



# STATISTICS AND PROBABILITY

## CLAST MATHEMATICS COMPETENCIES

- ID1: Identify information contained in bar, line, and circle graphs
- ID2: Determine the mean, median, and mode of a set of numbers
- ID3: Use the fundamental counting principle
- IID1: Recognize properties and interrelationships among the mean, median, and mode in a variety of distributions
- IID2: Choose the most appropriate procedure for selecting an unbiased sample from a target population
- IID3: Identify the probability of a specified outcome in an experiment
- IIID1: Infer relations and makes accurate predictions from studying statistical data
- IVD1: Interpret real-world data involving frequency and cumulative frequency tables
- IVD2: Solve real-world problem involving probabilities

## 4.1 SAMPLING, LINE, BAR AND CIRCLE GRAPHS

You have heard of the Gallup, Harris, and USA Today polls. Based on these polls, predictions are made about events ranging from presidential elections to consumer behavior. Since it is impossible to interview every voter or consumer, these polls rely on selecting an **unbiased sample** from a **target population** and are then used to make inferences regarding the entire population.

### A. Selecting Unbiased Samples

#### Objective IID2

#### CLAST SAMPLE PROBLEMS

1. The board of education wants to find out if high school coaches are pleased with a new district high school athletic program. Which procedure would be most appropriate for obtaining a statistically unbiased sample?
  - A. Using numbers selected at random, choose some coaches from a computerized numbered list of the most successful coaches in the district.
  - B. Survey the coaches who are attending a coaches convention.
  - C. Put the names of all high school coaches in the district in a hat, choose several names, and then survey all coaches in those selected schools.
  - D. Have all principal ask the coaches in their school to call the board of education and give their opinions

T	TERMINOLOGY -- UNBIASED SAMPLING	
	<i>RANDOM SAMPLE</i>	<i>EXAMPLES</i>
	<p>A <b>random sample</b> is a sample selected in such a way that every member of the <b>target population</b> (the entire group from which the sample is selected) has an equal chance of being chosen. If we select a <b>random sample</b> from a population, we say that we have an <b>unbiased sample</b> for the population.</p>	<p>The following procedures <b>do not</b> select a random sample for the entire population of Florida. Do you see why?</p> <ol style="list-style-type: none"> <li>1. Select every third woman entering a department store in New York.</li> <li>2. Select every third man entering a bar.</li> <li>3. Select every third person entering an opera concert.</li> </ol>

**ANSWERS**      1. C

1	SELECTING AN UNBIASED SAMPLE	
	<i>RULE</i>	<i>EXAMPLES</i>
	<p>To select a <b>random sample</b> for a population:</p> <ol style="list-style-type: none"> <li>1. Select members at random from the <b>target</b> population only.</li> <li>2. Eliminate samples that do not give <b>every</b> member of the population an equal chance of being chosen.</li> </ol>	<p>Selecting every third woman entering a department store in New York does not give a random sample representing the population in Florida, since it includes members from a different target population.</p> <p>Selecting every third man entering a bar or every third person entering an opera concert does not give a random sample of the entire population because not everyone has an equal chance of being selected.</p>

### □ CLAST EXAMPLE

- Example**
1. A college president wants to find out which courses are popular with students. What procedure would be most appropriate for obtaining an unbiased sample of students?
    - A. Survey a random sample of students from the English department.
    - B. Survey the first hundred students from an alphabetical listing.
    - C. Survey a random sample of students from a list of the entire student body.
    - D. Have students voluntarily mail in their preference.

- Solution**
- Examine each of the choices and make sure they satisfy the two conditions for an unbiased sample.
- Eliminate A and B because they do **not** give every member of the population an equal chance of being chosen. (Only students in English or in the first hundred on the list have a chance.)
- Eliminate D because not all students will mail their preference. (They may not be able to afford the postage, for example.)
- The correct answer is **C**. It selects persons from the target population (students) and it gives **every** student an equal chance of being chosen.

**Note:** If the response does not contain the word "random" it is probably incorrect.

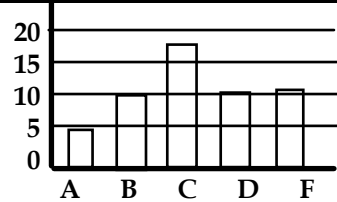
After learning how to find an unbiased sample of a population, we need to present the data we have obtained in a meaningful manner. This is done by means of a **graph**. The CLAST discusses three types of graphs: **line, bar and circle**. Here are the type of questions you will encounter in the CLAST.

## B. Line, Bar and Circle Graphs

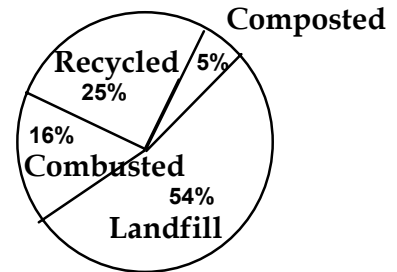
### Objective ID1

### CLAST SAMPLE PROBLEMS

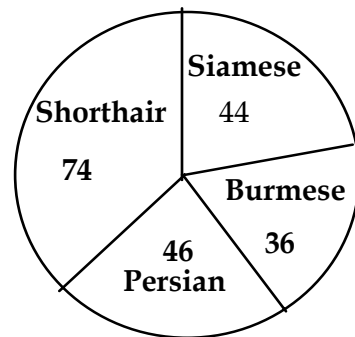
1. The graph to the right shows the grade distribution in a mathematics test. How many more students made a C than made an A?



2. The circle graph to the right represents the disposition of municipal waste in the year 2000. What percent of the municipal waste is recycled?



3. The circle graphs to the right shows the number and classification of cats entered in a cat show. What percent of the cats entered are Persian?

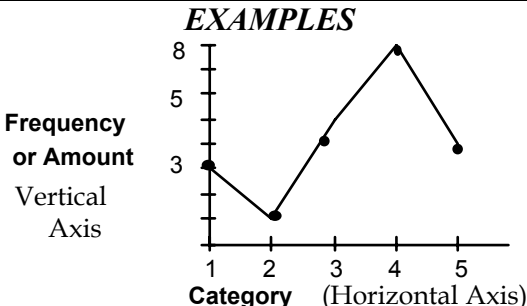


### ANSWERS

1. 13

2. 25%

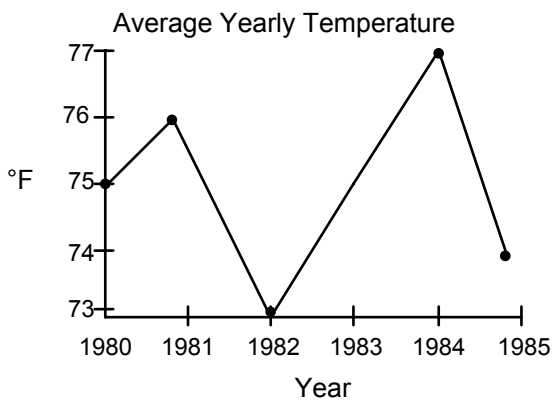
3. 23%

<b>2</b>	<b>INTERPRETING LINE GRAPHS</b>
<p><b>TO ANALYZE LINE GRAPHS</b></p> <ol style="list-style-type: none"> <li>1. Identify the category (horizontal axis) or categories to which the question refers.</li> <li>2. Identify the frequencies (vertical axis) for each of the categories. (Points or dots are used to mark the frequencies.)</li> <li>3. Use the information in 1 and 2 to answer the questions.             <ol style="list-style-type: none"> <li>(a) Find the frequency for a category.</li> <li>(b) Find "the sum of" or "the difference" between the frequencies of two categories.</li> <li>(c) Find the maximum or minimum points on the graph.</li> </ol> </li> </ol>	<p style="text-align: center;"><b>EXAMPLES</b></p>  <ol style="list-style-type: none"> <li>1. The categories are 1, 2, 3, 4 and 5</li> <li>2. The difference between the frequencies of categories 1 and 2, is <math>3 - 1 = 2</math></li> <li>3. The maximum is 8 (for category 4.) The minimum is 1 (for category 2.)</li> </ol>

□ **CLAST EXAMPLES**

**Example**

2. The graph shows the yearly average temperature from 1980 to 1985. What is the difference between the highest and lowest average yearly temperatures?



- |         |         |
|---------|---------|
| A. 73°F | B. 77°F |
| C. 1°F  | D. 4°F  |

**Solution**

1. The question refers to the temperatures from 1980 to 1985.
2. The temperatures for the respective years are: 75, 76, 73, 77 and 74.
3. The highest temperature is 77°F and the lowest is 73°F. The difference is

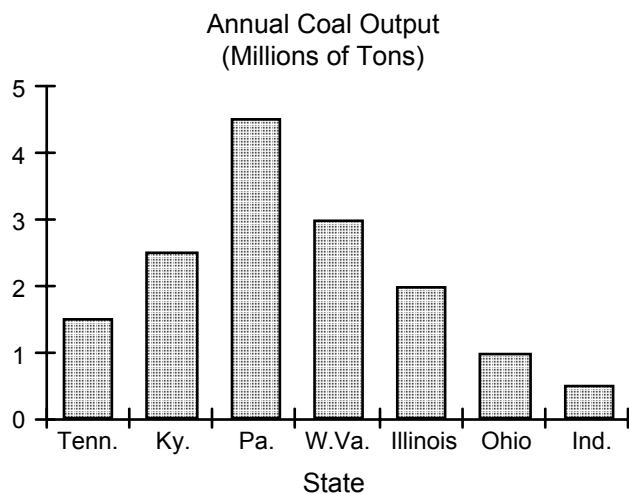
$$77^{\circ}\text{F} - 73^{\circ}\text{F} = 4^{\circ}\text{F}$$

The answer is **D**.

**Note:** The lowest point for a category (even when it intersects the horizontal axis) is not necessarily 0.

**Example**

3. The graph represents the average coal output for seven states. How much higher is the output (in tons) for Pennsylvania (Pa) than for West Virginia (W. Va.)?



- A. 500,000 tons      B. 1,000,000 tons  
C. 1,500,000 tons      D. 2,000,000 tons

**Solution**

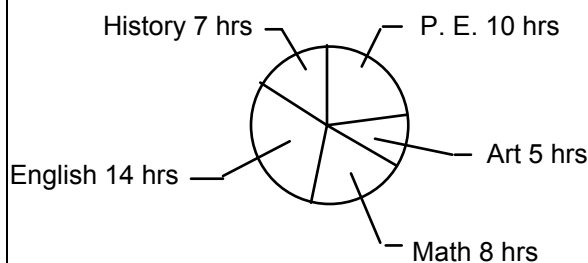
- The categories are the states.
- The outputs are represented by the bars drawn above each category.
- We want to find out how much higher the output is for Pennsylvania than for West Virginia. The bar above Pa. is 4.5 units and the bar above W. Va. is 3 units. To find how much higher the Pa. bar is, find the difference;  
 $4.5 - 3 = 1.5$ . Since the answer is in Millions of Tons (see the top of the diagram), the answer should be 1.5 million tons or 1,500,000 tons. The answer is C.

**Note:** Line graphs and bar graphs are interpreted similarly.

3

**INTERPRETING CIRCLE GRAPHS****TO ANALYZE A CIRCLE GRAPH**

- Identify the category (or categories) to which the question refers.
- Identify the amount for each of the categories.
- Use the information in 1 and 2 to answer the questions.
  - Find the sum or difference of the amounts for two categories.
  - Find the percent or portion of the total amount for a category.

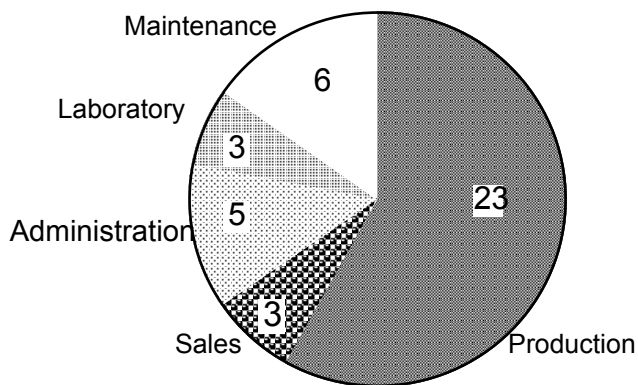
**EXAMPLES**

- The categories are History, P. E., Art, Math and English and the amounts are the number of hours a student has in each.
- The number of hours for Math is 8.
- The *difference* in hours between Math and English, for example, is  $14 - 8 = 6$ . The *total* number of hours is 44.

□ **CLAST EXAMPLE****Example**

4. The number of people employed in different work areas in a manufacturing plant are represented by the circle graph. What percent of the people work in Sales and Administration combined?

Number of Employees by Work Area



- A. 25%   B. 20%   C. 12.5%   D. 7.5%

**Solution**

- The categories are production, sales, administration, laboratory and maintenance.
- The numbers in these categories are: 23, 3, 5, 3 and 6, respectively.
- The *number* of people employed in sales and administration combined (the sum) is  $3 + 5 = 8$ . Since the answers are given as percents, we have to see what percent 8 is of the total  $23 + 3 + 5 + 3 + 6$  or 40. The percent is:

$$\frac{8}{40} = \frac{1}{5} = 0.20 = 20\%$$

The answer is **B**.

*Note:* If you have forgotten how to do percents, review Section 1.4.

