SECTION 15.2 First-Order Linear Differential Equations

First-Order Linear Differential Equations • Bernoulli Equations • Applications

First-Order Linear Differential Equations

In this section, you will see how integrating factors help to solve a very important class of first-order differential equations—first-order *linear* differential equations.

Definition of First-Order Linear Differential Equation

A first-order linear differential equation is an equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where P and Q are continuous functions of x. This first-order linear differential equation is said to be in **standard form.**

To solve a first-order linear differential equation, you can use an integrating factor $u(x)$, which converts the left side into the derivative of the product $u(x)y$. That is, you need a factor $u(x)$ such that

$$\begin{aligned}
u(x)\frac{dy}{dx} + u(x)P(x)y &= \frac{d[u(x)y]}{dx}\\
u(x)y' + u(x)P(x)y &= u(x)y' + yu'(x)\\
u(x)P(x)y &= yu'(x)\\
P(x) &= \frac{u'(x)}{u(x)}\\
\ln|u(x)| &= \int P(x)\,dx + C_1\\
u(x) &= Ce^{\int P(x)\,dx}.
\end{aligned}$$

Anna Johnson Pell Wheeler (1883–1966)

Anna Johnson Pell Wheeler was awarded a master's degree from the University of Iowa for her thesis *The Extension of Galois Theory to Linear Differential Equations* in 1904. Influenced by David Hilbert, she worked on integral equations while studying infinite linear spaces.

Because you don't need the most general integrating factor, let $C = 1$. Multiplying the original equation $y' + P(x)y = Q(x)$ by $u(x) = e^{\int P(x)dx}$ produces

$$\begin{aligned}
y'e^{\int P(x)\,dx} + yP(x)e^{\int P(x)\,dx} &= Q(x)e^{\int P(x)\,dx}\\
\frac{d}{dx}\left[ye^{\int P(x)\,dx}\right] &= Q(x)e^{\int P(x)\,dx}.
\end{aligned}$$

The general solution is given by

$$ye^{\int P(x)\,dx} = \int Q(x)e^{\int P(x)\,dx}\,dx + C.$$

THEOREM 15.3 Solution of a First-Order Linear Differential Equation

An integrating factor for the first-order linear differential equation

$$y' + P(x)y = Q(x)$$

is $u(x) = e^{\int P(x)\,dx}$. The solution of the differential equation is

$$ye^{\int P(x)\,dx} = \int Q(x)e^{\int P(x)\,dx}\,dx + C.$$

STUDY TIP Rather than memorizing this formula, just remember that multiplication by the integrating factor $e^{\int P(x)\,dx}$ converts the left side of the differential equation into the derivative of the product $ye^{\int P(x)\,dx}$.

EXAMPLE 1 Solving a First-Order Linear Differential Equation

Find the general solution of

$$xy' - 2y = x^2.$$

Solution The *standard form* of the given equation is

$$y' + P(x)y = Q(x)$$

$$y' - \left(\frac{2}{x}\right)y = x. \qquad \text{Standard form}$$

Thus, $P(x) = -2/x$, and you have

$$\int P(x)\,dx = -\int \frac{2}{x}\,dx = -\ln x^2$$

$$e^{\int P(x)\,dx} = e^{-\ln x^2} = \frac{1}{x^2}. \qquad \text{Integrating factor}$$

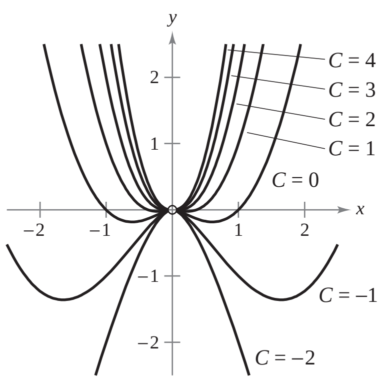

Figure 15.5

Therefore, multiplying both sides of the standard form by $1/x^2$ yields

$$\frac{y'}{x^2} - \frac{2y}{x^3} = \frac{1}{x}$$

$$\frac{d}{dx}\left[\frac{y}{x^2}\right] = \frac{1}{x}$$

$$\frac{y}{x^2} = \int \frac{1}{x}\,dx$$

$$\frac{y}{x^2} = \ln|x| + C$$

$$y = x^2(\ln|x| + C). \qquad \text{General solution}$$

Several solution curves (for $C = -2, -1, 0, 1, 2, 3,$ and 4) are shown in Figure 15.5.

