

GLOSSARY

Absolute maximum The output value of the highest point on a graph over a given input interval or over all possible input values. An absolute maximum point either is a local maximum point or occurs at an endpoint of the given input interval. (Also see *extreme point*.)

Absolute minimum The output value of the lowest point on a graph over a given input interval or over all possible input values. An absolute minimum point is a local minimum point occurs at an endpoint of the given input interval. (Also see *extreme point*.)

Accumulated change The accumulated change in a quantity is represented as the area of a region between the rate-of-change function for that quantity and the horizontal axis. In the case where the rate-of-change function is negative, the accumulated change in the quantity is the negative of the area of the region between the rate-of-change function and the horizontal axis.

Accumulation function A function of the form $\int_a^x f(t) dt$ where the lower limit a is a constant (called the *starting value* of the accumulation) and the upper limit x is a variable. An accumulation function gives a formula that can be use to calculate the accumulated change in a quantity.

Aligning data A renumbering process by which data values are shifted. Large input values are often renumbered to make them smaller before fitting functions to data. Large input values (such as years) should always be aligned before fitting exponential or logistic functions to data.

Amplitude The amplitude of a periodic function is one-half the difference between the largest output value and the smallest output value. Amplitude is a positive quantity, and a negative sign in front of the amplitude indicates that the graph is reflected across the horizontal axis.

Angle The amount of rotation in a turn. An angle can be thought of as the counterclockwise rotation of its initial side into its terminal side. The size of an angle is the measure of this rotation, and a negative sign in front of the size indicates clockwise rotation. (See standard position.)

Antiderivative A function F is an antiderivative of another function f if the derivative of F is f . If F is an antiderivative of f , where both F and f have input x and C is an arbitrary constant, then $y = F(x) + C$ is called a *general antiderivative* of f and is denoted by $\int f(x) dx$. A general antiderivative is a group of infinitely many functions. A *specific antiderivative* is an antiderivative with a particular constant C . An accumulation function is a specific antiderivative.

Annual percentage rate (APR) The percentage used in calculating interest each compounding period. In an investment context, the annual percentage rate (or *nominal rate*) is the advertised rate of interest, $100r\%$.

Annual percentage yield (APY) A percentage by which an investment grows over one year. Unlike APR, APY indicates the affect of the compounding periods. APY will be larger than APR any time interest is compounded more frequently than once a year. In an investment context with an exponential function of the form $A(t) = ab^t$, the annual percentage yield (or *effective rate*) is equal to the constant percentage change $(b - 1) \cdot 100\%$. The APY for the exponential function $A(t) = Pe^{rt}$ can be found by first converting Pe^{rt} to Pb^t . When comparing compound interest rates, we should use the APY rather than the APR because annual percentage rates do not reflect the compounding periods.

Approximate change in a function The rate of change of the function times a small change in the input of the function. That is, for the function f with input variable x , the approximate change in f is $f'(x) \cdot h$ where h represents the small change in x . The exact change is $f(x + h) - f(x)$.

Average cost The total production cost divided by the number of units produced.

Average rate of change The amount that a quantity changes over an interval divided by the length of the interval. That is, if a quantity changes from a value of m to a value of n over a certain interval, the average rate of change equals $\frac{n - m}{\text{length of interval}}$. The average rate of change is the slope of the secant line. Average rates of change have labels of output units per input unit.

Average value of a function A number that equals the accumulated change in a function f over an input interval from a to b divided by the length of the interval and is calculated as $\frac{1}{b-a} \int_a^b f(x) dx$.

Biological stream A continuous flow of individuals into an existing population over time. (We call the elements of any population individuals.)

Break-even point The number of units (produced or sold) for which revenue equals cost so that profit is zero.

Change If a quantity changes from a value of m to a value of n over a certain interval, then the change in the quantity is $n - m$.

Composition A method of combining two functions in which the output of one function (called the *inside function*) is used as the input of the other function (called the *outside function*). The symbol for the output of a composite function at the input x is $(f \circ g)(x)$ or $f(g(x))$, where g is the inside function and f is the outside function.

Compound interest formulas Exponential formulas that are used to determine the amount $A(t)$ accumulated in an account after t years when P dollars are initially invested. If the nominal interest rate is $100r\%$ compounded n times a year, then the accumulated amount is given by $A = P\left(1 + \frac{r}{n}\right)^{nt}$ dollars. If the interest is compounded continuously at a nominal rate of $100r\%$, then the accumulated amount is $A = Pe^{rt}$ dollars.

Concavity A description of the curvature of a graph. A graph is *concave up* at a point if the tangent to that point lies below the graph near the point of tangency and *concave down* if the line tangent to that point lies above the graph near the point of tangency. A point where the concavity of a graph changes is called an inflection point.

Constant dollars Dollar values that have been adjusted for inflation by means of price indexes to eliminate inflationary factors and allow direct comparison across years. Conversion to constant dollars for a given year t may be calculated as the current dollar value multiplied by the purchasing power of the dollar based on a dollar value of \$1 in year t .

Consumer price index (CPI) A measure that is 100 times the ratio obtained by comparing the current cost of a specified group of goods and services to the cost of comparable items determined at an earlier date.

Consumers' expenditure The actual amount spent by consumers for a certain quantity of goods or services. The consumers' expenditure equals the market price times the quantity in demand.