**C. Relations and Predictions from Statistical Data****Objective IID1****CLAST SAMPLE PROBLEMS**

1. The profits realized by a certain business over a 5-year period are given below:

1. \$20,000   2. \$32,000   3. \$28,000   4. \$40,000   5. \$44,000

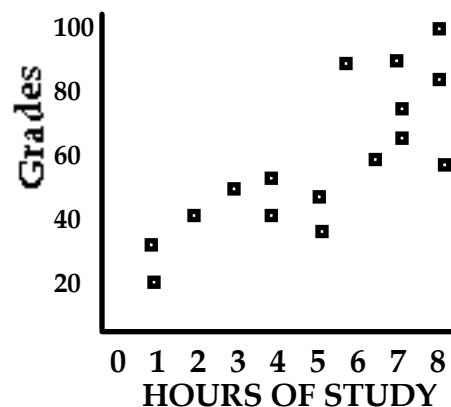
Which statement best describes the trend in profits?

- There is an increasing trend in profit from the first to the fifth year
- Profits began to stabilize the second year
- There is no trend in the profits
- Profits are steadily declining after the fourth year

**ANSWERS**   1. A

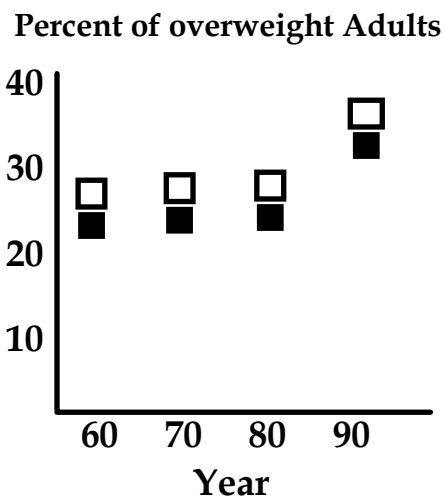
**Objective IID1****CLAST SAMPLE PROBLEMS (CONT)**

2. The graph to the right depicts the number of hours a group of 16 students spent studying for a test during the week prior to the test and the grades they made on the test. Which of the following best describes the relationship between the number of hours spent studying and the test grade?



- A. There appears to be a negative association between study hours and test grade
- B. Increasing the number of study hours causes the grade to be higher
- C. There appears to be a positive association between the number of study hours and the test grade
- D. The number of hours spent studying during the week prior to the test does not provide the information needed to predict grades.

3. The graph to the right depicts the percent of overweight male (shaded squares) and female (blank squares) adults from 1960 until 1990. Which of the following best describes the relationship between the year (time) and the percent of overweight adults?



- A. A relationship between being overweight and time exists *only* for males.
- B. There appears to be no relationship between time and being overweight
- C. Incidence of being overweight tends to increase as time increases. A stronger relationship exists for males than for females.
- D. Incidence of being overweight tends to increase as time increases. The relationship is about the same for males and females.

**ANSWERS** 2. C

3. D



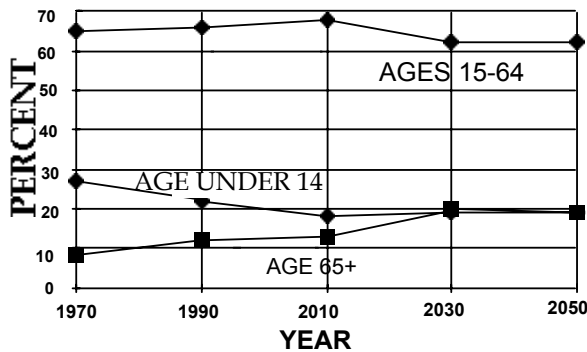
Now that we know how to interpret a graph, we are ready to use our knowledge to discover **relations** that may be suggested by the graphs as well as making predictions based on the information contained in these graphs or from data given in table form.

□ **CLAST EXAMPLES**

**Example**

5. The graph shows the percent of persons in different age groups for several years. Which of the following best describes the trend in the "Age Under 14" group?

**PROJECTED AGE STRUCTURES IN THE U. S.**



- A. The percent of persons in the group steadily decreased.
- B. The percent of persons in the group decreases until 2010 and then levels off.
- C. There is no trend for this group.
- D. The percent of persons in the group increased from 1970 to 1990.

**Example**

6. Referring to the graph in Example 5, which is the best estimate of the percent of persons under 14 in the year 2000?

- A. 15%
- B. 20%
- C. 25%
- D. 30%

**Solution**

Since these are three different categories involved (15-64, Under 14 and 65+) make sure you select the correct graph, the one in the middle.

1. The years are 1970, 1990, 2010, 2030 and 2050.
2. The percents for the "Age Under 14" are, approximately, 29%, 21%, 19%, 19% and 19%
3. Thus, the group decreases until 2010 then levels off.

The correct answer is **B**.

*Note:* This type of question asks for a trend in a *single* variable. Look for a pattern in the graph indicating:

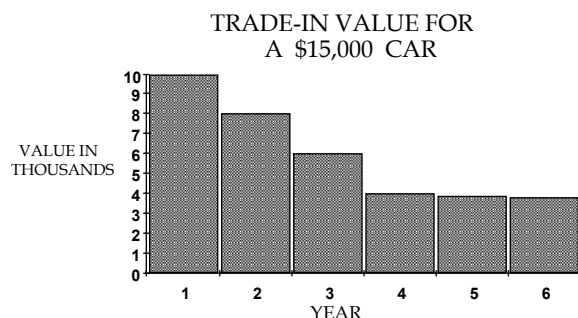
- (a) an increase,
- (b) a decrease, or
- (c) a leveling off.

**Solution**

This is a different type of question where you are asked for an *estimate*. Since the year 2000 is between 1990 and 2010, the graph shows that the answer must be between 19% and 21%. Thus, the best estimate is 20%. The answer is **B**.

**Example**

7. Consider the following graph showing the value of a \$15,000 car after 1, 2, 3, 4, 5 and 6 years. In what year did the price of the car begin to stabilize?



- A. 6      B. 5      C. 4      D. 3

**Solution**

This Example is similar to Example 5 but uses a bar graph instead of a line graph.

1. The categories are the numbers of years elapsed, 1, 2, 3, 4, 5 and 6.
2. The frequencies are the approximate values of the car (in thousand of dollars).  
10, 8, 6, 4, 3.9, 3.8
3. In the fourth year the price began to level off.

The answer is **C**.

In Example 7 we discerned a trend in the value of the car by looking at the graph. We can find trends involving a single variable such as salary increases, or number of births by looking at the data itself as it is done in the next Example.

**CLAST EXAMPLE****Example**

8. The profits realized by Jackie's Place over a period of time are as shown:

Year	Profits
1990	\$15,000
1991	\$24,000
1992	\$21,000
1993	\$30,000
1994	\$33,000
1995	\$37,500
1996	\$33,500

Which statement best describes the trend in profits?

- A. There is an increasing trend in profits from 1990 to 1995.  
 B. Profits began to stabilize in 1991.  
 C. There is no trend in profits  
 D. Profits are steadily declining into the 1990's.

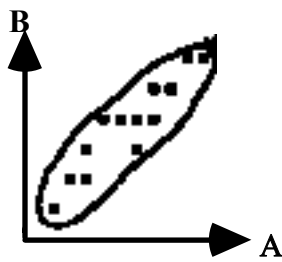
**Solution**

This Example is similar to Example 7 but uses the raw data instead of graphing the data.

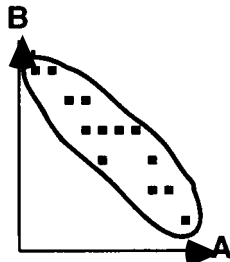
1. The categories are the years, 1990, 1991, and so on.
2. The frequencies are the profits realized. \$15,000, \$21,000 and so on.
3. Since there is a consistent increase over the time period represented, the correct answer is **A**.

The last two types of questions about statistical trends deal with relations between two or more variables. If two variables both increase or both decrease these variables have a *positive association* and their graph will slant *upward* from left to right. Thus, when you increase the number of hours of study and your grade goes up you may say that there is a *positive* relationship between the two variables: number of hours you study and grade. On the other hand, if one variable increases and the other variable decreases, these variables have a *negative association* and their graph will slant *downward* from left to right. For example, if the time it takes you to walk a mile decreases as the number of practice hours increase, we say that there is a *negative* relationship between the time it takes you to walk a mile and the number of practice hours. Of course, if there is no consistent trend (increasing or decreasing), we say that there is no relationship between the variables. You can see these relationships better by drawing a close-fitting oval around the graphs as shown.

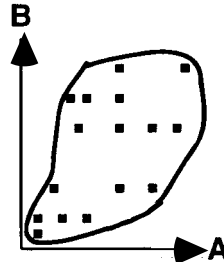
**Strong Positive Relationship**



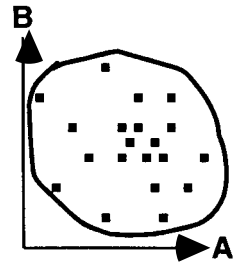
**Strong Negative Relationship**



**Weak Positive Relationship**



**No Relationship**

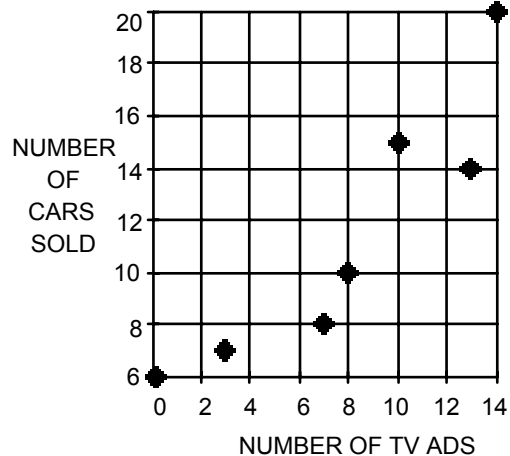


□ **CLAST EXAMPLES**

**Example**

9. The graph at the right depicts the number of TV ads shown and the number of cars sold during a 14-week period. Which of the following best describes the relationship between the number of TV ads shown and the number of cars sold?
- A. There is no apparent association between car sales and TV ads.
  - B. Increased number of TV advertisements caused the increase in car sales.
  - C. Increasing the number of cars sold caused the number of TV advertisements to increase.
  - D. There is an association between the number of TV ads shown and the number of cars sold.

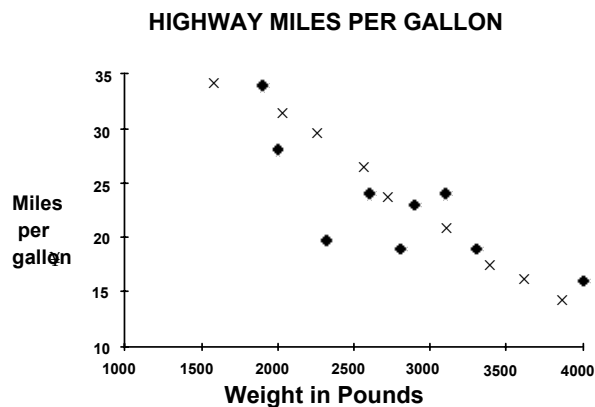
**Solution**



Eliminate B and C, since there is not enough information to establish cause and effect. As the number of TV ads increases from 0 to 14, the number of car sold increases from 6 to 20. Thus, we have to discard A and select **D** for the correct answer.

**Example**

10. The graph at the right depicts the weight and the highway miles per gallon for domestic (×) and foreign (•) cars. Which of the following best describes the relationship between car weight and highway miles per gallon?
- Heavier cars tend to get fewer miles per gallon than lighter cars; a stronger relationship exists for foreign cars than for domestic cars.
  - Heavier cars tend to get fewer miles per gallon than lighter cars; a stronger relationship exists for domestic cars than for foreign cars.
  - A relationship between weight and highway miles per gallon exists only for domestic cars.
  - A relationship between weight and highway miles per gallon exists only for foreign cars.

**Solution**

In this Example we have two categories, domestic cars (×) and foreign cars (•). You can see from the graph that heavier cars, both foreign and domestic, get fewer miles per gallon. Eliminate C and D. The relationship for domestic cars (those marked with the (×)) is stronger because, in every instance, the heavier the car, the fewer miles per gallon it gets. The foreign cars (marked with the (•)) have more variation as shown by the "scattered" pattern. The answer is **B**.

**Example**

11. The table shows the Grade Point Average (GPA) and the number of hours taken by a large group of students. Which of the following best describes the relationship between the GPA and the number of hours taken?
- Increasing the number of hours causes a decrease in the GPA.
  - Increasing the student's GPA causes an increase in the number of hours taken.
  - There appears to be a negative association between the number of hours taken and GPA.
  - The number of hours taken does not provide the information needed to predict GPA.

**Solution**

Hours	GPA
10	3.6
11	3.4
12	3.2
13	3.2
14	3.0
15	2.8
16	2.2
17	2.2
18	2.0

There is not enough information to show cause and effect, so we eliminate A and B.

As you can see from the table, as the number of hours taken increases the GPA decreases. Thus, the correct answer is **C**.

**Note:** Responses containing the word "cause" or "makes" or "produces" are usually incorrect.

## Section 4.1 Exercises

### □ WARM-UPS A

The supervisor of elections in a certain city needs to estimate the average number of voters per house in its city. Refer to the procedure indicating how to select an unbiased sample (Rule 1) and determine what is wrong with the given procedure.

1. Survey all residents of the largest subdivision in the city.
2. Select the largest grocery store in the state and survey a random sample of its customers.