EXAMPLE 2 Solving a First-Order Linear Differential Equation

Find the general solution of

$$y' - y\tan t = 1, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

Solution The equation is already in the standard form $y' + P(t)y = Q(t)$. Thus, $P(t) = -\tan t$, and

$$\int P(t)\,dt = -\int \tan t\,dt = \ln|\cos t|$$

which implies that the integrating factor is

$$e^{\int P(t)\,dt} = e^{\ln|\cos t|}$$
$$= |\cos t|. \qquad \text{Integrating factor}$$

A quick check shows that $\cos t$ is also an integrating factor. Thus, multiplying $y' - y\tan t = 1$ by $\cos t$ produces

$$\frac{d}{dt}[y\cos t] = \cos t$$
$$y\cos t = \int \cos t\,dt$$
$$y\cos t = \sin t + C$$
$$y = \tan t + C\sec t. \qquad \text{General solution}$$

Several solution curves are shown in Figure 15.6.

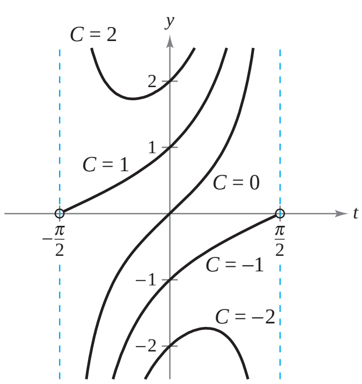

Figure 15.6

Bernoulli Equations

A well-known *nonlinear* equation that reduces to a linear one with an appropriate substitution is the **Bernoulli equation,** named after James Bernoulli (1654–1705).

$$y' + P(x)y = Q(x)y^n \qquad \text{Bernoulli equation}$$

This equation is linear if $n = 0$, and has separable variables if $n = 1$. Thus, in the following development, assume that $n \neq 0$ and $n \neq 1$. Begin by multiplying by y^{-n} and $(1 - n)$ to obtain

$$y^{-n}y' + P(x)y^{1-n} = Q(x)$$
$$(1-n)y^{-n}y' + (1-n)P(x)y^{1-n} = (1-n)Q(x)$$
$$\frac{d}{dx}[y^{1-n}] + (1-n)P(x)y^{1-n} = (1-n)Q(x)$$

which is a linear equation in the variable y^{1-n}. Letting $z = y^{1-n}$ produces the linear equation

$$\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x).$$

Finally, by Theorem 15.3, the *general solution of the Bernoulli equation* is

$$y^{1-n}e^{\int(1-n)P(x)\,dx} = \int (1-n)Q(x)e^{\int(1-n)P(x)\,dx}\,dx + C.$$

EXAMPLE 3 Solving a Bernoulli Equation

Find the general solution of $y' + xy = xe^{-x^2}y^{-3}$.

