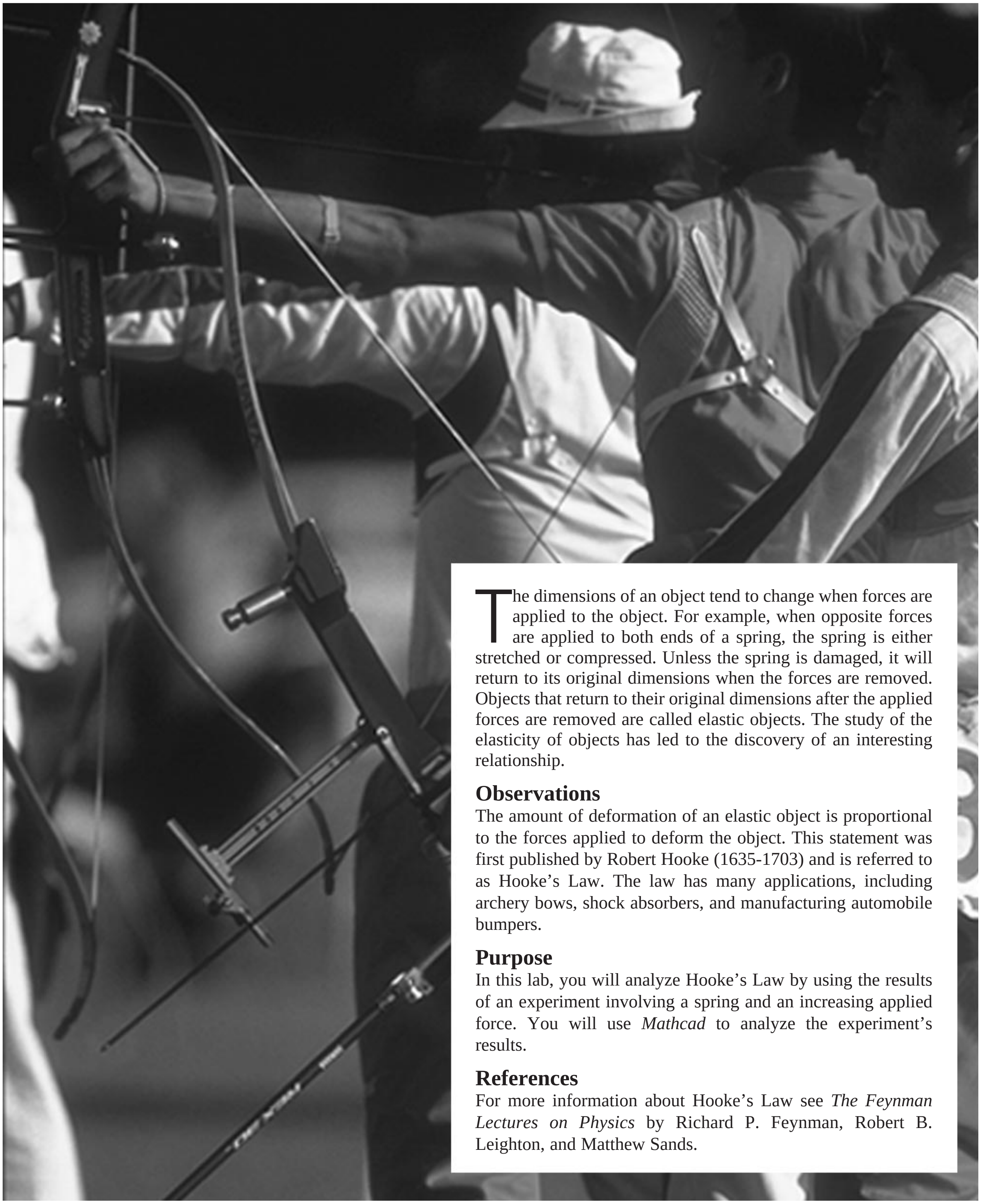
LAB 8

Stretching a Spring

Hooke's Law



The dimensions of an object tend to change when forces are applied to the object. For example, when opposite forces are applied to both ends of a spring, the spring is either stretched or compressed. Unless the spring is damaged, it will return to its original dimensions when the forces are removed. Objects that return to their original dimensions after the applied forces are removed are called elastic objects. The study of the elasticity of objects has led to the discovery of an interesting relationship.

Observations

The amount of deformation of an elastic object is proportional to the forces applied to deform the object. This statement was first published by Robert Hooke (1635-1703) and is referred to as Hooke's Law. The law has many applications, including archery bows, shock absorbers, and manufacturing automobile bumpers.

Purpose

In this lab, you will analyze Hooke's Law by using the results of an experiment involving a spring and an increasing applied force. You will use *Mathcad* to analyze the experiment's results.