### □ CLAST PRACTICE A

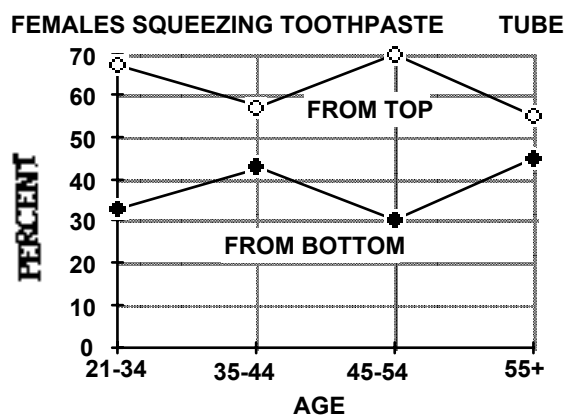
### PRACTICE PROBLEMS: Chapter 4, # 1, 2

3. The vice-president of ABC corporation needs to estimate the average over-time pay per person in any given week. What procedure would be most appropriate for obtaining a statistically unbiased sample of the corporation's employees?
  - A. Have employees voluntarily list their over-time hours on a bulletin board.
  - B. Select the largest department in the corporation and survey a random sample of its employees.
  - C. Randomly select a sample from all employees.
  - D. Survey all the newest employees.
4. A city's school board needs to estimate the average number of school-age children per household in its city. What procedure would be most appropriate for selecting a statistically unbiased sample?
  - A. Randomly select geographic regions of the city and then survey a random sample of people within the regions.
  - B. Select the largest grocery store in the city and survey a random sample of its customers.
  - C. Survey a random sample of the people who work at a local medical center.
  - D. Survey all residents of the largest subdivision in the city.

□ WARM-UPS B

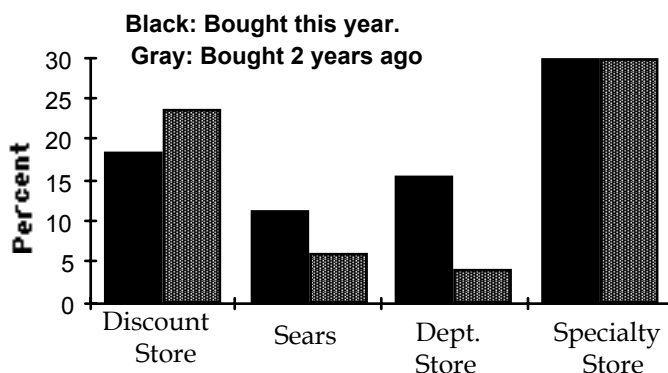
**Referring to the graph at the right:**

5. What percent of the females in the 35-44 age group squeeze the toothpaste tube from the bottom?
6. What percent of the females in the 21-34 age group squeeze the toothpaste tube from the top?
7. What age group has the largest percent of top-tube squeezers?
8. What age group has the largest percent of bottom-tube squeezers?
9. In which age group is the percent of top and bottom tube squeezers closest?
10. What is the greatest percent difference between top and bottom tube squeezers and in which age group does this difference occur?



11. What type of store sold the smallest percent of 35 mm cameras 2 years ago?
12. What type of store sold the largest percent of 35 mm cameras this year?
13. What type of store has the largest percent difference between this year and 2 years ago?
14. What type of store had stable sales?

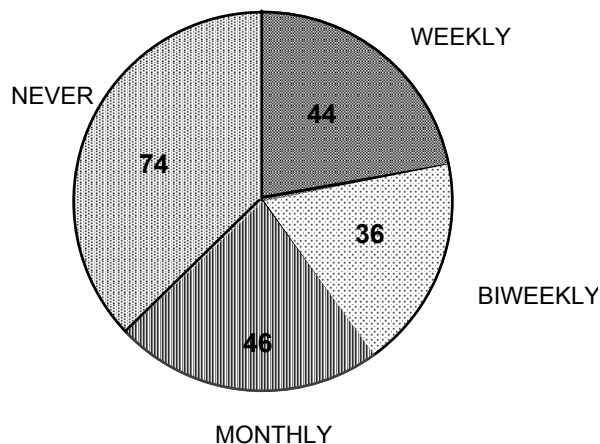
Where 35 mm cameras were bought



**Referring to the circle graph at the right:**

15. What percent of the people never washed their cars?
16. What percent of the people washed their cars monthly?
17. What percent of the people washed their cars weekly or biweekly?
18. What was the percent difference between people that washed their cars weekly and those that washed them biweekly?
19. In what category did the largest percent of the people fall?

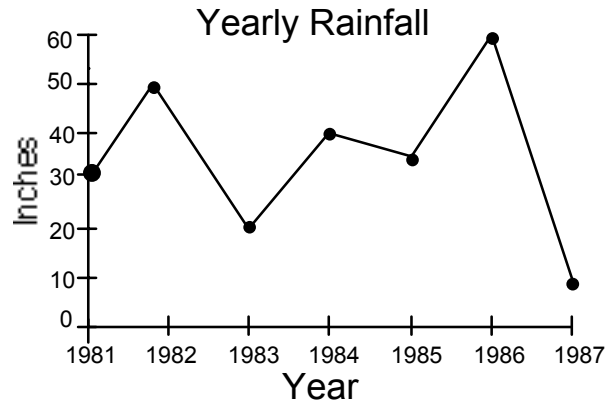
HOW OFTEN DO YOU WASH YOUR CAR?



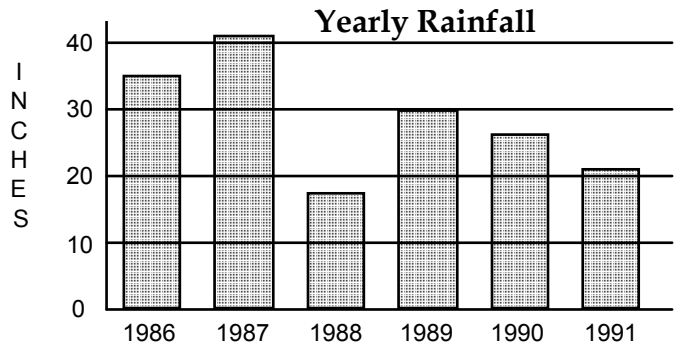
□ **CLAST PRACTICE B**

20. Referring to the graph at the right, in what year was the rainfall the lowest?
- A. 1981                                      B. 1983  
 C. 1986                                      D. 1987
21. What is the difference (in inches) between the lowest and the highest yearly rainfalls?
- A. 10 inches                                  B. 50 inches  
 C. 30 inches                                  D. 40 inches

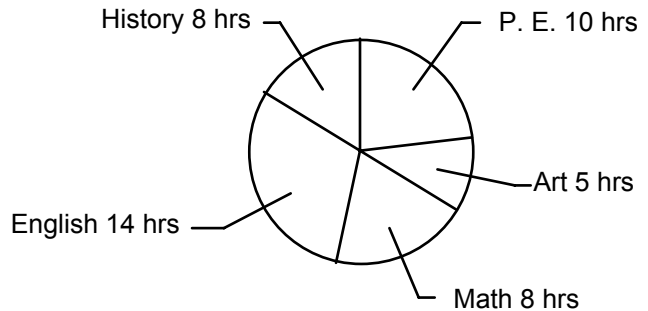
**PRACTICE PROBLEMS: Chapter 4, # 3-5**



22. The graph at the right represents the yearly rainfall in inches for 1986-1991. Find the rainfall for 1989.
- A. 21 inches                                  B. 18 inches  
 C. 30 inches                                  D. 26 inches



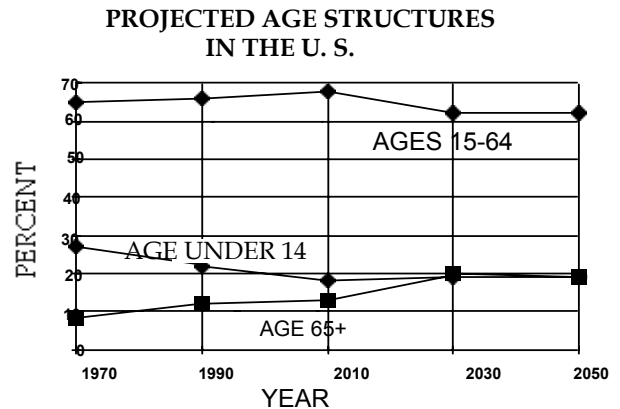
23. The circle graph at the right shows the number of hours required in each discipline of a college core curriculum. What percent of the hours are in P. E. and Math combined?
- A. 18%    B.  $\frac{2}{5}$     C. 40%    D. 33%



□ **WARM-UPS C**

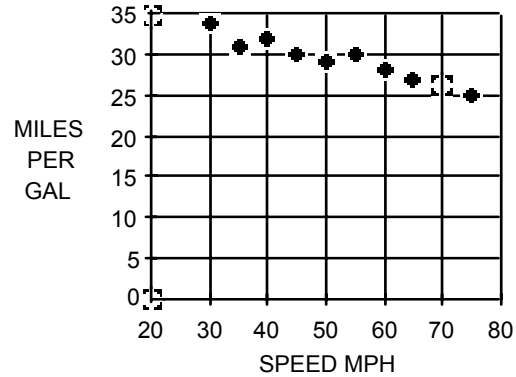
**Referring to the graph at the right:**

24. Describe the trend from 1970 to 2030 for the "Age 65+" group.
25. Describe the trend from 2030 to 2050 for the "Age 65+" group.
26. Which group shows the largest decrease from 1970 to 2050?
27. Which group shows the largest increase from 1970 to 2050?



**Referring to the graph at the right:**

28. What would you predict for the mileage when the speed is 70 mph?
29. What mileage would you predict for 80 mph?
30. What is the best description for the relationship between the number of miles per gallon and the speed in mph?

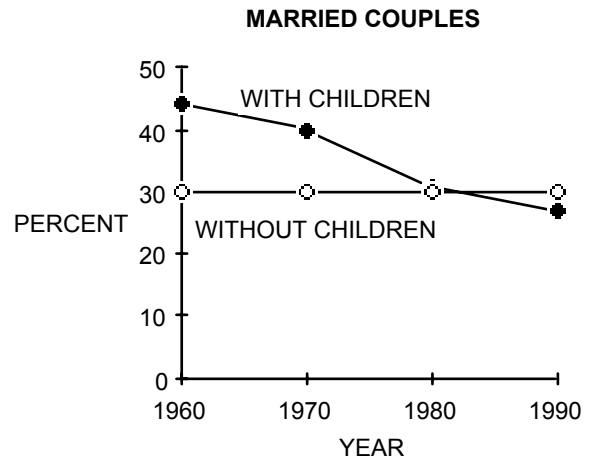


**CLAST PRACTICE C**

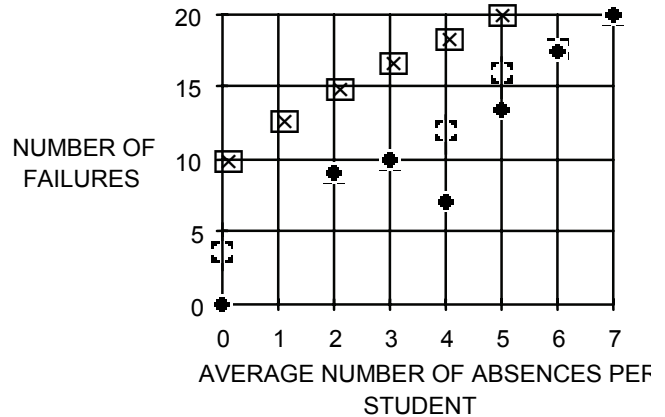
**Referring to the graph at the right:**

31. Which of the following best describes the relationship from 1960 to 1990 for married couples with children?
  - A. The number of married couples with children stabilized.
  - B. The number of married couples with children stayed the same.
  - C. The number of married couples with children decreased.
  - D. The number of married couples with children increased.

**PRACTICE PROBLEMS: Chapter 4, # 6**



32. The graph at the right depicts the number of absences and failures in Math (x) and English (•). Which of the following best describes the relationship between the number of failures and the number of absences?



- A. An excessive number of absences causes an excessive number of failures: a stronger relationship exists in Math than in English.
- B. An increase in the number of absences tends to indicate an increase in the number of failures; a stronger relationship exists in Math than English.
- C. A relationship between the number of absences and failures exists only for Math.
- D. A relationship between the number of absences and failures exists only for English.



33. The table at the right shows the number of servers and the sales (in dollars) for Pizza Hot.

Day	Number of Servers	Sales (in dollars)
Sun.	4	350
Mon.	4	432
Tues.	5	428
Wed.	6	500
Thurs.	6	510
Fri.	9	900
Sat.	12	1430

Which of the following best describes the relationship between the number of servers and the sales revenue?

- A. Increasing the number of servers increases sales.
- B. The number of servers does not provide enough information to predict sales revenue.
- C. Increased sales causes management to have more servers.
- D. There appears to be a positive association between the number of servers and the sales revenue.

### EXTRA CLAST PRACTICE

34. The yearly enrollment at a university is given below. Which of the following best describes the trend in enrollment?

Year	Enrollment
1988	16,460
1989	18,024
1990	17,952
1991	19,246
1992	20,400
1993	21,524

- A. There is no trend in enrollment.
- B. Enrollment is increasing into the 1990's.
- C. Enrollment leveled off in the early 1990's.
- D. Enrollment is remaining about the same.

35. The table shows the average height of a group of children during their first 5 years. Which of the following best describes the potential for the average height of children?

Age	Height
1	24"
2	28"
3	33"
4	39"
5	42"
	?
	?

- A. Children will be taller at 7 years than at 5 years.
- B. The trend is highly unpredictable.
- C. A child could some day be 240" tall
- D. A child will never be taller than 60"

## 4.2 THE MEAN, THE MEDIAN AND THE MODE

We have already learned how to select an *unbiased sample* from a population. We shall now represent this sample by a single quantity: the *mean*, the *median* or the *mode*.