Solution For this Bernoulli equation, let $n = -3$, and use the substitution

$$z = y^4 \qquad \text{Let } z = y^{1-n} = y^{1-(-3)}.$$
$$z' = 4y^3y'. \qquad \text{Differentiate.}$$

Multiplying the original equation by $4y^3$ produces

$$y' + xy = xe^{-x^2}y^{-3} \qquad \text{Original equation}$$
$$4y^3y' + 4xy^4 = 4xe^{-x^2} \qquad \text{Multiply both sides by } 4y^3.$$
$$z' + 4xz = 4xe^{-x^2}. \qquad \text{Linear equation: } z' + P(x)z = Q(x)$$

This equation is linear in z. Using $P(x) = 4x$ produces

$$\int P(x)\,dx = \int 4x\,dx = 2x^2$$

which implies that e^{2x^2} is an integrating factor. Multiplying the linear equation by this factor produces

$$z' + 4xz = 4xe^{-x^2} \qquad \text{Linear equation}$$
$$z'e^{2x^2} + 4xze^{2x^2} = 4xe^{x^2} \qquad \text{Exact equation}$$
$$\frac{d}{dx}[ze^{2x^2}] = 4xe^{x^2} \qquad \text{Write left side as total differential.}$$
$$ze^{2x^2} = \int 4xe^{x^2}\,dx \qquad \text{Integrate both sides.}$$
$$ze^{2x^2} = 2e^{x^2} + C$$
$$z = 2e^{-x^2} + Ce^{-2x^2}. \qquad \text{Divide both sides by } e^{2x^2}.$$

Finally, substituting $z = y^4$, the general solution is

$$y^4 = 2e^{-x^2} + Ce^{-2x^2}. \qquad \text{General solution}$$

So far you have studied several types of first-order differential equations. Of these, the separable variables case is usually the simplest, and solution by an integrating factor is usually a last resort.

Summary of First-Order Differential Equations

	Method	*Form of Equation*
1.	Separable variables:	$M(x)\,dx + N(y)\,dy = 0$
2.	Homogeneous:	$M(x, y)\,dx + N(x, y)\,dy = 0$, where M and N are nth-degree homogeneous
3.	Exact:	$M(x, y)\,dx + N(x, y)\,dy = 0$, where $\partial M/\partial y = \partial N/\partial x$
4.	Integrating factor:	$u(x, y)M(x, y)\,dx + u(x, y)N(x, y)\,dy = 0$ is exact
5.	Linear:	$y' + P(x)y = Q(x)$
6.	Bernoulli equation:	$y' + P(x)y = Q(x)y^n$

Applications

One type of problem that can be described in terms of a differential equation involves chemical mixtures, as illustrated in the next example.

EXAMPLE 4 A Mixture Problem

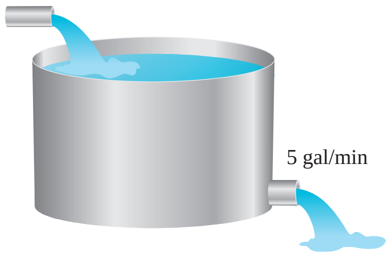

Figure 15.7

A tank contains 50 gallons of a solution composed of 90% water and 10% alcohol. A second solution containing 50% water and 50% alcohol is added to the tank at the rate of 4 gallons per minute. As the second solution is being added, the tank is being drained at the rate of 5 gallons per minute, as shown in Figure 15.7. Assuming the solution in the tank is stirred constantly, how much alcohol is in the tank after 10 minutes?

Solution Let y be the number of gallons of alcohol in the tank at any time t. You know that $y = 5$ when $t = 0$. Because the number of gallons of solution in the tank at any time is $50 - t$, and the tank loses 5 gallons of solution per minute, it must lose

$$\left(\frac{5}{50-t}\right)y$$

gallons of alcohol per minute. Furthermore, because the tank is gaining 2 gallons of alcohol per minute, the rate of change of alcohol in the tank is given by

$$\frac{dy}{dt} = 2 - \left(\frac{5}{50-t}\right)y \quad \Rightarrow \quad \frac{dy}{dt} + \left(\frac{5}{50-t}\right)y = 2.$$

