 5$ in on the stack. Press ◀ ▲ (STACK) NXT DUP2 several times to duplicate the two functions so you won't have to enter them again.	
Press × to obtain the product function $f(x) * g(x)$. This function can now be graphed, used in the solve application, and so forth. You can then drop it and form another function combination with the functions left on the stack.	

1.3.1 GRAPHING PIECEWISE CONTINUOUS FUNCTIONS Piecewise continuous functions are used throughout the text. It is often helpful to use your calculator to graph and evaluate outputs of piecewise continuous functions. Consider the following example.

The population of West Virginia from 1985 through 1993 can be modeled by

$$P(t) = \begin{cases} -23.514t + 3903.667 \text{ thousand people when } 85 \le t < 90 \\ 9.1t + 972.6 \text{ thousand people when } 90 \le t \le 93 \end{cases}$$

where t is the number of years since 1900.

Enter the function $P(t)$ in EQ by entering the pieces and where they are defined in parentheses using a multiplication sign and addition sign as indicated on the screen shown to the right. Use ▷ to move outside of ")" when you finish entering one portion of the equation. (Note: The parentheses are obtained with ◁ ÷ and the inequality symbols are on the menu when you press PRG TEST .)	 Each piece of the function and its corresponding input must be enclosed in parentheses.
Notice that the function is defined only when the input is between 85 and 93. You could find $P(85)$ and $P(93)$ to help you set the vertical view. However, we choose to let the calculator set the vertical view. Go to the PLOT application, use NXT CALC to pick up the equation from the stack, and set the horizontal view 85 93. Check AUTOSCALE and graph $P(x)$.	
(To draw the plot in "dot" mode -- that is, the points are not connected as the graph draws, press OPTS on the plot screen menu and be sure that CONNECT is not checked. Press OK to return to the previous screen.) Check AUTOSCALE, and graph $P(x)$.	
Because the two pieces are close together at $x = 90$, you may need to take a closer look to see the break. Two of the ways this can be done use ZOOM BOXZ or ZOOM BOXZ .	
You can find output values using any of the previously-mentioned methods. For example, the screen to the right shows the solve application used to find $P(87)$ and $P(92)$.	

1.4 Linear Functions and Models

Actual real-world data is used throughout *Calculus Concepts*. It is necessary that you use your calculator to find a curve that models the data. Be very careful when you enter the

data in your calculator because your model and all of your results depend on the values that you enter!

1.4.1 ENTERING DATA There are several ways to input data in your calculator. Data you enter in the HP-48 is stored in the statistical matrix called ΣDAT. Different sets of data, each called ΣDAT, can be in different directories of your calculator. Therefore, it is important that you be in the directory you want to hold the data before entering any data.

- If you have changed directories, you should always recall ΣDAT to the stack by pressing [ΣDAT] to be sure you are working with the correct data.

  There are several ways to input data in the HP-48. Two of these, entering data using the stack interface and using the screen interface, are discussed below. Try both methods, and then choose the one you prefer.

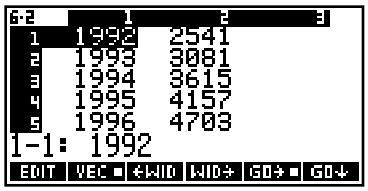
We will explore data entry with the following data:

Year	1992	1993	1994	1995	1996	1997
Tax	2541	3081	3615	4157	4703	5242

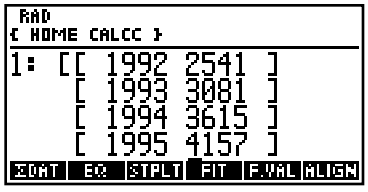
The ΣDAT matrix contains a row for each data *point* and a column for each *variable*. The number of rows that are entered equals the number of data points. Thus, to enter the data above, you will use five rows with x in the first column and y in the second column.