### A. Finding the Mean, Median and Mode

#### Objective ID2      CLAST SAMPLE PROBLEMS

1. Find the mean, median and mode of:      4   4   8   12   17   21

1	FINDING THE MEAN, THE MEDIAN AND THE MODE	
	<p style="text-align: center;"><b>FINDING THE MODE</b></p> <p>The <b>mode</b> of a set of numbers is the number occurring most <i>frequently</i> in the set. (Remember <b>mode = most.</b>)</p> <p>If no number in a set occurs more than once, there is <u>no</u> mode.</p> <p>If several numbers occur an equal number of times and more than all the rest, <u>all</u> these numbers are modes.</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <p>The <b>mode</b> of the numbers in the sample 2, 3, 4, 1, 3, 4, 5, 6, 4 is <b>4</b>. (4 occurs most frequently: 3 times.)</p> <p>The sample 1, 3, 5, 7, 9 has <u>no</u> mode. (No number occurs more than once.)</p> <p>The sample 1, 2, 3, 2, 3, 1, 4, 3, 4, 2, 1 has three modes: <b>1, 2, and 3</b>. (Each occurs three times.)</p>
	<p style="text-align: center;"><b>FINDING THE MEAN</b></p> <p>The <b>mean</b> (average) of a set of numbers is the sum of the numbers in the set divided by the number of elements in the set.</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <p>The <b>mean</b> of the data in the sample 10, 5, 2, 4, 5, 3, 6 is is <math>\frac{10 + 5 + 2 + 4 + 5 + 3 + 6}{7} = \frac{35}{7} = 5</math></p> <p>We divided by 7 because there are 7 numbers in the data sample.</p>
	<p style="text-align: center;"><b>FINDING THE MEDIAN</b></p> <p>The <b>median</b> of a set of numbers is the number in the middle when the numbers are arranged in order of magnitude. Note: This procedure works when we have an <i>odd</i> number of items in the sample.</p> <p>If there is no single middle number the <b>median</b> is the average of the two middle numbers. Note: Use this procedure when there is an <i>even</i> number of items in the sample.</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <p>The median of: 9, 9, 6, 8, 9, 6, 9, 4, 5 is <b>8</b> the middle number when the numbers are arranged in order of magnitude.</p> <p style="text-align: center;">9 9 9 9 <span style="border: 1px solid black; padding: 0 5px;">8</span> 6 6 5 4</p> <p>The median of: 3, 2, 5, 6, 5, 4 is <b>4.5</b>. Since we have an even number of items in the data sample, arrange them in order of magnitude and take the average of the two middle numbers 4 and 5.</p> <p style="text-align: center;">2 3 4 5 5 6 <math>\frac{4 + 5}{2} = 4.5</math></p>

**ANSWERS**      1. Mean: 11; Median: 10; Mode: 4

□ **CLAST EXAMPLE**

**Example**

1. Find the mean, median and mode of the data in the following sample:

6, 15, 24, 23, 29, 22, 21, 29, 29

- A. 22, 23, 29                      B. 17.5, 22, 29  
 C. 29, 23, 22                      D. 23, 22, 29

**Solution**

The mean is:

$$\frac{6 + 15 + 24 + 23 + 29 + 22 + 21 + 29 + 29}{9} = \frac{198}{9} = 22$$

You could stop here! The only possible answer is **A**. Let us check the rest.

The median is the middle number, 23

6 15 21 22 **23** 24 29 29 29

The mode is the most frequently occurring number, 29, so the answer is indeed **A**.

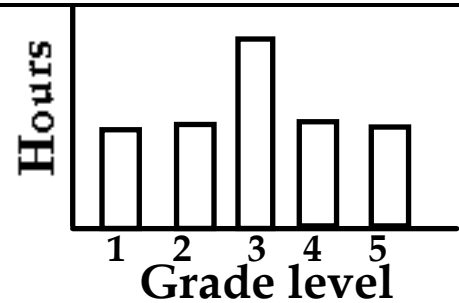
The *mean*, *median* and *mode* of a sample give us an idea of where the "center" of the sample is and, because of this, are called *measures of central tendency*. The next CLAST competency does not ask for the calculation of the mean, median and mode but rather the relationships that can be established among these measurements by examining their graphs.

**B. Relationships among the Means, Medians and Modes**

**Objective IID1**

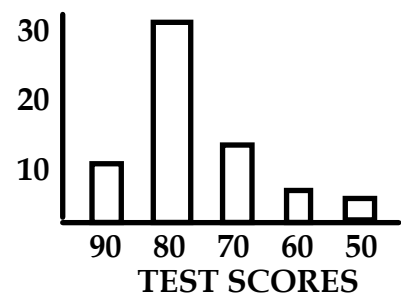
**CLAST SAMPLE PROBLEMS**

1. The distribution to the right gives the number of hours that elementary school students spent watching TV during a certain week. Select the statement that is true about the distribution of hours.



- A. The mean is equal to the median  
 B. The median is less than the mean  
 C. The mean is greater than the median  
 D. The mean is greater than the mode

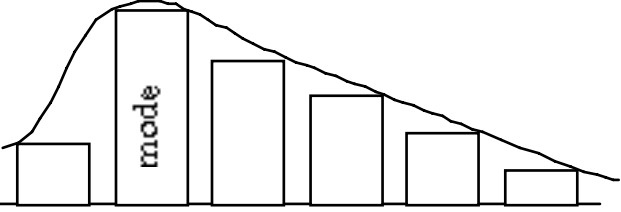
2. The graph to the right represents the grades made by students in a class. Which of the following is true about the grade distribution?



- A. The median is less than the mode  
 B. The median is less than the mean  
 C. The mode and the median are the same  
 D. The mode is less than the mean

3. A class of 40 students received their tests scores. All but three received an 80. The remaining students received 89, 75 and 76. What is the relationship between the mean, median and mode of the scores?

**ANSWERS**      1. A                      2. C                      3. Mean = Median = Mode

<b>2</b>	<b>RELATIONSHIPS AMONG THE MEANS, MEDIANS AND MODES</b>
<p><b>FOR A RIGHT-SKEWED DISTRIBUTION</b></p> <p>To discover the relationship between the mean, median and mode, write:</p> <p>_____ <span style="border: 1px solid black; padding: 2px;">median</span> _____</p> <p>If the graph of the distribution is <b>skewed right</b>, the <i>mode</i> (highest bar) must be to the <i>left</i> of the median (middle). This forces the mean be to the <i>right</i> of the median. Using the inequality sign <math>&lt;</math>, write:</p> <p style="text-align: center;"><u>Mode</u> &lt; <span style="border: 1px solid black; padding: 2px;">median</span> &lt; <u>Mean</u></p>	<p style="text-align: center;"><b>EXAMPLES</b></p>  <p>In a <i>right skewed distribution</i>, the following relationship exists between the mean, median and mode:</p> <p style="text-align: center;">Mode &lt; <span style="border: 1px solid black; padding: 2px;">Median</span> &lt; Mean</p> <p><i>Note:</i> Skewed right distributions are sometimes called <b>pulled right</b> distributions.</p>

□ **CLAST EXAMPLE**

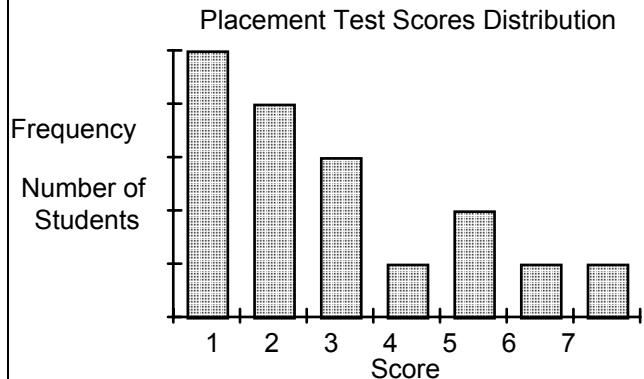
**Example**

2. The graph below represents the distribution of scores on a placement test for students at Central High School. Select the statement that is true about the distribution of scores.
- A. The median is less than the mode.
- B. The mean is greater than the mode.
- C. The mean is less than the mode.
- D. The mean and the median are the same.

Remember to start the problem by writing:

\_\_\_\_\_ median \_\_\_\_\_

**Solution**

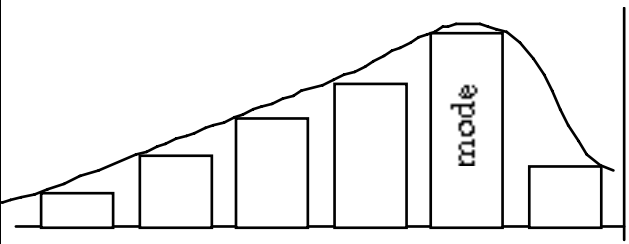


Since the mode is to the left of the middle in the graph, we write "mode" to the left of the word "median" and "mean" to the right. We then have: mode < median < mean

Now, look at the responses. **A** says that the median is less than the mode. This is not consistent with our information,

$$\text{mode} < \text{median} < \text{mean}$$

Since the mean is greater than the mode, the correct response is **B**.

<b>3</b>	<b>RELATIONSHIPS AMONG THE MEANS, MEDIANS AND MODES</b>
<p><b>FOR A LEFT-SKEWED DISTRIBUTION</b></p> <p>To discover the relationship between the mean, median and mode, write:</p> <p>_____ <span style="border: 1px solid black; padding: 2px;">median</span> _____</p> <p>If the graph of the distribution is <b>skewed left</b>, the <i>mode</i> ( highest bar) must be to the <i>right</i> of the median (middle). This forces the mean to be to the <i>left</i> of the median. Using the inequality sign <math>&lt;</math>, write:</p> <p style="text-align: center;"><u>Mean</u> <math>&lt;</math> <span style="border: 1px solid black; padding: 2px;">median</span> <math>&lt;</math> <u>Mode</u></p>	<p style="text-align: center;"><b>EXAMPLES</b></p>  <p>In a <i>left skewed distribution</i>, the following relationship exists between the mean, median and mode:</p> <p style="text-align: center;">Mean <math>&lt;</math> <span style="border: 1px solid black; padding: 2px;">Median</span> <math>&lt;</math> Mode</p> <p><i>Note:</i> Skewed left distributions are sometimes called <b>pulled left</b> distributions.</p>

□ **CLAST EXAMPLES**

**Example**

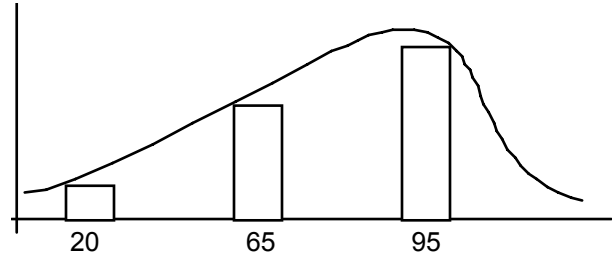
**3.** In a literature class, half the students scored 95 on an achievement test. Most of the remaining students scored 65, except for a few students who scored 20. Which of the following statements is true about the distribution of the scores?

- A. The mode equals the mean
- B. The median is greater than the mode
- C. The mode is greater than the mean
- D. The mean is greater than the mode.

**Note:** Since half the students scored 95, the *mode* is 95. The average (*mean*) of the students making 95 (half of the students) and those making 65, is **less** than 95 (the *mode*) so the mode (95) is greater than the mean. The answer is **C**.

**Solution**

This time, we do not have a diagram, so we sketch the situation as shown.



Now, we write \_\_\_\_\_ median \_\_\_\_\_

Since the mode is to the right of the middle,

$$\text{mean} < \text{median} < \text{mode}$$

This makes **C** the correct choice.

The answer "The mean is less than the median" is also correct, but it is not one of the choices.

**Example**

4. Pedro did some comparison shopping on his favorite brand of jeans. Over half of the stores priced the jeans at \$28.00. Most of the remaining stores priced them at \$29.00 except for a few stores who charged \$30.00. Which of the following statements is true about the distribution of the prices?
- A. The mean and the mode are the same.
- B. The mean is greater than the mode.
- C. The mean is less than the mode.
- D. The mean is less than the median.

**Solution**

Here we do not have a diagram either but we can deduce the following important fact:

**If a particular category contains more than half of the data, the mode = the median.**

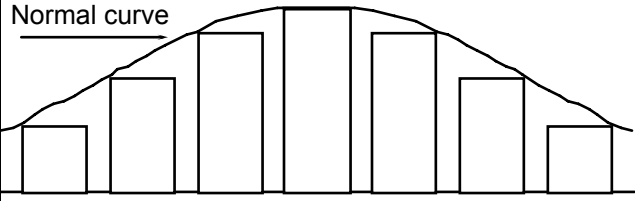
Unfortunately, this is not one of the choices.

Now, let us think about the *mean*. Since over half of the stores priced the jeans at \$28 and most of the remaining stores priced them at \$29, when you take the average of the stores charging \$28 (more than half of the stores) and the stores that priced them at \$29, that average (mean) will certainly be more than \$28. Thus, we can write:

$$\text{Mode} = \text{Median} = \$28 < \text{Mean}$$

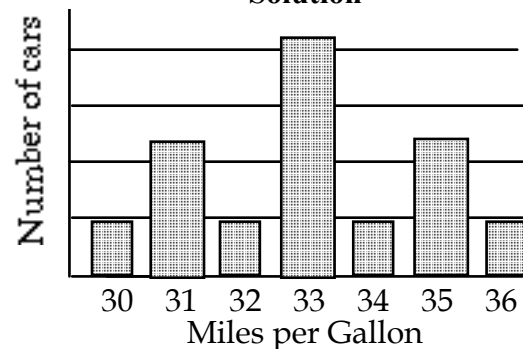
The answer is **B**.