To solve this linear equation, let $P(t) = 5/(50 - t)$ and obtain

$$\int P(t)\,dt + \int \frac{5}{50-t}\,dt = -5 \ln|50 - t|.$$

Because $t < 50$, you can drop the absolute value signs and conclude that

$$e^{\int P(t)\,dt} = e^{-5\ln(50-t)} = \frac{1}{(50-t)^5}.$$

Thus, the general solution is

$$\frac{y}{(50-t)^5} = \int \frac{2}{(50-t)^5}\,dt = \frac{1}{2(50-t)^4} + C$$

$$y = \frac{50-t}{2} + C(50-t)^5.$$

Because $y = 5$ when $t = 0$, you have

$$5 = \frac{50}{2} + C(50)^5 \quad \Rightarrow \quad -\frac{20}{50^5} = C$$

which means that the particular solution is

$$y = \frac{50-t}{2} - 20\left(\frac{50-t}{50}\right)^5.$$

Finally, when $t = 10$, the amount of alcohol in the tank is

$$y = \frac{50-10}{2} - 20\left(\frac{50-10}{50}\right)^5 = 13.45 \text{ gal}$$

which represents a solution containing 33.6% alcohol.

In most falling-body problems discussed so far in the text, we have neglected air resistance. The next example includes this factor. In the example, the air resistance on the falling object is assumed to be proportional to its velocity v. If g is the gravitational constant, the downward force F on a falling object of mass m is given by the difference $mg - kv$. But by Newton's Second Law of Motion, you know that $F = ma = m(dv/dt)$, which yields the following differential equation.

$$m\frac{dv}{dt} = mg - kv \quad \Rightarrow \quad \frac{dv}{dt} + \frac{k}{m}v = g$$

EXAMPLE 5 A Falling Object with Air Resistance

An object of mass m is dropped from a hovering helicopter. Find its velocity as a function of time t, assuming that the air resistance is proportional to the velocity of the object.

Solution The velocity v satisfies the equation

$$\frac{dv}{dt} + \frac{kv}{m} = g$$

where g is the gravitational constant and k is the constant of proportionality. Letting $b = k/m$, you can *separate variables* to obtain

$$dv = (g - bv)\,dt$$
$$\int \frac{dv}{g - bv} = \int dt$$
$$-\frac{1}{b}\ln|g - bv| = t + C_1$$
$$\ln|g - bv| = -bt - bC_1$$
$$g - bv = Ce^{-bt}.$$

Because the object was dropped, $v = 0$ when $t = 0$; thus $g = C$, and it follows that

$$-bv = -g + ge^{-bt} \quad \Rightarrow \quad v = \frac{g - ge^{-bt}}{b} = \frac{mg}{k}(1 - e^{-kt/m}).$$

NOTE Notice in Example 5 that the velocity approaches a limit of mg/k as a result of the air resistance. For falling-body problems in which air resistance is neglected, the velocity increases without bound.

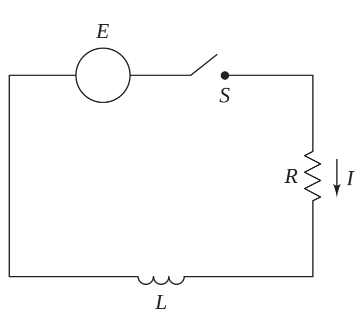

Figure 15.8

A simple electrical circuit consists of electric current I (in amperes), a resistance R (in ohms), an inductance L (in henrys), and a constant electromotive force E (in volts), as shown in Figure 15.8. According to Kirchhoff's Second Law, if the switch S is closed when $t = 0$, the applied electromotive force (voltage) is equal to the sum of the voltage drops in the rest of the circuit. This in turn means that the current I satisfies the differential equation

$$L\frac{dI}{dt} + RI = E.$$

EXAMPLE 6 An Electric Circuit Problem

Find the current I as a function of time t (in seconds), given that I satisfies the differential equation $L(dI/dt) + RI = \sin 2t$, where R and L are nonzero constants.