To enter two-variable statistical data using screen interface:

- Press [EDIT] to enter the MatrixWriter application, and press [GO↓] .
- Key in the first x-data value (1992) press [ENTER] , key in the next x-data value (1993), press [ENTER] , etc. until you have entered all the x-data (i.e., the years) in the first column. Press [▷] to move to the top of the second column.

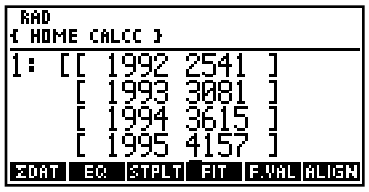
Key in the y-data values in the same manner, remembering to press [ENTER] after typing each value.	

- Press [ENTER] to copy the data to the stack. Press [ENTER] again to make another copy of the data on the stack in case you need it later.
- Press [VAR] [↰] [ΣDAT] to make the entered data the current ΣDAT matrix. If you do not store the data and an error results when you use the data, it is lost and you will have to reenter it.

Press [ΣDAT] to view the data matrix. (On your calculator screen, the last row is hidden by the menu.)	

To enter two-variable statistical data using stack interface:

- Press ↰ 5 (STAT) and DATA to bring up the statistics data entry menu.
- Press CLΣ *before* beginning data entry. If you do not do this, the new data you enter will be appended to data already in the calculator.
- Press ↰ × ([ ]) to let the HP-48 know you are entering two-variable data.
- Type in 1992, the first *x*-data value, press SPC , and then type in 2541, the first *y*-data value. Press Σ+ to enter the first row of ΣDAT.
- After the first row is entered, do not use brackets. Key in 1983 3081, the next data point, separating the *x*- and *y*-values with SPC . Press Σ+ . Continue in this manner until all 5 rows are entered.
- Press ΣDAT at any time during data entry to view the matrix.

Press VAR ΣDAT to see the data. (Your last row is hidden by the menu.) ΣDAT will be stored in the directory that is currently chosen in your VAR menu.	

1.4.2 EDITING DATA If you incorrectly type a data value before you press ENTER , use ← to delete the unwanted character(s) and type the correct value.

To check your data entry, press ΣDAT and ▽. The data appears in the Matrix Writer and you can move around and view the values with ◁, ▷, △, and/or ▽. If a value needs to be corrected, move the cursor to highlight that value. Type the correct value and press ENTER . Press ENTER to return to the stack. *You must now press* ↰ ΣDAT *to store the edited matrix as ΣDAT -- otherwise, your corrections are not saved.*

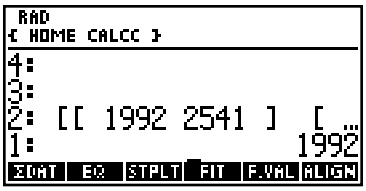
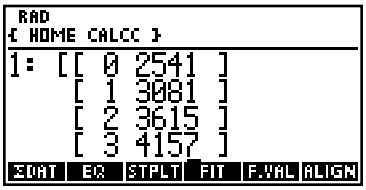
To *insert* a row in the data matrix, highlight the row above which the new row will appear. Press NXT +ROW , and then type in the values to be inserted. To delete a row, highlight the row to be deleted, and then press −ROW .

1.4.3 DELETING OLD DATA If you use the stack interface method to enter new data in your calculator, you must first delete any previously-entered data from your current VAR menu by pressing ↰ 5 (STAT) DATA CLΣ . If you use the screen interface method, it is not necessary to delete previously-entered data. It is automatically replaced with the new data when you press ↰ ΣDAT to store the data in the current VAR menu.

1.4.4 ALIGNING DATA Suppose you want the first column of ΣDAT to contain the number of years since a certain year (here, 1992) instead of actual years. That is, you want to *align* the input data. Use program ALIGN to align the first-column values. (See the HP Appendix).