**Note:** Examples 3 and 4 should remind you that: when **half or more than half** of the measurements fall into one category, the **mode** represents that category!

4	RELATIONSHIPS AMONG THE MEAN, MEDIAN AND MODE
<p><b>FOR A SYMMETRICAL DISTRIBUTION</b></p> <p>If the graph of the distribution is <b>symmetric (normal)</b>,</p> <p style="text-align: center;">Mean = <span style="border: 1px solid black; padding: 2px;">median</span> = mode</p>	<p><b>EXAMPLES</b></p>  <p>Normal curve</p> <p style="text-align: center;">mean = median = mode</p>

**CLAST EXAMPLE****Example**

5. The average miles per gallon obtained by a certain car are shown in the graph. Select the statement that is true about the distribution of the data.
- A. The mean is greater than the median.
- B. The mean equals the median.
- C. The mean is greater than the mode.
- D. The mode is less than the median.

**Solution**

Since the distribution is symmetric,  
Mean = Median = Mode

The answer is **B**.

We have already learned how to find the relationship between the mean, median and mode when the bar graph for the situation is provided or can be deduced from the given information. We are now ready to make inferences about the mean, median and mode using *frequency* and *cumulative frequency* tables.

### C. Applications of the Mean, Median and Mode

Objective IVD1	CLAST SAMPLE PROBLEMS	
1. The table to the right gives score and percentile rank of students taking a science test.	<b>SCORE</b>	<b>PERCENTILE RANK</b>
	80	99
A. What percent of the students scored between 60 and 80?	70	92
	60	66
	50	38
B. What percent of the scores are below 70?	40	21
	30	12
C. What percent of the scores are above 70?	20	2

A *frequency* table is a table displaying data and the *percent* or *proportion* of the time that each of the data items appears. The procedure for finding the mean, median and mode from one of these frequency tables is similar to the procedures we have used.

5	FINDING THE MEAN, MEDIAN AND MODE FROM A FREQUENCY TABLE													
<b>RULE</b>	<b>EXAMPLES</b>													
To find the <b>mode</b> from a frequency table, find the largest proportion (percent) in the table.	The table shows the proportion of persons having the indicated number of bedrooms in their homes.													
The <b>mode</b> for the number of bedrooms in the table at right is <b>3</b> , because 0.37 of the persons (the highest proportion) have <b>3</b> bedrooms.	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Number of bedrooms</th> <th style="text-align: center;">Proportion</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">0</td> <td style="text-align: center;">0.02</td> </tr> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">0.15</td> </tr> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">0.32</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">0.37</td> </tr> <tr> <td style="text-align: center;">4+</td> <td style="text-align: center;">0.14</td> </tr> </tbody> </table>		Number of bedrooms	Proportion	0	0.02	1	0.15	2	0.32	3	0.37	4+	0.14
Number of bedrooms	Proportion													
0	0.02													
1	0.15													
2	0.32													
3	0.37													
4+	0.14													
To find the <b>median</b> for a frequency table, add the proportions (percent) starting at the bottom until the sum is more than 0.50. The number for the last category is the <b>median</b> .	The <b>median</b> for the number of bedrooms is also <b>3</b> , since $0.14 + 0.37 = 0.51$ which exceeds 0.50. The category whose percent we added to exceed 0.50 was <b>3</b> .													
To find the <b>mean</b> for a frequency table:	$0 \cdot 0.02 + 1 \cdot 0.15 + 2 \cdot 0.32 + 3 \cdot 0.37 + 4 \cdot 0.14 = 2.46$ This means that the average number of bedrooms per home is 2.46.													
1. Multiply the value of the data by its proportion.														
2. Add the results														

<b>ANSWERS</b>	1. A. 33%	B. 92%	C. 7%
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## □ CLAST EXAMPLES

**Example**

6. The table shows the distribution of the number of children in all families and in Hispanic families for the year 1990. What was the median number of children for Hispanic families?

A. 0      B. 1      C. 0.5      D. 1.5

Make sure you look under the category "Hispanics" in the third column and remember that you have to start adding the proportions from the bottom. The category whose proportion is used to make the sum more than 0.50 is the answer!

**Example**

7. Use the Table in Example 6 to find the average (mean) number of children for all families.

A. 0      B. 0.86      C. 0.50      D. 1.50

A different type of CLAST question asks for the percent of the data falling *at*, *between*, *below* or *above* certain categories. We illustrate these types of problems next.

## □ CLAST EXAMPLES

**Example**

8. The table shows the percent distribution of households by income level in 1990. What percent of the families have income of at least \$35,000?

A. 47      B. 53      C. 26      D. 37

Since we want the percent of families that make *at least* \$35,000 we must consider the persons whose income start at \$35,000 (17%) and those with incomes above \$35,000 (13% + 7%).

**Solution**

Number of Children	Proportion All Families	Proportion Hispanics
0	0.51	0.36
1	0.21	0.24
2	0.19	0.22
3 or more	0.09	0.18

Add the proportions under "Hispanic" starting from the bottom. The "3 or more category" is 0.18 and the "2" category is 0.22.

Now,  $0.18 + 0.22 = 0.40$ , which is not more than 0.50. Add the "1" category, 0.24 to 0.40. Since the total is over 0.50, the answer is "1", the category used to go over 0.50. The correct response is **B**.

**Solution**

Multiply the numbers in column 1 by the numbers in column 2 and add. We have:

$$0 \cdot 0.51 = 0$$

$$1 \cdot 0.21 = 0.21$$

$$2 \cdot 0.19 = 0.38$$

$$3 \cdot 0.09 = 0.27$$

The sum is: 0.86 and the answer is **B**.

**Solution**

Income Level	Percent of families
0 - \$4999	6
\$5000 - \$9999	11
\$10,000 - \$14,999	10
\$15,000 - \$24,999	19
\$25,000 - \$34,999	16
\$35,000 - \$49,999	17
\$50,000 - \$74,999	13
\$75,000 and over	7

17% of the persons are in the \$35,000 - \$49,999 category. Count all of those and also those with higher income to obtain:

$17 + 13 + 7 = 37\%$ . The answer is **D**.



**Example**

9. Use the table of Example 8 to identify the amount for which at least 20% of the families have higher income.

- A. \$25,000                      B. \$35,000  
 C. \$50,000                      D. \$75,000

**Solution**

The percent of families in the last two categories add up to 20%. Those families make more than \$50,000 (the first value in the category). The answer is **C**.

The last type of CLAST problem uses cumulative frequency tables. Two common cumulative frequency tables are *percentiles* and *quartiles* tables. Here are some examples.

<b>6</b>	<b>PERCENTILES</b>	
	<i>PERCENTILES</i>	<i>EXAMPLES</i>
	The <b>p</b> percentile of a distribution is a value such that <b>p</b> % of the data will fall <i>at or below</i> it.	If you score in the 89%th percentile on a test, then 89% of the scores were <i>below</i> yours.

□ **CLAST EXAMPLE**

**Example**

10. The scores on the Mathematics Placement Examination have been scaled with the scores listed corresponding to the indicated percentile ranks. What percent of the students taking the test scored between 15 and 20?

- A. 72      B. 50      C. 20      D. 8

Remember that since the percentile rank for 20 is 80, a score of 20 means that 80% of the scores were below 20.

**Solution**

Score	Percentile Rank
25	99
20	80
15	72
10	50
5	30
1	1

The percentile ranks corresponding to 20 and 15 are 80 and 72, respectively. Since 80% of the scores are less than 20 and 72% of the scores are less than 15, the percent of the students scoring between 15 and 20 is  $80 - 72 = 8$ . The answer is **D**.

## Section 4.2 Exercises

### □ WARM-UPS A

- Find the mean in the sample: 7, 29, 15, 4, 3, 3, 3, 5, 3
- Find the median for the sample: 1, 21, 2, 19, 14, 2, 20, 28, 2, 18
- What is the mode in the sample? 1, 14, 10, 6, 4, 30, 29, 6, 27, 21, 6

### □ CLAST PRACTICE A

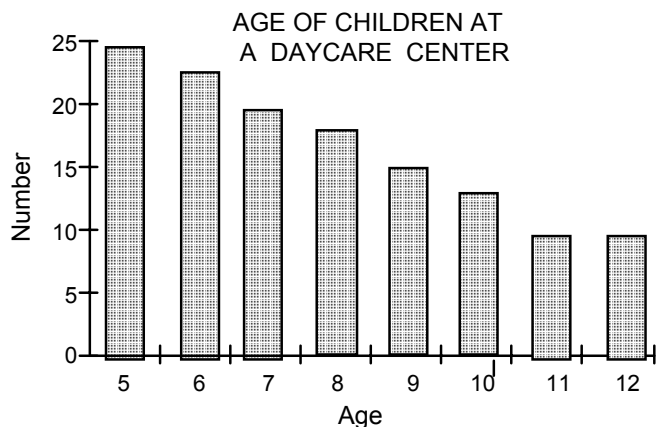
### PRACTICE PROBLEMS: Chapter 4, # 7-10

- What is the mean in the following sample? 8, 1, 10, 4, 1, 1, 14, 26, 25  
A. 1            B. 8            C. 10            D. 13.5
- What is the mean in the following sample? 3, 16, 23, 14, 24, 21, 13, 24, 24  
A. 21            B. 24            C. 13.5            D. 18
- What is the median in the following sample? 2, 18, 22, 9, 22, 12, 22, 6, 22  
A. 18            B. 22            C. 15            D. 12
- What is the median in the following sample? 30, 15, 6, 4, 6, 6, 17, 27, 30, 27  
A. 27            B. 16            C. 30            D. 15.5
- What is the mode in the following sample? 2, 30, 3, 2, 30, 2, 9, 10  
A. 30            B. 11            C. 2            D. 10

### □ WARM-UPS B

The diagram will be used in Problems 9-12

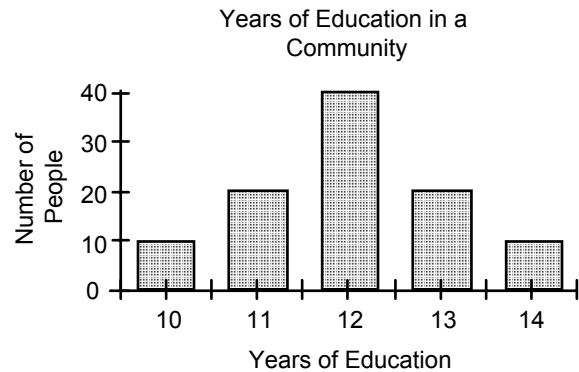
- What is the relationship between the mean and the median?
- What is the relationship between the mean and the mode?
- What is the relationship between the median and the mode?
- What is the mode for the data?



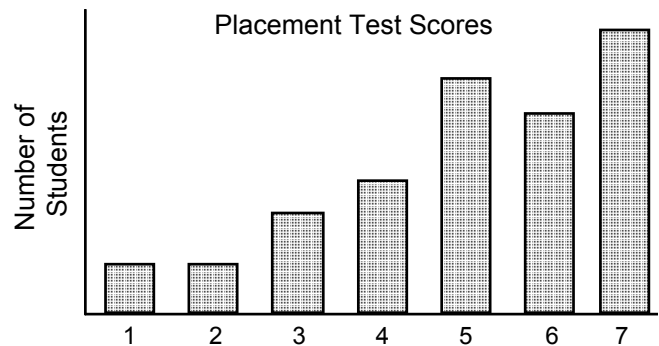
□ **CLAST PRACTICE B**

**PRACTICE PROBLEMS: Chapter 4, # 11-12**

13. The graph at the right represents the distribution of years of education in a small community. Select the statement that is true about this distribution.
- A. The mode is greater than the mean.
  - B. The mean is greater than the median.
  - C. The mode approximately equals the mean.
  - D. The median is greater than the mode.



14. The graph at the right represents the distribution of scores on a placement test for students at Central High school. Select the statement that is true about this distribution.
- A. The mean is greater than the mode.
  - B. The mean and the median are the same.
  - C. The median is greater than the mode.
  - D. The mean is less than the mode.



15. In a mathematics class, half the students scored 74 on an achievement test. Most of the remaining students scored 78, except for a few students who scored 22. Which of the following statements is true about the distribution of the scores?
- A. The mean and the median are the same.
  - B. The mean is less than the median.
  - C. The median is greater than the mode.
  - D. The mean is greater than the mode.
16. Members of a foreign language class were asked how many foreign language courses they had taken. Half the class had taken three semesters. Of the remaining students, most had taken two semesters except for a few students who had taken no foreign language course. Which of the following statements is true about the distribution of responses?
- A. The mode is equal to the median.
  - B. The mode is greater than the mean.
  - C. The mean is greater than the mode.
  - D. The mode is equal to the mean.

□ **WARM-UPS C**

**The Table shows the percent distribution for the size of households in 1970**

17. Find the mode for the household size.  
 18. Find the median for the household size.  
 19. Find the mean for the household size.

Size of Household	Percent
1	25
2	32
3	17
4	16
5	7
6 or more	3

**The Table shows the result of a survey in which people were asked to count the number of unmatched socks found in their drawer.**

20. What was the median number of unmatched socks found in the drawer by males?

Number of Un-matched Socks	Percent (Males)	Percent (Females)
0	38	81
1	5	4
2	25	2
3	13	4
4 or more	19	9

21. What was the median number of unmatched socks found in the drawer by females?  
 22. What was the mean number of unmatched socks found in the drawer by males?  
 23. What was the mean number of unmatched socks found in the drawer by females?

