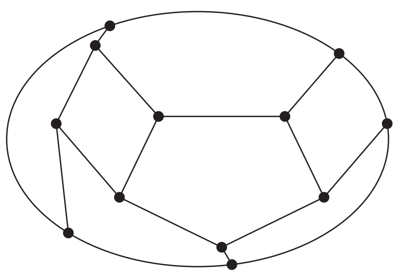
Answers to Odd-Numbered Problems

CHAPTER 15

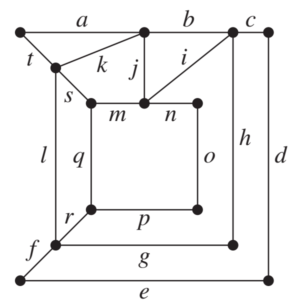
Exercises 15.1

1. The degrees of vertices A, B, C, D, and E are 1, 1, 1, 0 and 3, respectively.
3. The degrees of vertices A, B, C, and D are 5, 3, 4, and 6, respectively.
5. Not a subgraph (Each edge needs two endpoints.)
7. Subgraph
9. Path from *A* to *D*
11. Circuit based at *A*
13. Path from *A* to *G*
15. Not a path
17. Connected
19. Disconnected
21. Complete
23. Not complete
25. Complete
27. Simple
29. Simple
33. Represent each computer as a *vertex* and the connecting wires as *edges* as shown in the diagram below.



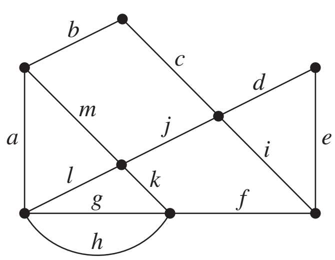
Exercises 15.2

1. Euler path from *D* to *B*
3. Euler circuit based at *A*
5. Each vertex has even degree, so the graph has an Euler circuit based at each vertex.
7. Each vertex has even degree, so the graph has an Euler circuit based at each vertex.
9. Each vertex has even degree, so the graph has an Euler circuit based at each vertex.
11. There are many correct solutions; for example, [*a, b, c, d, e, f, g, i, j, h*] is an Euler circuit.
13. There are many correct solutions; for example, [*a, b, e, f, h, g, d, c*] is an Euler circuit.
15. There are many correct solutions; for example [*a, b, c, d, e, f, g, h, i*] is an Euler circuit.
17. There are many correct solutions; for example [*a, b, c, d, e, f, g, h, i, j, k, l*] is an Euler circuit.
19. We represent the streets as labeled edges and the intersections as vertices. The result is shown.



There are many correct solutions that start at either vertex of odd degree. For example, [*s, t, a, b, c, d, e, f, g, h, i, j, k, l, r, p, o, n, m, q*] is a possible Euler path.

21. We represent the streets as labeled edges and the intersections as vertices. The result is shown.



There are many correct solutions that start at either vertex of odd degree. For example, [*a, l, m, b, c, j, k, g, h, f, i, d, e*] is a possible Euler path.

25. There are 3 regions, 5 edges, and 4 vertices. Thus, $X(G) = 3 - 5 + 4 = 2$.
27. There are 9 regions, 12 edges, and 5 vertices. Thus, $X(G) = 9 - 12 + 5 = 2$.

Exercises 15.3

1. Hamilton circuit
3. Neither
5. The length of the path is $8 + 3 + 2 + 4 = 17$.
7. The length of the path is $5 + 5 + 7 + 4 = 21$.
9. Recall that the graph K_4 has $(4 - 1)!/2 = 3$ distinct Hamilton circuits based at vertex *A*. Their lengths are $5 + 7 + 5 + 5 = 22$, $5 + 3 + 5 + 4 = 17$, and $5 + 3 + 7 + 4 = 19$.

11. The nearest neighbor method determines the circuit with length 2 + 2 + 4 + 6 = 14.

13. There is more than one answer. One possibility is the circuit with length 2 + 1 + 1 + 5 + 2 = 11.

15. There is more than one answer. One possibility is the circuit with length 2 + 1 + 1 + 1 + 9 + 2 = 16.

17. The most efficient delivery route has length 8 + 12 + 19 + 11 = 50.

19. Using the nearest neighbor method, we get 15 + 13 + 17 + 25 = 70.

25. There are numerous answers depending on your choice of starting point.

27. There are numerous answers depending on your choice of starting point.

Exercises 15.4

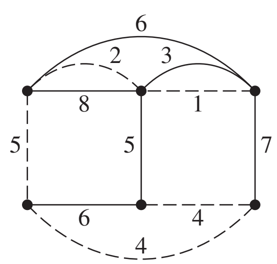
1. Tree **3.** Not a tree

5. There are many correct answers. One possibility is {*a, b, c*}.

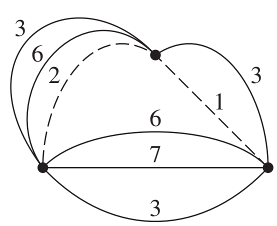
7. There are many correct answers. One possibility is {*a, b, c, d*}.

9. There are many correct answers. One possibility is {*a, b, c, d*}.

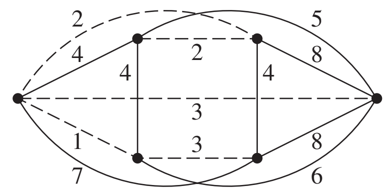
11. The graph below indicates a minimal spanning tree (there is more than one). Its weight is 1 + 2 + 4 + 4 + 5 = 16.



13. Applying Kruskal's algorithm provides the solution indicated in the graph below. Its weight is 1 + 2 = 3.



15. Applying Kruskal's algorithm provides the solution indicated in the graph below. Its weight is 1 + 2 + 2 + 3 + 3 = 11.



17. Applying Kruskal's algorithm, we find that the most economical way to link all the cities is a network of length 3.8 + 4.9 + 5.4 + 7.8 = 21.9, representing a cost of $21.9 million.

19. The minimal cost is 126 + 132 + 135 + 141 = $534.

25. Neither Euler or Hamilton circuits nor paths would be appropriate here because both require that every edge be used. For this problem, we are not concerned about edges and, in fact, if we can leave out an edge and still connect all the cities, it will be cheaper and, thus, more desirable. The tree shown is a possible answer! Is it the cheapest way? Can you find a better way?