Before running the program,

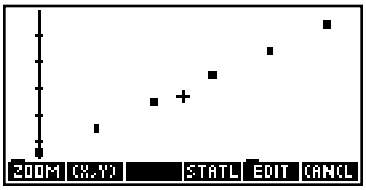
have ΣDAT on level 2 of the stack
have the number you want to subtract from each input on level 1

Press ΣDAT and then type 1 9 92 ENTER .	
Press ALIGN and view the aligned data. Press ΣDAT and notice that the program has stored the aligned data as the new ΣDAT. (Press ▽ to see all five data points.)	

1.4.5 PLOTTING DATA The HP-48 has a built-in scatter plot command accessed with ↰ 5 (STAT) PLOT SCATR . Try it if you like, but you will find it much more convenient to use program STPLT to draw a scatter plot using large dots and to automatically choose the horizontal and vertical views.

Let us draw a scatter plot of the data that was aligned in Section 1.4.4 of this *Guide*.

Before running program STPLT, have ΣDAT on level 1 of the stack.

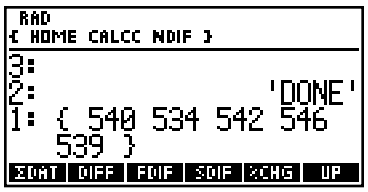
Press STPLT . (You may have to press VAR and/or NXT to find STPLT on your menu.)	

- Because the dots the calculator uses to plot data are sometimes difficult to see when overdrawing the model of best fit, the program places a small box around each data point. The boxes may appear a slightly different size in different views.
- Even though the HP-48 generally allows you to *store* data to any name you want, *you must have the data entered in ΣDAT when using any program given in this Guide or when using any of the statistics functions in your calculator.*
- It is not possible to trace a scatter plot drawn on the HP-48.

1.4.6 FINDING FIRST DIFFERENCES When the input values are evenly spaced, use program DIFF to compute first differences in the output values. If the data are perfectly linear (*i.e.*, every data point falls on the graph of the line), the first differences in the output values are constant. If the first differences are "close" to constant, this is an indication that a linear model *may* be appropriate.

So that your VAR menu does not get too cluttered, create a new directory, NDIF, in the CALCC directory to hold program DIFF and the information it presents. (See "Setup" at the beginning of Part C and the HP Calculator Appendix for instructions. If the programs have been transferred to your calculator, this has already been done.)

Before running program DIFF, have ΣDAT on level 1 of the stack.

Press NDIF followed by DIFF . When the 'DONE' message appears, the first differences in the output values have been computed and placed in the list obtained by pressing FDIF . Press UP to return to the CALCC directory.	

- Program DIFF **should not** be used for data with input values that are *not* evenly spaced. First differences give no information about a possible linear fit to data with inputs that are not the same distance apart.
- Don't be concerned with the results appearing in SDIF and %CHG -- they are used in later sections.

1.4.7 FINDING A LINEAR MODEL Use your calculator to obtain the linear model that best fits two-variable data. Even though the HP-48 has a built-in routine for fitting a linear model, you will find it easier to use the general polynomial model- fitting program PFIT. The model used by this program is of the form

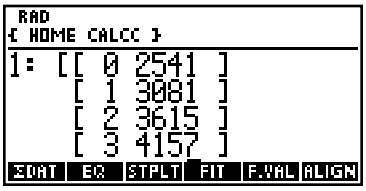
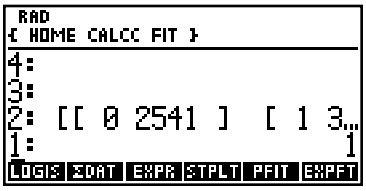
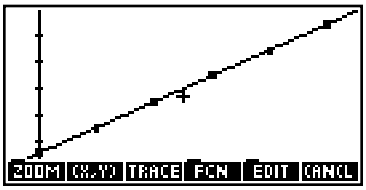
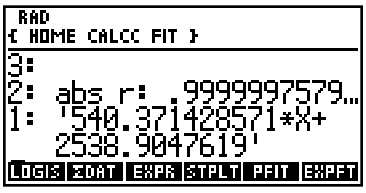
$$y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 .$$

Thus, the linear model (n=1) found by PFIT is of the standard form $y = ax + b$.