Solution In standard form, the given linear equation is

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{1}{L}\sin 2t.$$

Let $P(t) = R/L$, so that $e^{\int P(t)\,dt} = e^{(R/L)t}$, and, by Theorem 15.3,

$$Ie^{(R/L)t} = \frac{1}{L}\int e^{(R/L)t}\sin 2t\,dt$$
$$= \frac{1}{4L^2 + R^2}e^{(R/L)t}(R\sin 2t - 2L\cos 2t) + C.$$

Thus, the general solution is

$$I = e^{-(R/L)t}\left[\frac{1}{4L^2 + R^2}e^{(R/L)t}(R\sin 2t - 2L\cos 2t) + C\right]$$
$$I = \frac{1}{4L^2 + R^2}(R\sin 2t - 2L\cos 2t) + Ce^{-(R/L)t}.$$

EXERCISES FOR SECTION 15.2

True or False? **In Exercises 1 and 2, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

1. $y' + x\sqrt{y} = x^2$ is a first-order linear differential equation.

2. $y' + xy = e^x y$ is a first-order linear differential equation.

In Exercises 3 and 4, (a) sketch an approximate solution of the differential equation satisfying the initial condition by hand on the direction field, (b) find the particular solution that satisfies the initial condition, and (c) use a graphing utility to graph the particular solution. Compare the graph with the hand-drawn graph of part (a).

Differential Equation	*Initial Condition*
3. $\dfrac{dy}{dx} = e^x - y$	$(0, 1)$
4. $y' + 2y = \sin x$	$(0, 4)$

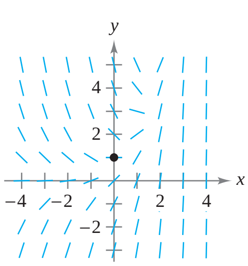

Figure for 3

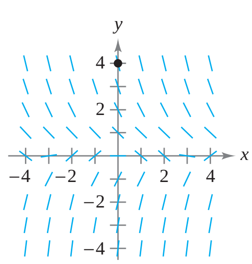

Figure for 4

In Exercises 5–12, solve the first-order linear differential equation.

5. $\dfrac{dy}{dx} + \left(\dfrac{1}{x}\right)y = 3x + 4$

6. $\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = 3x + 1$

7. $y' - y = \cos x$

8. $y' + 2xy = 2x$

9. $(3y + \sin 2x)\,dx - dy = 0$

10. $(y - 1)\sin x\,dx - dy = 0$

11. $(x - 1)y' + y = x^2 - 1$

12. $y' + 5y = e^{5x}$

In Exercises 13–18, find the particular solution of the differential equation that satisfies the boundary condition.

Differential Equation	*Boundary Condition*
13. $y'\cos^2 x + y - 1 = 0$	$y(0) = 5$
14. $x^3y' + 2y = e^{1/x^2}$	$y(1) = e$
15. $y' + y\tan x = \sec x + \cos x$	$y(0) = 1$
16. $y' + y\sec x = \sec x$	$y(0) = 4$
17. $y' + \left(\dfrac{1}{x}\right)y = 0$	$y(2) = 2$
18. $y' + (2x - 1)y = 0$	$y(1) = 2$

In Exercises 19–24, solve the Bernoulli differential equation.

19. $y' + 3x^2y = x^2y^3$

20. $y' + 2xy = xy^2$

21. $y' + \left(\frac{1}{x}\right)y = xy^2$

22. $y' + \left(\frac{1}{x}\right)y = x\sqrt{y}$

23. $y' - y = x^3\sqrt[3]{y}$

24. $yy' - 2y^2 = e^x$

In Exercises 25–28, (a) use a graphing utility to graph the direction field for the differential equation, (b) find the particular solutions of the differential equation passing through the specified points, and (c) use a graphing utility to graph the particular solutions on the direction field.