**The table shows the distribution of the total family income of blacks in 1990.**

24. What percent of the families have incomes of at least \$50,000?  
 25. What percent of the families have incomes of less than \$10,000?  
 26. What percent of the families have incomes between \$25,000 and \$49,999?

Income Level	Percent of families
0 - \$4999	12
\$5000 - \$9999	14
\$10,000 - \$14,999	11
\$15,000 - \$24,999	20
\$25,000 - \$34,999	14
\$35,000 - \$49,999	15
\$50,000 - \$74,999	10
\$75,000 and over	4

27. Identify the amount above which 29% of the families have higher income.  
 28. Identify the amount below which 26% of the families have lower income.

**CLAST PRACTICE C**

The table shows the results of a survey asking smokers the number of times they have tried to quit smoking.

**PRACTICE PROBLEMS: Chapter 4, #13**

Number of Times	Percent (Males)	Percent (Females)
0	1	5
1	16	15
2	13	29
3	23	20
4	47	31

29. What is the mode for the number of times females tried to quit smoking?

- A. 1      B. 2      C. 3      D. 4

30. What was the median number of times males tried to quit smoking?

- A. 1              B. 2              C. 3              D. 4

31. What was the mean number of times females tried to quit smoking?

- A. 2.61          B. 2.57          C. 2.99          D. 3

The table shows the result of a survey in which respondents were asked to count the number of different cereal boxes they had in their pantry.

Number of Boxes	Percent
0	4
1-2	45
3-4	26
5-6	19
7 or more	6

32. What percent of the families had more than 2 boxes?

- A. 4      B. 45      C. 49      D. 51

33. What percent of the families had less than 5 boxes?

- A. 26          B. 25          C. 75          D. 49

The Table shows the grades and the percentile rank for a Mathematics Test.

Grade	Percentile Rank
95	99
90	85
85	82
80	78
75	70
70	60
65	50
60	40

34. Find the grade such that 78% scored less than that grade

- A. 80      B. 85      C. 90      D. 75

35. Find the grade such that 40% of the scores are above it

- A. 80      B. 85      C. 90      D. 70.

36. What percent of the students made a grade between 75 and 90?

- A. 20          B. 15          C. 70          D. 85

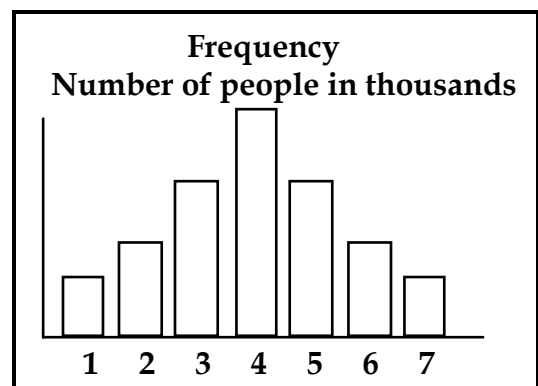
The table shows the distribution of families by income in Chicago, Illinois

Income Level	Percent of families
0 - \$9,999	3
\$10,000 - \$14,999	4
\$15,000 - \$19,999	23
\$20,000 - \$24,999	43
\$25,000 - \$34,999	10
\$35,000 - \$49,999	8
\$50,000 - \$79,999	5
\$80,000 - \$119,000	3
\$120,000 and over	1

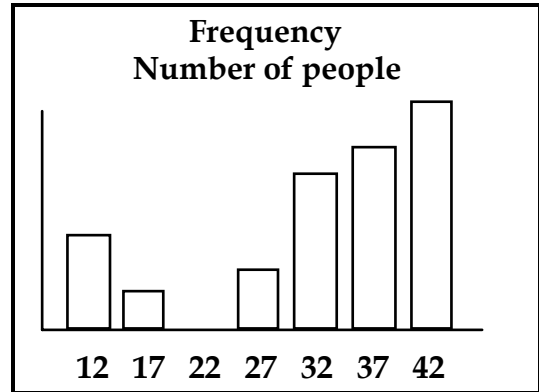
37. Identify the amount below which 73% of the families in Chicago have lower incomes.
- A. \$15,000                      B. \$20,000  
C. \$35,000                      D. \$25,000

### EXTRA CLAST PRACTICE

38. On an algebra test, half of the students scored 75. Of the remaining students, some scored 70 and the same number scored 80. Select the statement that is true about the distribution of test scores.
- A. The median is greater than the mode.                      B. The mode is less than the mean.  
C. The mode is equal to the mean.                              D. The mean is greater than the mode.
39. At Spoc's Music half of the CD's cost \$12. Most of the remaining CD's cost \$9, except for a few that are on sale for \$6. Which of the following is true about the distribution of the cost of CD's?
- A. The mean is equal to the median.                      B. The mean is less than the median.  
C. The mean is greater than the mode.                      D. The mode is less than the median.
40. The graph shows the population of a city over a seven-year period. Select the statement that is true about the distribution.
- A. The mode is equal to the mean.  
B. The median is less than the mode.  
C. The mean is greater than the median.  
D. The mode is greater than the mean.



41. The graph represents the ages of a random selection of people who had the flu in a certain year. Select the statement that is true about the distribution of ages.



- A. The median and the mode are the same.
- B. The median is greater than the mode.
- C. The mode and the mean are the same.
- D. The mode is greater than the mean.
42. Tyrone found out that over half of the stores he surveyed sold his favorite brand of tennis shoes for \$125. Most of the remaining stores priced the shoes at \$120, except for a few who charged \$100. Which of the following is true about the distribution of prices?
- A. The mean and the mode are the same.
- B. The mean is greater than the mode.
- C. The mean is less than the mode.
- D. The mean is greater than the median.

## 4.3 COUNTING PROBLEMS AND PROBABILITY

Suppose you are having breakfast at a restaurant. You can have your eggs fried, scrambled or poached and your toast can be whole wheat or rye. How many choices do you have for your eggs and toast? If the server picks one of the choices at random, what is the probability that you will end up with your favorite selection? To solve these two problems you have to know how to *count* the ways in which an event can happen and how to calculate the *probability* of an event. These are two of the CLAST competencies and we study them next.

### A. Using the Fundamental Counting Principle

#### Objective ID3

#### CLAST SAMPLE PROBLEMS

1. There are three airlines that fly from A to B and four that fly from B to C. Assuming all flights are available, how many different choices do you have to fly from A to C connecting at B?
2. 5 chairs are arranged in a row at the theater. In how many ways can 5 people be seated in these 5 chairs?
3. 5 automobiles are entered in a race. Assuming there are no ties, in how many ways can the first 3 finishers come in?
4. How many faculty committees consisting of 3 members can be formed if there are 6 faculty members available?
5. 6 males and 4 females have applied for work at a company. How many groups of 5 employees consisting of 3 males and 2 females are possible?

1	<b>THE FUNDAMENTAL COUNTING PRINCIPLE (FCP)</b>	
	<i><b>RULE</b></i>	<i><b>EXAMPLES</b></i>
	<p>If an event can occur in <b>m</b> ways and then a second event can occur in <b>n</b> ways and a third event can occur in <b>r</b> ways and so on, then the sequence of events can occur in</p> <p style="text-align: center;"><b><math>m \times n \times r</math> ways</b></p>	<p>If you can have your eggs fried, scrambled or poached (3 choices) and your toast can be whole wheat or rye (2 choices), the number of choices that you have for your eggs and toast is:</p> <p style="text-align: center;"><b><math>2 \times 3 = 6</math></b></p>

**ANSWERS**    1. 12    2. 120    3. 60    4. 20    5. 120



The Fundamental Counting Principle (FCP) is used to count the number of possible outcomes for a sequence of events. Here is how it is done.

2	<b>USING THE FUNDAMENTAL COUNTING PRINCIPLE</b>	
<b><i>PROCEDURE TO USE THE FCP</i></b>	<b><i>EXAMPLES</i></b>	
<p>To count the number of ways in which a sequence of events can happen:</p> <ol style="list-style-type: none"> <li>1. Draw a blank (____) to represent each of the individual events.</li> <li>2. Fill each of the blanks with the number of ways in which the individual event can occur.</li> <li>3. Multiply the numbers in the blanks.</li> </ol>	<p>A prospective computer buyer can choose from three different models (IBM, Mac or a clone) each coming with four different types of software and either with or without a laser printer. How many options are available?</p> <ol style="list-style-type: none"> <li>1. Draw a blank to represent each of the individual events.</li> </ol> $\underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$ <p style="text-align: center;"><b>Computer      Software      Printer</b></p> <ol style="list-style-type: none"> <li>2. Fill each of the blanks with the number of ways in which the event can occur: 3 computers, 4 types of software and 2 ways to choose the printer.</li> <li>3. Multiply the numbers in the blanks</li> </ol> $\frac{\underline{3}}{\text{Computer}} \times \frac{\underline{4}}{\text{Software}} \times \frac{\underline{2}}{\text{Printer}} = 24$ <p>There are 24 choices.</p>	

□ **CLAST EXAMPLES**

**Example**

1. Students are asked to rank 4 instructors from best to worst. How many different ways can the four instructors be ranked?

A. 1      B. 4      C. 64      D. 24

In this Example the number of choices *decreases* by 1 each time since, for example, you have 4 choices for number one, but after you select an instructor to fill that slot you have only 3 choices for number 2.

**Solution**

1. There are *four* individual events.

$$\underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$$

**Pick # 1      Pick # 2      Pick # 3      Pick # 4**

2. We have four choices for #1. We then have three choices for #2, two choices for #3 and only one choice for #4.
3.  $\frac{4}{\text{Pick \# 1}} \times \frac{3}{\text{Pick \# 2}} \times \frac{2}{\text{Pick \# 3}} \times \frac{1}{\text{Pick \# 4}} = 24$

The answer is **D**.

**Example**

2. Alpha airline has 3 flights from Florida to Atlanta and 6 flights from Atlanta to New York. In how many ways can you fly Alpha from Florida to New York stopping in Atlanta?

A. 36      B. 720      C. 18      D. 4320

**Solution**

1. There are *two* events. Selecting a flight to Atlanta and then selecting a flight to New York.

Atlanta      New York

2. We have three choices for going to Atlanta and then six choices for going to New York.

$$3. \frac{3}{\text{Atlanta}} \times \frac{6}{\text{New York}} = 18$$

The answer is **C**.

**Example**

3. A customer can order a hamburger with or without each of the following condiments: onions, mustard, catsup, pickle, and mayonnaise. How many different combinations of these options are available?

A. 1      B. 8      C. 16      D. 32

**Solution**

1. There are *five* events. Selecting onions, mustard, catsup, pickle and mayonnaise.  
2. Each of the events has 2 choices: Yes; No

Y or N   Y or N   Y or N   Y or N   Y or N

$$3. \frac{2}{\text{Y or N}} \times \frac{2}{\text{Y or N}} \times \frac{2}{\text{Y or N}} \times \frac{2}{\text{Y or N}} \times \frac{2}{\text{Y or N}} = 32$$

The answer is **D**.

The second type of CLAST counting competency requires counting the number of outcomes of an event that involves subsets, or *combinations*, of objects. When dealing with combinations, the **order** of the objects does not matter. For example, a committee consisting of Bob, Chad and Dana is the same committee as the one consisting of Dana, Chad and Bob because rearranging (ordering) the people in a different manner does not yield a different result. How many ways can groups (combinations) of 2 letters be chosen from the letters {a, b, c, d}? Since we only have 4 letters, we can list the 6 combinations as: ab, ac, ad, bc, bd and cd. Note that the number of possible arrangements of the two letters chosen from the 4 letters is:  $\frac{4 \times 3}{2} = 12$

Here they are:      ab                  ba                  ac                  ca                  ad                  da  
                                 bc                  cb                  bd                  db                  cd                  dc

Since there are only **6** different groups of combinations or letters, not 12, each of the combinations have been counted 2 times instead of once. These extra arrangements can be eliminated by

dividing  $12 = 4 \times 3$  by  $2 = 2 \times 1$  and writing:  $\frac{4 \times 3}{2 \times 1} = 6$

Here is the procedure we need:

3	<b>COMPUTING THE NUMBER OF COMBINATIONS</b>	
	<b>RULE</b>	<b>EXAMPLE</b>
	<ol style="list-style-type: none"> <li>1. Find the total number of objects from which you can choose (larger number).</li> <li>2. Find the number of objects to be included in each group (smaller number).</li> <li>3. The number of blanks on the numerator and denominator is the smaller number. Place a multiplication sign between each blank.</li> <li>4. Fill all of the blanks in the numerator starting with the larger number and going down by one. Fill the blanks in the denominator starting with the smaller number and going down by one.</li> <li>5. Reduce the fractions by canceling out common factors and then multiply.</li> </ol>	<p>The number of committees consisting of 3 persons that can be formed if there are 6 persons to choose from is:</p> $\frac{\overset{2}{6} \times \overset{2}{5} \times \overset{2}{4}}{\underset{1}{3} \times \underset{1}{2} \times \underset{1}{1}} = 20$ <ol style="list-style-type: none"> <li>1. Here the larger number is <b>6</b>, and</li> <li>2. The smaller number is <b>3</b></li> <li>3. There are 3 blanks in the numerator and 3 blanks in the denominator.</li> <li>4. The blanks in the numerator were filled starting with the larger number (6), going down by one and placing a multiplication sign between the numbers. The denominator was filled starting with the smaller number (3), going down by one, and placing a multiplication sign between the numbers.</li> <li>5. The final fraction was then reduced and the result multiplied. Thus, we can form 20 committees of 3 if there are 6 persons to choose from.</li> </ol>

To write the combination formula in a more succinct manner, we introduce *factorial notation*.