References

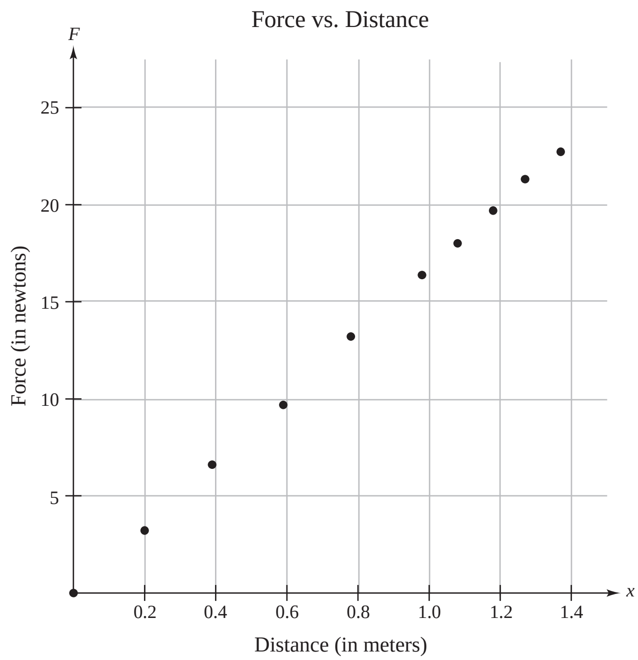
For more information about Hooke's Law see *The Feynman Lectures on Physics* by Richard P. Feynman, Robert B. Leighton, and Matthew Sands.

Data

A force F is applied to a spring and the distance x of the spring is recorded. F is measured in newtons and x is measured in meters. The results of the experiment are shown in the table below.

Distance, *x*	0.00	0.20	0.39	0.59	0.78	0.98	1.08	1.18	1.27	1.37
Force, *F*	0.00	3.22	6.61	9.69	13.21	16.38	18.01	19.70	21.32	22.73

A scatter plot of the data is given below.



The data in the table and the scatter plot are stored in the *Mathcad* file called LAB08.MCD.

Exercises

Name ______________________________

Date ____________________ Class ____________

Instructor ______________________________

1. ***Representing Hooke's Law Mathematically.*** Let F be the force required to stretch or compress an object (within its elastic limits) and let x be the distance that the object is stretched or compressed from its original length. Then Hooke's Law can be represented mathematically by

$$F = kx$$

where k is a constant of proportionality. Verbally describe the graph of this equation. What does k represent in the graph of this equation?

2. ***Calculating the Constant of Proportionality.*** Explain how you would calculate the value of k in the formula $F = kx$ using the data from the spring experiment as described in this lab's Data. Then calculate the value of k. Use *Mathcad* to graph $F = kx$ and the data points. Is the model a good fit to the data? Why or why not? If the model is not a good fit, refine your method for calculating the value of k to obtain a model that is a good fit to the data.

3. ***Is it Unique?*** Do you think the value of k that you found in Exercise 2 can be used for other springs? Why or why not?

4. ***Exceeding the Elastic Limit.*** Hooke's Law says that if an object is within its elastic limits, the force F required to stretch or compress an object is proportional to the distance x that the object is stretched or compressed from its original length. What do you think would happen if a force was applied to the spring sufficient to stretch the spring beyond its elastic limit? Would the relationship between F and x still be linear?

5. ***Work.*** If an object is moved along a straight line by a continuously varying force F, then the work W done by the force as the object is moved from $x = a$ to $x = b$ is

$$W = \int_a^b F dx.$$

Rewrite this integral using the force equation you found in Exercise 2. What do you think the graph of W looks like? Describe the relationship between work done to stretch the spring and the distance x the spring is stretched. Is the relationship constant, linear, or quadratic? Explain your reasoning.

6. *Calculating Work.* In Exercise 5 you found an integral for the work done in stretching the spring described in this lab's Data. Complete the table by evaluating this integral using *Mathcad* and the following distances from the spring's original length. Sketch a graph showing distance on the horizontal axis and work on the vertical axis. Does the graph confirm your answer to Exercise 5? If not, how would you answer Exercise 5 now?

Distance (in meters)	Work
3	
6	
9	
12	
15	

7. ***Calculating Additional Work.*** Determine the work done when a force of 0.20 newton stretches the spring described in this lab's Data to a length of 3.22 meters. Should the work done in stretching the spring from $x = 3.22$ meters to $x = 6.44$ meters be more than, the same as, or less than this? Explain.

8. ***Spring in Motion.*** Suppose a rigid object of mass m is attached to the end of a spring and causes a displacement. Assume the spring's mass is negligible compared to m. If the object is pulled down and released, then the resulting oscillations are a product of two opposing forces—the spring force $F = kx$ and the weight mg of the object. You can use a differential equation to find the position of the object as a function of time. According to Newton's Second Law of Motion, the force acting on the weight is ma, where $a = d^2x/dt^2$ is the acceleration. Assuming the motion is undamped—that is, that there are no other forces acting on the object—then $m(d^2x/dt^2) = -kx$ and you have

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0.$$

Now suppose a force of 16.59 newtons stretches a spring 1 meter from its natural length. The spring is then stretched an additional 5 meters and released with an initial velocity of 7 meters per second. Use *Mathcad* to graph its solution. Describe the motion of the spring as the time t increases.