Before running program PFIT,

have ΣDAT on level 2 of the stack

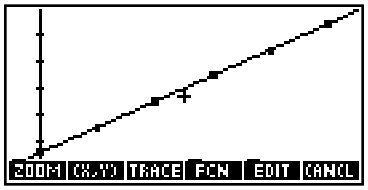
have the value of n on level 1 of the stack

Press ΣDAT and check that the data is as shown on the right. (Remember, to see all the data, press ▽ when ΣDAT is on the stack. Press ENTER to return to the stack.) Press FIT to enter the FIT directory. Store ΣDAT in the FIT directory with ← ΣDAT .	
Press ΣDAT to place the data on the stack. Enter 1 to tell the calculator that you want to fit a linear model.	
Press PFIT . A scatter plot of the data first appears on the screen. Press ON and the program overdraws the graph of the linear model on the scatter plot.	
You are automatically returned to the stack after the graph is drawn. The "$ax + b$" portion of the linear model of best fit for the aligned tax data entered in Section 1.4.4 of this *Guide* is displayed on the stack. (The absolute value of the correlation coefficient, r, is in level 2. Ignore the value of r -- we do not use it in this course.)	

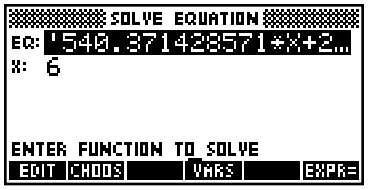
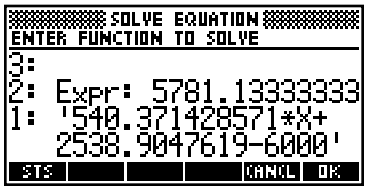
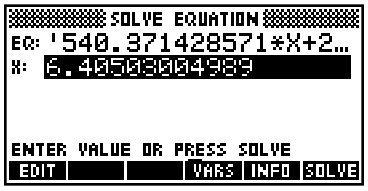
1.4.8 PASTING A MODEL INTO THE FUNCTION LIST The coefficients of the model found by the calculator should *not* be rounded. This is not a problem because program PFIT has automatically stored the model in the function location EQ.

Press EQ after executing program PFIT to recall the model. (You may need to press NXT first.) The equation remains in until EQ you run another model-fitting program or until you manually store another equation in that location.

1.4.9 GRAPHING A MODEL After finding a model, you should always graph it on a scatter plot of the data to verify that the model provides a good fit to the data. Program PFIT automatically draws the graph of the model on the scatter plot of the data. You can recall this graph at any time until you draw another graph.

Press ◁ after executing program PFIT. The last-drawn graph returns to the screen. All of the menu options at the bottom of the screen are available.	

1.4.10 PREDICTIONS USING A MODEL You could use one of the methods described in Sections 1.1.7 or 1.2.1 of this *Guide* to evaluate the linear model at the desired input value. Remember, if you have aligned your data, the input value at which you evaluate the model may not be the value given in the question you are asked.

Predict the tax owed in 1998 where the tax is found using the linear model computed from the data given in Section 1.4.1 of this *Guide*: Note that 1998 is six years since 1992, so $x = 6$. Enter 6 in the X location, press △ to move to the EQ location, and press EXPR= . The output is copied to the stack. The 1998 tax is predicted to be about \$5781.	 Tax = 540.37143t + 2538.90476 dollars where t is the number of years since 1992.
Predict the year in which the tax will be \$6000. Press NXT CALC and type 6000 ENTER . Press − to convert the expression into one that equals 0 at the desired input. (You could also use EDIT to change the expression.) Press OK .	
With the X: field highlighted, press SOLVE to obtain the value shown to the right. Remember that the data is aligned, so the answer is about 1992 + 6.4 or near the middle of 1999.	