	Differential Equation	*Points*
25.	$\frac{dy}{dx} - \frac{1}{x}y = x^2$	$(-2, 4),\ (2, 8)$
26.	$\frac{dy}{dx} + 2xy = x^3$	$\left(0, \frac{7}{2}\right),\ \left(0, -\frac{1}{2}\right)$
27.	$\frac{dy}{dx} + (\cot x)y = x$	$(1, 1),\ (3, -1)$
28.	$\frac{dy}{dx} + 2xy = xy^2$	$(0, 3),\ (0, 1)$

Electrical Circuits **In Exercises 29–32, use the differential equation for electrical circuits given by**

$$L\frac{dI}{dt} + RI = E.$$

In this equation, I is the current, R is the resistance, L is the inductance, and E is the electromotive force (voltage).

29. Solve the differential equation given a constant voltage E_0.

30. Use the result of Exercise 29 to find the equation for the current if $I(0) = 0$, $E_0 = 110$ volts, $R = 550$ ohms, and $L = 4$ henrys. When does the current reach 90% of its limiting value?

31. Solve the differential equation given a periodic electromotive force $E_0 \sin \omega t$.

32. Verify that the solution of Exercise 31 can be written in the form

$$I = ce^{-(R/L)t} + \frac{E_0}{\sqrt{R^2 + \omega^2L^2}}\sin(\omega t + \phi)$$

where ϕ, the phase angle, is given by $\arctan(-\omega L/R)$. (Note that the exponential term approaches 0 as $t \to \infty$. This implies that the current approaches a periodic function.)

33. ***Population Growth*** When predicting population growth, demographers must consider birth and death rates as well as the net change caused by the difference between the rates of immigration and emigration. Let P be the population at time t and let N be the net increase per unit time resulting from the difference between immigration and emigration. Thus, the rate of growth of the population is given by

$$\frac{dP}{dt} = kP + N, \qquad N \text{ is constant.}$$

Solve this differential equation to find P as a function of time if at time $t = 0$ the size of the population is P_0.

34. ***Investment Growth*** A large corporation starts at time $t = 0$ to continuously invest part of its receipts at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. Thus, the rate of growth of the amount A in the fund is given by

$$\frac{dA}{dt} = rA + P$$

where $A = 0$ when $t = 0$. Solve this differential equation for A as a function of t.

Investment Growth **In Exercises 35 and 36, use the result of Exercise 34.**

35. Find A for the following.

(a) $P = \$100{,}000$, $r = 6\%$, and $t = 5$ years

(b) $P = \$250{,}000$, $r = 5\%$, and $t = 10$ years

36. Find t if the corporation needs \$800,000 and it can invest \$75,000 per year in a fund earning 8% interest compounded continuously.

37. ***Intravenous Feeding*** Glucose is added intravenously to the bloodstream at the rate of q units per minute, and the body removes glucose from the bloodstream at a rate proportional to the amount present. Assume $Q(t)$ is the amount of glucose in the bloodstream at time t.

(a) Determine the differential equation describing the rate of change with respect to time of glucose in the bloodstream.

(b) Solve the differential equation from part (a), letting $Q = Q_0$ when $t = 0$.

(c) Find the limit of $Q(t)$ as $t \to \infty$.

38. ***Learning Curve*** The management at a certain factory has found that the maximum number of units a worker can produce in a day is 30. The rate of increase in the number of units N produced with respect to time t in days by a new employee is proportional to $30 - N$.

(a) Determine the differential equation describing the rate of change of performance with respect to time.

(b) Solve the differential equation from part (a).

(c) Find the particular solution for a new employee who produced ten units on the first day at the factory and 19 units on the twentieth day.

Mixture **In Exercises 39–44, consider a tank that at time $t = 0$ contains v_0 gallons of a solution of which, by weight, q_0 pounds is soluble concentrate. Another solution containing q_1 pounds of the concentrate per gallon is running into the tank at the rate of r_1 gallons per minute. The solution in the tank is kept well stirred and is withdrawn at the rate of r_2 gallons per minute.**

39. If Q is the amount of concentrate in the solution at any time t, show that

$$\frac{dQ}{dt} + \frac{r_2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1.$$

40. If Q is the amount of concentrate in the solution at any time t, write the differential equation for the rate of change of Q with respect to t if $r_1 = r_2 = r$.