T	<b>TERMINOLOGY -- FACTORIAL NOTATION</b>	
	<b><i>n</i> FACTORIAL</b>	<b>EXAMPLES</b>
	<p><i>n</i> factorial, denoted by <i>n!</i>, is defined as:</p> $n! = n \cdot (n - 1) \cdot (n - 2) \dots \cdot 1$ <p>We also define <b>0! = 1</b></p>	$1! = 1 \qquad 2! = 2 \cdot 1 = 2$ $3! = 3 \cdot 2 \cdot 1 = 6 \qquad 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ <p>Note that: <math>7! = 7 \cdot 6! = 7 \cdot 6 \cdot 5! = 7 \cdot 6 \cdot 5 \cdot 4!</math></p> <p>and <math>\frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 7 \cdot 6 \cdot 5 = 210</math></p>

We are now ready to discuss the combination formula using factorial notation.

4	COMBINATION FORMULA $C(n, r)$
<p style="text-align: center;"><b>RULE</b></p> <p>The number of subsets (<i>combinations</i>) of <math>r</math> objects that can be made from a set of <math>n</math> objects is:</p> $C(n, r) = \frac{n!}{r!(n-r)!}$ <p style="text-align: center;">Available      Used</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <p>What is the number of subsets of 4 coins that can be made from a set of 6 coins? Here <math>r = 4</math>, <math>n = 6</math> and</p> $C(6, 4) = \frac{6!}{4!(6-4)!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot 2!} = \frac{30}{2} = 15$ <p>Note: If you use the rule given in box 2, we get <math>\frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1} = 15</math>, same answer!</p>
<p>Note that this formula can be simplified.</p> $C(n, r) = \frac{n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-r+1) \cdot \cancel{(n-r)!}}{r! \cdot \cancel{(n-r)!}}$ $= \frac{n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-r+1)}{r \cdot (r-1) \cdot (r-2) \cdot \dots \cdot 1}$ <p>The <i>numerator</i> is a product starting with <math>n</math> and descending to one more than the difference of <math>n</math> and <math>r</math>. The <i>denominator</i> is <math>r!</math></p> <p>Make sure you simplify this fraction before doing the multiplication.</p>	<p>Find <math>C(6, 4)</math> using this formula.</p> <p>Here <math>r = 4</math>, <math>n = 6</math> and <math>n - r + 1 = 6 - 4 + 1 = 3</math> The product in the numerator starts with <math>n = 6</math> and ends with <math>n - r + 1 = 3</math>. The denominator is <math>4!</math>. Thus,</p> $C(6, 4) = \frac{\overset{3}{6} \cdot 5 \cdot \cancel{4} \cdot \cancel{3}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 15$ <p>The same answer as before.</p> <p>Note that we simplified the fraction before multiplying.</p>

Note: If you prefer the formula for computing combinations given in box 2, use it!

#### □ CLAST EXAMPLE

- |  | Example           |
|--|-------------------|
| <p>4. The equal opportunity committee consists of two men and two women. How many different committees can be formed if 5 men and 6 women are willing to serve on the committee?</p> |                   |
| <p>A.     4</p>  | <p>B.     11</p>  |
| <p>C.     30</p>   | <p>D.     150</p> |

- Solution**
- We still use the FCP. There are two events:
- \_\_\_\_\_                      \_\_\_\_\_  
Select 2 men              Select 2 women
  - There are  $C(5, 2)$  ways to select the men and  $C(6, 2)$  ways to select the women.
  - We have to find  $C(5, 2) \times C(6, 2)$   
For  $C(5, 2)$ ,  $n = 5$ ,  $r = 2$ ,  $n - r + 1 = 4$   
For  $C(6, 2)$ ,  $n = 6$ ,  $r = 2$ ,  $n - r + 1 = 5$   
Thus,  $C(5, 2) \square C(6, 2) = \frac{5 \cdot 4}{2 \cdot 1} \times \frac{6 \cdot 5}{2 \cdot 1}$   
 $= 10 \times 15 = 150$
- The answer is **D**.

## B. Computing Probabilities

### Objective IID3

### CLAST SAMPLE PROBLEMS

- A student is taking English, Math and Science. The student has 3 English books, 2 Math books and 4 Science books. The student selects one book at random. Find:
  - The probability that the book selected is a Science book.
  - The probability that the book selected is not a Math book
- The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are written on slips of paper and placed in a hat. One number is picked at random. Find:
  - The probability that the number picked is less than 10
  - The probability that the number picked is more than 10
- Five percent of the computer chips manufactured by a certain company are defective. A chip is selected at random from a shipment of chips. What is the probability that the chip is not defective?
- The probability that a college student chosen at random at University A is absent on a given day is 0.05. If two students are chosen at random at University A, what is the probability that they are both absent?
- A shipment of 6 computer chips contains 2 defective chips. Three chips are chosen at random and without replacement. Find the probability of choosing 3 good chips.
- During their freshman year, 85% of the students take English, 70% take Math and 65% take both. If a student is chosen at random, what is the probability that the student took an English or a Math course during the freshman year?

The counting techniques we have studied are used to compute the probability of an event. Here is the rule we need.

5	FINDING THE PROBABILITY OF AN EVENT	
<b>RULE</b>	<b>EXAMPLE</b>	
<p>The probability <math>P(E)</math> that event E will occur is computed as follows:</p> <ol style="list-style-type: none"> <li>Find the number of favorable ways in which E can occur.</li> <li>Find the total number of possible outcomes.</li> <li><math>P(E) = \frac{\text{number of favorable ways that E can occur}}{\text{total number of possible outcomes}}</math></li> </ol> <p>Note that your answer <math>P(E)</math> must be between 0 and 1, that is, <math>0 \leq P(E) \leq 1</math></p>	<p>A student randomly selects a day of the week for studying. What is the probability that the student will select a week-end day (Saturday or Sunday)?</p> <ol style="list-style-type: none"> <li>Let W be the event in which the student selects a week-end day. There are 2 ways in which W can occur.</li> <li>There are 7 possible outcomes (You can pick any of the 7 days of the week)</li> <li><math>P(W) = \frac{2}{7}</math></li> </ol>	

### ANSWERS

- |                                      |                  |                           |
|--------------------------------------|------------------|---------------------------|
| 1. A. $\frac{4}{9}$ B. $\frac{7}{9}$ | 2. A. 1 B. 0     | 3. 0.95                   |
| 4. 0.0025                            | 5. $\frac{1}{5}$ | 6. 0.90 or $\frac{9}{10}$ |

The CLAST asks three types of questions:

1. Given an event A, find the probability of A not occurring.
2. Given two events A and B, find the probability of A and B
3. Given two events A and B, find the probability of A or B

6	<b>FINDING THE PROBABILITY OF A <u>not</u> OCCURRING</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
<p>The probability that an event A will <b>not</b> occur is given by</p> $\mathbf{P(\text{not } A) = 1 - P(A)}.$ <p>This means that to find the probability of A <u>not</u> occurring you subtract the probability of A from 1.</p>	<p>If the probability that it will rain tomorrow is <math>\frac{7}{10}</math>, the probability that it will <u>not</u> rain tomorrow is <math>1 - \frac{7}{10} = \frac{3}{10}</math>.</p>	
7	<b>FINDING THE PROBABILITY OF A <u>and</u> B</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
<p>If A and B are two <i>independent</i> events</p> $\mathbf{P(A \text{ and } B) = P(A) \cdot P(B)}$ <p>This means that to calculate the probability of two independent events A and B you <b>multiply</b> their individual probabilities.</p> <p><i>Note:</i> The probabilities can be given as fractions or as percents that can be converted to fractions.</p>	<p>Ten percent of the tires produced by a tire company are defective. If two tires are selected at random. What is the probability that they are both good?</p> <p>Since ten percent of the tires are defective, <math>P(D) = \frac{10}{100} = \frac{1}{10}</math> and thus, the probability that the tire is good is <math>P(G) = 1 - \frac{1}{10} = \frac{9}{10}</math>.</p> <p>The probability that both tires are good is:</p> $P(G) \cdot P(G) = \frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$	
<p>If A and B are <i>dependent</i> events (the occurrence of one event affects the probability of the other event),</p> $\mathbf{P(A \text{ and } B) = P(A) \cdot P(B A)}$ <p>where <math>P(B A)</math> means the probability of B after A has occurred.</p>	<p>A car lot has ten cars, three of which are red. If two cars are sold on a particular day, what is the probability that they are both red?</p> <p>Let <math>P(F)</math> be the probability that the first car is red and <math>P(S)</math> the probability that the second car is red. We need <math>P(F \text{ and } S)</math>.</p> <p>This time, the events are not independent. (Selecting a red car first with probability <math>\frac{3}{10}</math> changes the probability of selecting a red car next to <math>\frac{2}{9}</math>, since 2 out of the 9 cars left are red). Thus, <math>P(F) = \frac{3}{10}</math>, <math>P(S F) = \frac{2}{9}</math></p> <p>and <math>P(F \text{ and } S) = P(F) \cdot P(S F) = \frac{3}{10} \cdot \frac{2}{9} = \frac{1}{15}</math></p>	

## □ CLAST EXAMPLES

**Example**

5. A survey at a particular college indicated that 90% of the students taking the Essay portion of the CLAST passed. If only 70% of those taking the Math portion passed, what is the probability that a randomly selected student at this college will fail both the Essay and the Math portion of the CLAST?

- A.  $\frac{63}{100}$                       B.  $\frac{1}{100}$   
 C.  $\frac{3}{100}$                         D.  $\frac{63}{1000}$

**Example**

6. Ten percent of the students at a University have blue eyes. If two students are selected at random from this University, what is the probability that at least one of them has blue eyes?

- A.  $\frac{19}{100}$                         B.  $\frac{81}{100}$   
 C.  $\frac{1}{5}$                             D.  $\frac{1}{10}$

Hint: Let L be the event of selecting at least one blue eyed student.

$$P(L) = 1 - P(\text{not } L)$$

What does "not L" mean?

**Solution**

Since the probability of passing the Essay is  $\frac{9}{10}$  the probability of failing it is  $\frac{1}{10}$ .

Similarly, the probability of failing the Math is  $1 - \frac{7}{10} = \frac{3}{10}$ . Thus, the probability of failing both the Essay and the Math is:

$$\begin{aligned} P(\text{FE and FM}) &= P(\text{FE}) \cdot P(\text{FM}) \\ &= \frac{1}{10} \cdot \frac{3}{10} \\ &= \frac{3}{100} \end{aligned}$$

The answer is C.

**Solution**

Let L be the event in which at least one of the students has blue eyes. Then "not L" is the event in which neither of the students has blue eyes. This means that the first student does not have blue eyes and the second student does not have blue eyes. Thus,

$$\begin{aligned} P(L) &= 1 - P(\text{not } L) \\ &= 1 - P(\text{not B}) \cdot P(\text{not B}) \\ &= 1 - \frac{9}{10} \cdot \frac{9}{10} \\ &= 1 - \frac{81}{100} \\ &= \frac{19}{100} \end{aligned}$$

The answer is A.

We have studied two types of events connected by the word "**and**": *independent* and *dependent*. Events connected by the word "**or**" can sometimes be "mutually exclusive". For example, passing the Math and passing the Essay portions of the CLAST are not mutually exclusive, but passing the Math portion and failing the Math portion are. Here is the rule we need to find the probability of events connected by the word "or".

8	FINDING THE PROBABILITY OF A or B	
<b>RULE</b>	<b>EXAMPLE</b>	
<p>If A and B are events,</p> <p><b><math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math></b></p> <p>When <b><math>P(A \text{ and } B) = 0</math></b>, A and B are called mutually exclusive and</p> <p style="text-align: center;"><b><math>P(A \text{ or } B) = P(A) + P(B)</math>.</b></p>	<p>If the probability of passing Math is 70%, the probability of passing English 80% and the probability of passing both Math and English is 60%, the probability of passing Math or English is</p> <p><math>P(M \text{ or } E) = P(M) + P(E) - P(M \text{ and } E)</math>  <math>= 70\% + 80\% - 60\% = 90\%</math></p>	

□ **CLAST EXAMPLE****Example**

7. Two common sources of protein for US adults are beans and meat. If 75% of US adults eat meat, 80% eat beans and 70% eat both meat and beans, what is the probability that a randomly selected adult in the US eats meat or beans?

- A.  $\frac{4}{5}$                       B.  $\frac{3}{4}$
- C.  $\frac{17}{20}$                       D.  $\frac{3}{20}$

**Solution**

We want to find :

$$P(M \text{ or } B) = P(M) + P(B) - P(M \text{ and } B)$$

$$= 75\% + 80\% - 70\% = 85\%$$

Since all the answers are given as fractions, change 85% to a fraction in reduced form.

$$85\% = \frac{85}{100} = \frac{17}{20}$$

The answer is C.

## C. Applications of Probability

### Objective IVD2

### CLAST SAMPLE PROBLEMS

- The table shows the method used to ship packages in two weight classes. What is the probability that a randomly selected package:
  - is less than 2 pounds and shipped **US**?
  - is not shipped **UPS**?
- Referring to the table, if it is known that a package was not shipped **UPS** or **FedX**, what is the probability that the package weighed at least 2 pounds?
- ABC company records show that 60% of payments are made by credit card, 15% cash and 25% check. How many of the next 500 payments would one expect to be made with cash?