1.4.11 COPYING GRAPHS TO PAPER Your instructor may ask you to copy what is on your graphics screen to paper. If so, use the following to more accurately perform this task.

Press ◁ to return the graph to the screen. Press TRACE to trace the graph. (Use a ruler to connect the linear model points.)	Use ▷ and ◁ to locate several values that are as "nice" as possible and mark those points on your paper.

1.4.12 WHAT IS "BEST FIT'? Even though the HP-48 easily computes the values a and b for the best fitting linear model $y = ax + b$, it is important to understand the method of least-squares and the conditions necessary for its application if you intend to use this model. You can explore the process of finding the line of best fit with program LSLNE. (Program LSLNE is given in the HP-48 Appendix.) For your investigations of the least-squares process with this program, it is better to use data that is not perfectly linear and data for which you do *not* know the best-fitting line.

Before using LSLNE, enter your data in ΣDAT. Next, use program STPLT to draw a scatter plot and view the data. You may want to reset X-TIC and Y-TIC so that you can use the tick marks to help identify points on the graphics screen.

Run program LSLNE by pressing [LSLN]. The program first draws a scatter plot of your data. You should view the graph and estimate the slope and y-intercept of *some* line you think will go "through" the data. (You should not expect to guess the best fit line on your first try!) Press [ON] to continue the program. You are then asked for your *estimates* of the slope and y-intercept of the line of best fit. At the Input slope A prompt, type your guess for the slope of the line and press [ENTER]. At the Input y intercept B prompt, type your guess for where the line crosses the vertical axis and press [ENTER]. (The line the program fits to the data is of the form $y = A\,x + B$.)

You are next shown a graph of your line on the scatter plot with the errors displayed as vertical line segments on the graph. After you again press [ON], your guesses for the slope and y-intercept and the sum of squares of errors, SSE, are displayed on the stack. Press [◁], view the graph and decide whether you want to move the y-intercept of the line or change its slope to improve the fit to the data.

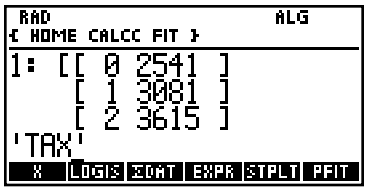
Run the program again by pressing [LSLN]. After you enter another guess for the y-intercept and/or slope, the process of viewing your line, the errors, and display of SSE is repeated. If the new value of SSE is smaller than the SSE for your first guess, you have improved the fit. When you feel you have found an SSE value close to the minimum, you should press [↰] [5] (STAT) [FIT] [LR]. The y-intercept and slope of the best-fitting line for these data are displayed. Store these values on the stack in *A* (the slope) and *B* (the y-intercept). (Don't worry about the tags on the numbers.) Again execute program LSLNE (to find the minimum SSE). At the Input slope A prompt, type [α] A and press [ENTER]. At the Input y intercept B prompt, type [α] B and press [ENTER]. You now see the graph of the best-fitting line (and the errors for the best-fitting line) overdrawn on the scatter plot. [ON] returns the y-intercept, slope, and minimum SSE to the stack.

- If the line that is drawn is horizontal, you probably have an X on some menu in your CALCC directory and the function has been evaluated at that X value. Press [UP] and search for any Xs and purge them. Rerun program LSLNE.

Use program LSLINE to explore finding the line of best fit.

1.4.13 NAMING DATA MATRICES (optional) You may or may not want to use the additional features given below for data entered on your calculator. You can name data and store it in the calculator memory for later recall.

For instance, suppose you wanted to call the tax data ΣDAT matrix by another name and save it in your calculator.

Press ΣDAT to return the aligned data to the stack. (If you wanted to store the original data, this process should be followed before aligning the data.) Press ' , type in a name by pressing and *holding down* α while you press the alphabet keys. Press STO .	
TAX appears on your menu. When you press TAX , the data you stored appears on the stack.