41. A 200-gallon tank is full of a solution containing 25 pounds of concentrate. Starting at time $t = 0$, distilled water is admitted to the tank at a rate of 10 gallons per minute, and the well-stirred solution is withdrawn at the same rate.

(a) Find the amount of concentrate Q in the solution as a function of t.

(b) Find the time at which the amount of concentrate in the tank reaches 15 pounds.

(c) Find the quantity of the concentrate in the solution as $t \to \infty$.

42. Repeat Exercise 41, assuming that the solution entering the tank contains 0.05 pound of concentrate per gallon.

43. A 200-gallon tank is half full of distilled water. At time $t = 0$, a solution containing 0.5 pound of concentrate per gallon enters the tank at the rate of 5 gallons per minute, and the well-stirred mixture is withdrawn at the rate of 3 gallons per minute.

(a) At what time will the tank be full?

(b) At the time the tank is full, how many pounds of concentrate will it contain?

44. Repeat Exercise 43, assuming that the solution entering the tank contains 1 pound of concentrate per gallon.

In Exercises 45–48, match the differential equation with its solution.

	Differential Equation	*Solution*
45.	$y' - 2x = 0$	(a) $y = Ce^{x^2}$
46.	$y' - 2y = 0$	(b) $y = -\frac{1}{2} + Ce^{x^2}$
47.	$y' - 2xy = 0$	(c) $y = x^2 + C$
48.	$y' - 2xy = x$	(d) $y = Ce^{2x}$

In Exercises 49–64, solve the first-order differential equation by any appropriate method.

49. $\dfrac{dy}{dx} = \dfrac{e^{2x+y}}{e^{x-y}}$

50. $\dfrac{dy}{dx} = \dfrac{x + 1}{y(y + 2)}$

51. $(1 + y^2)\,dx + (2xy + y + 2)\,dy = 0$

52. $(1 + 2e^{2x+y})\,dx + e^{2x+y}\,dy = 0$

53. $y \cos x - \cos x + \dfrac{dy}{dx} = 0$

54. $(x + 1)\dfrac{dy}{dx} = e^x - y$

55. $(x^2 + \cos y)\dfrac{dy}{dx} = -2xy$

56. $y' = 2x\sqrt{1 - y^2}$

57. $(3y^2 + 4xy)\,dx + (2xy + x^2)\,dy = 0$

58. $(x + y)\,dx - x\,dy = 0$

59. $(2y - e^x)\,dx + x\,dy = 0$

60. $(y^2 + xy)\,dx - x^2\,dy = 0$

61. $(x^2y^4 - 1)\,dx + x^3y^3\,dy = 0$

62. $y\,dx + (3x + 4y)\,dy = 0$

63. $3y\,dx - (x^2 + 3x + y^2)\,dy = 0$

64. $x\,dx + (y + e^y)(x^2 + 1)\,dy = 0$

SECTION PROJECT

Weight Loss A person's weight depends on both the amount of calories consumed and the energy used. Moreover, the amount of energy used depends on a person's weight—the average amount of energy used by a person is 17.5 calories per pound per day. Thus, the more weight a person loses, the less energy the person uses (assuming that the person maintains a constant level of activity). An equation that can be used to model weight loss is

$$\left(\frac{dw}{dt}\right) = \frac{C}{3500} - \frac{17.5}{3500}w$$

where w is the person's weight (in pounds), t is the time in days, and C is the constant daily calorie consumption.

(a) Find the general solution of the differential equation.

(b) Consider a person who weighs 180 pounds and begins a diet of 2500 calories per day. How long will it take the person to lose 10 pounds? How long will it take the person to lose 35 pounds?

(c) Use a graphing utility to graph the solution. What is the "limiting" weight of the person?

(d) Repeat parts (b) and (c) for a person who weighs 200 pounds when the diet is started.