	<u>UPS</u>	<u>FedX</u>	<u>US</u>
2 pounds and over	27%	8%	11%
Less than 2 pounds	32%	7%	15%

### ANSWERS

1. A. 15%    B.  $\frac{41}{100}$  or 0.41                      2.  $\frac{11}{36}$                       3. 75



The frequencies of many events are given in a table as percents, decimals or whole numbers. All of the preceding rules still apply in finding probabilities. For example, the table gives the number of males and females in a survey falling into various salary classifications.

The probability that a person selected at random from those surveyed:

Had a low salary is:  $\frac{240}{1500} = \frac{4}{25}$

Was a female is:  $\frac{660}{1500} = \frac{11}{25}$

Was not a female is:  $1 - \frac{660}{1500} = \frac{14}{25}$

SALARY	SEX		Totals
	Male	Female	
Low	40	200	240
Average	300	160	460
High	500	300	800
Totals 840	660	1500	

□ **CLAST EXAMPLES**

**Example**

8. The table gives the percent of students at a university by sex and student classification.

	Freshmen	Sophomore	Junior	
<b>Senior</b>				
Male	16%	13%	10%	11%
Female	14%	15%	12%	9%

Find the probability that a randomly selected university student is a senior.

- A. 0.20
- B. 0.30
- C. 0.52
- D. 0.49

**Example**

9. Referring to the table in Example 8, find the probability that a randomly selected university student is a NOT a senior, if it is known that the student is a male.

- A. 0.80
- B. 0.11
- C. 0.22
- D. 0.78

**Solution**

11% + 9% = 20% of the students are seniors.

Thus,  $P(\text{Senior}) = \frac{20}{100}$

Since all answers are given as decimals, we have to convert  $\frac{20}{100}$  to a decimal.

$\frac{20}{100} = 0.20$  and the answer is **A**.

**Solution**

The percent of males is 16+13+10+11 = 50  
Of these 11% are seniors, which means that 39% are not, so the probability of randomly selecting a student that is NOT a senior, if the

student is male, is  $\frac{39}{50} = \frac{78}{100} = 0.78$

The answer is **D**.

**Example**

10. The following is a distribution of the causes of fires nationally:

<u>CAUSE</u>	<u>PERCENT OF ALL FIRES</u>
Cooking	31
Smoking	15
Appliances	18
Heating System	9
Electrical System	8
Other	19

If it is known that a fire has NOT been caused by the heating system or electrical system, find that probability that it was caused by appliances.

- A.  $\frac{31}{83}$                       B.  $\frac{18}{83}$   
 C.  $\frac{31}{100}$                       D.  $\frac{9}{50}$

**Solution**

If the fire was NOT caused by the heating (9%) or the electrical (8%) systems there is an 83% (100% - 9% - 8%) probability that the fire was caused by something else. Of that 83%, 18% were caused by appliances, thus, the probability that a fire was caused by appliances if it is known that the fire has NOT been caused by the heating or electrical system is:

$$\frac{18\%}{100\% - 9\% - 8\%} = \frac{18}{83}$$

The answer is **B**.

Note that in this type of problem the denominator is obtained by subtracting from 100% the percents corresponding to the eliminated categories.

**Example**

11. Referring to the Table in Example 10, of the next 200 fires how many would you expect to be caused by Cooking or Heating systems?

- A. 40                              B. 80  
 C. 120                              D. 240

**Solution**

The percent of all fires caused by Cooking or Heating is 31% + 9% = 40%. Thus, if there are 200 fires, the number expected to be caused by Cooking or Heating is:

$$40\% \text{ of } 200 = 0.40 \square 200 = 80$$

The correct answer is **B**.

## Section 4.3 Exercises

### □ WARM-UPS A

- Five restaurants are to be ranked from best to worst. How many different rankings are possible?
- A student has 6 books for her six classes. In how many different ways can she stack them in a pile containing the six books?
- There are 4 cars entered in a race. If there are no ties, in how many ways can the cars finish?
- A diner can select from 5 entrees and 3 desserts at a local restaurant. How many different meals consisting of an entree and a dessert can a diner select?
- A customer can buy a house with or without the following options: pool, two car garage, security system, den and sprinkling system. How many different combinations of these options are available?
- Five brands of cheese are to be tested in pairs. How many tests have to be made in order to cover all possible pairs?
- A pizza parlor has five different toppings for their pizzas. How many different pizzas with three toppings are possible?
- A swim team consisting of two swimmers and two divers is to be selected from 5 swimmers and 4 divers. How many different teams are possible?

### □ CLAST PRACTICE A

### PRACTICE PROBLEMS: Chapter 4, #14-15

- In how many different ways can four persons be lined up at the check-out counter at the supermarket?
 

A. 4                      B. 10                      C. 16                      D. 24
- Five runners are competing in a race. If there are no ties, in how many ways can the race end?
 

A. 5                      B. 20                      C. 120                      D. 25
- Five runners are competing in a race. If there are no ties, in how many ways can first, second and third place be awarded?
 

A. 5                      B. 60                      C. 120                      D. 125

12. There are five qualified applicants for two executive jobs. How many different sets of two applicants can be selected from the five?
- A. 5                      B. 25                      C. 10                      D. 20
13. From a group of eight vans and seven cars, a car and a van are to be selected for purchase. How many different sets of cars and vans are possible?
- A. 112                      B. 28                      C. 84                      D. 56
14. A University requires that a student take 2 math courses from 5 offered by the Mathematics department and 3 courses from 6 offered by the English department. How many combinations are possible for the 5 courses, 2 Math and 3 English?
- A. 200                      B. 6                      C. 30                      D. 600
15. A panel of judges is to consist of 2 women and 5 men. A list of potential judges has 5 women and 8 men on it. How many different panels could be created from this list?
- A. 10                      B. 112                      C. 560                      D. 70

□ **WARM-UPS B**

16. A desk has five drawers. Your pen is in one of them. If a drawer is selected at random, what is the probability that your pen is in it?
17. Referring to Exercise 16, find the probability that your pen is not in the chosen drawer.
18. You go to the movies with 4 other friends. Two persons from your group are selected to get free tickets. What is the probability that you are NOT one of the persons selected?
19. Referring to Exercise 18, what is the probability that you are one of the persons selected?
20. A restaurant has 5 flavors of ice cream and 2 kinds of pie. If the server selects a piece of pie and ice cream at random, what is the probability that you get your favorite selection? (Assume your favorite selection is one of the available ones.)
21. Ten percent of the students in a certain class will not pass. If two students are selected at random from the class, what is the probability that both will pass?
22. A box contains ten computer disks, two of which are defective. If two disks are selected at random from the box, find the probability that they are both defective.

23. Three percent of the video games sold have audio problems, eight percent have either audio or video problems but only one percent have both audio and video trouble. If a video game is sold, what is the probability that it has video trouble?
24. A car dealership conducted a survey indicating that 45% of the customers had seen an advertisement for a particular car, 50% eventually bought the car and 25% had neither seen the ad nor did they buy the car. What is the probability that a person selected at random from those surveyed had seen the ad and bought the car?
25. The Florida Tourist Commission estimates that the probability that a person visiting Florida will visit Disney World, Busch Gardens, or both with probabilities 0.5, 0.3 and 0.2 respectively. What is the probability that a person visiting Florida will visit Disney World or Busch Gardens?

□ **CLAST PRACTICE B**

**PRACTICE PROBLEMS: Chapter 16-19**

26. The probability of getting an A in Mr. Z's class is 22%. What is the probability of NOT getting an A?
- A. 78%                      B. 56%                      C. 68%                      D. 88%
27. A box contains three red, six white and four blue balls. Two balls are drawn from the box at random without replacement. What is the probability that neither ball is red?
- A.  $\frac{10}{13} \times \frac{11}{12}$                       B.  $\frac{10}{13} \times \frac{9}{12}$                       C.  $\frac{10}{13} \times \frac{10}{13}$                       D.  $\frac{10}{13} \times \frac{2}{13}$
28. Two representatives are chosen at random from a group of 12 students that has 6 girls and 6 boys. What is the probability that the representatives selected are either both boys or both girls?
- A.  $\frac{1}{3} \times \frac{1}{3}$                       B.  $\frac{1}{3} + \frac{1}{3}$                       C.  $2 \times \frac{1}{2} \times \frac{5}{11}$                       D.  $\frac{1}{2} \times \frac{5}{11}$
- 29.** Research shows that in the US, 51% of adults aged 18-21 work full time, 27% are students, and 24% both work full time and are students. If you choose a person from this age group at random, what is the probability that the person works full time, is a student, or both?
- A. 78%                      B. 24%                      C. 54%                      D. 51%
30. Ten percent of the baseballs manufactured by a factory are defective. If two baseballs are selected at random from those produced by the company, what is the probability that both baseballs are defective?
- A.  $\frac{1}{5}$                       B.  $\frac{1}{100}$                       C.  $\frac{81}{100}$                       D.  $\frac{19}{100}$

31. Ten percent of the baseballs manufactured by a factory are defective. If two baseballs are selected at random from those produced by the company, what is the probability that neither baseball is defective?
- A.  $\frac{1}{5}$     B.  $\frac{1}{100}$     C.  $\frac{81}{100}$     D.  $\frac{19}{100}$
32. Ten percent of the baseballs manufactured by a factory are defective. If two baseballs are selected at random from those produced by the company, what is the probability that at least one of the baseballs is defective?
- A.  $\frac{1}{5}$     B.  $\frac{1}{100}$     C.  $\frac{81}{100}$     D.  $\frac{19}{100}$
33. A survey indicated that 30% of the customers buying a new computer, buy a clone. Of those, 10% buy a laser printer. What is the probability that a randomly selected computer customer buys a clone with a laser printer?
- A. 0.40    B. 0.60    C. 0.03    D. 0.30
34. Referring to Exercise 33, what is the probability that a randomly selected computer customer buys a clone without a laser printer
- A. 0.90    B. 0.27    C. 0.60    D. 0.63

□ **WARM-UPS C**

The table at the right shows the probability that there is a given number of people waiting in line to register for a course. Find the probability of having:

Number of persons in line	Probability
0	0.10
1	0.15
2	0.20
3	0.35
4 or more	0.20

35. More than 3 persons in line.
36. At least 1 person in line.
37. More than 3 or fewer than 1 persons in line.

**The table below will be used in Exercises 38-41**

The University Apartments has 1000 units classified by size and location as shown. Based on this table, what is the probability of selecting at random:

	B E D R O O M S		
	ONE	TWO	THREE
FIRST FLOOR	20%	30%	10%
SECOND FLOOR	15%	20%	5%

38. A first floor apartment?                      39. A first floor, three bedroom apartment?
40. A second floor apartment given that it is a one bedroom apartment?
41. A two or three bedroom apartment given that it is on the first floor?

**The table below will be used in Exercises 42-45**

An examination of Professor X's records for the last ten years shows the distribution of grades shown in the table. If one of Professor X's students is randomly selected, find the probability for the following:

42. The student received neither an A nor a B.
43. The student received a B if it is known that the student did not get a C or a D.
44. The student made better than a C.
45. If 200 of Professor X's students were selected at random, how many would be expected to have received a B or a D?

Grade	Percent of all grades
A	15
B	30
C	40
D	10
F	5

## □CLAST PRACTICE C

## PRACTICE PROBLEMS: Chapter 4, # 20-21

The table below will be used in Exercises 46-49

	Good Evaluations	Poor Evaluations
Tenured	36	84
Non-tenured	42	38

A survey of 200 instructors at a university revealed the data shown. Based on this data, what is the probability that:

46. An instructor received a good evaluation, given that the instructor was tenured?

- A.  $\frac{9}{50}$       B.  $\frac{18}{39}$       C.  $\frac{3}{10}$       D.  $\frac{3}{5}$

47. An instructor received a poor evaluation?

- A.  $\frac{61}{100}$       B.  $\frac{42}{61}$       C.  $\frac{21}{50}$       D.  $\frac{19}{40}$

48. An instructor received a poor evaluation given that the instructor was non-tenured?

- A.  $\frac{61}{100}$       B.  $\frac{42}{61}$       C.  $\frac{21}{50}$       D.  $\frac{19}{40}$

49. A non-tenured instructor received a good evaluation?

- A.  $\frac{21}{100}$       B.  $\frac{21}{40}$       C.  $\frac{19}{40}$       D.  $\frac{39}{40}$

50. The table gives a description of the students at a university based on sex and student classification.

	Freshmen	Sophomores	Juniors	Senior
Male	16%	13%	11%	11%
Female	14%	15%	11%	9%

Find the probability that a randomly selected university student is a senior.

- A. 0.51      B. 0.30      C. 0.20      D. 0.49



51. The table gives the distribution of the causes of fires nationally

Cause	Percent of all fires
Cooking	31
Smoking	15
Appliances	18
Heating System	9
Electrical system	8
Other	19

If it is known that a fire has NOT been caused by the heating or electrical system, find the probability that it was caused by cooking.

- A.  $\frac{31}{83}$     B.  $\frac{9}{50}$     C.  $\frac{31}{100}$     D.  $\frac{18}{83}$

### EXTRA CLAST PRACTICE

52. A buyer can purchase a computer with or without the following options: CD, color printer, high resolution monitor, video card. How many combinations of options are available?
- A. 24    B. 16    C. 8    D. 4
53. Five students entered the Math competition. In how many different ways can the \$100 first prize, \$50 second prize and \$20 third prize be awarded?
- A. 125    B. 60    C. 12    D. 3
54. Dinner for 7 at Wan Tom's restaurant consists of 3 items from column A and 4 items from column B. If column A has 5 choices and column B has 6 choices, how many different dinner combinations are possible?
- A. 150    B. 12    C. 25    D. 7