Consumers' surplus The amount that consumers are willing and able to spend but do not actually spend for a certain quantity of goods or services.

Consumers' willingness and ability to spend The maximum amount that consumers say they will spend and/or actually spend for a certain quantity of goods or services.

Continuous graph/function A continuous graph is an unbroken curve whose set of inputs is assumed to fill up an entire interval of values along the horizontal axis. A continuous graph can be drawn

without lifting the writing instrument from the page. A *smooth* continuous graph is one with no sharp points. A continuous function is a function whose graph is continuous. When modeling real-life situations, continuous functions could be used without restriction or could be discretely interpreted. More formally, a function is *continuous* at an input $x = c$ if and only if $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and $\lim_{x \rightarrow c} f(x) = f(c)$.

Continuous function used without restriction A continuous function for which inputs of any value make sense in context.

Continuous function with discrete interpretation A continuous function whose interpretation makes sense only at certain distinct points.

Contour curve A two-dimensional outline of a three-dimensional graph at a given output level. For a three-dimensional function f , the k -contour curve is the collection of all points (x, y) for which $f(x, y) = k$, where k is a constant. Contour curves are also called *level curves*.

Contour graph A graph of the contour curves $f(x, y) = k$ for several values of a constant k . Usually, the values of k are equally spaced.

Cosine The trigonometric function denoted by $\cos \theta$ whose output is x , the first coordinate of the point where the terminal side of the angle θ (centered at the origin) intercepts the unit circle. The cosine of angle θ , when θ is one of the angles in a right triangle, is defined as the length of the side adjacent to θ divided by the length of the hypotenuse.

Count data Totals that are reported for a specific time period but that are not cumulative because the counter that tallies the data during the period is reset to zero at the beginning of each period. When working with count data, accumulated change in a quantity is calculated by summing the output data.

Critical point A saddle point or a point corresponding to a relative maximum or relative minimum on a multivariable surface.

Cross section For a two-variable function, the curve resulting when the function is intersected with a plane. A cross section of a function will always have one less dimension (variable) than the function itself.

Cross-sectional function/model An equation describing a cross section of a multivariable function.

Cubic function/model A function of the form $f(x) = ax^3 + bx^2 + cx + d$ where $a, b, c,$ and d are constants and $a \neq 0$. Cubic functions have one change in concavity (*i.e.*, one inflection point) and no end behavior limiting values.

Cumulative density function An accumulation function of a probability density function. The cumulative density function shows how probabilities accumulate as the value of the random variable increases.

Cyclic function A periodic, continuous function that varies between two extremes. The part of the graph of the function that keeps repeating itself is called a *cycle* of the graph.

Data Real-world information recorded as numerical values.

Decreasing without bound A term applied to the output of a function that infinitely decreases in height. Decreasing without bound may describe either the end behavior of a function or the limiting value of a function as the input approaches a certain value.

Definite integral The definite integral of a continuous or piecewise continuous function f from $x = a$ to $x = b$ is defined as $\int_a^b f(x) dx = \lim_{h \rightarrow 0} [f(x_1) + f(x_2) + \dots + f(x_n)]\Delta x$ where x_1, x_2, \dots, x_n are the midpoints of n subintervals of length $\Delta x = \frac{b-a}{n}$ between a and b . If the graph of f lies above the x -axis between a and b , then the definite integral is the area of the region between f and the x -axis from a to b . If the graph of f lies below the x -axis between a and b , then the definite integral is the negative of the area (also called the *signed area*) of the region between f and the x -axis from a to b . Whenever F , an antiderivative of a continuous function f , is known, a more efficient method of evaluating a definite integral of f from a to b is given by the Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$.

Degree One of 360 equal parts into which a complete revolution is divided. Degree measure is one of the ways angles can be measured and is denoted by a small circle as a superscript (*e.g.*, 45°). For finer measurements, a degree can be divided into 60 equal parts called *minutes* and a minute can be divided into 60 equal parts called *seconds*.

Demand curve/function A graph or equation relating the quantity of goods or services that consumers demand and the price per unit of those goods or services. Mathematicians use price as input and quantity demanded as output; economists use quantity demanded as input and price per unit as output.

Derivative The mathematical term for an instantaneous rate of change. The terms *derivative*, *rate of change*, *instantaneous rate of change*, *slope of a curve*, and *slope of the line tangent to a curve* are synonymous. If $y = f(x)$, the derivative is defined by $f'(x) = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

Derivative graph See *slope graph*.

Determinant See *second-partials matrix*.

Differential equation An equation involving one or more derivatives. A *general solution* for a differential equation is a function that has derivatives that satisfy the differential equation, and a *particular solution* is a function obtained from the general solution and the initial conditions stated in a specific problem.

Direct proportionality For variables x and y , y is proportional to x if there exists some constant k such that $y = kx$. The terms *proportional* and *directly proportional* are used interchangeably. The constant k is referred to as the *constant of proportionality*.

Discrete Discrete information is represented by a scatter plot or a table of data. Discrete graphs are scatter plots of data. In some situations, continuous functions are interpreted discretely; that is, outputs of the function have meaning in the context of a real-life situation only at some, not all, input values in an interval.

Distribution See *probability distribution*.

Diverge A term applied to an improper integral for which the limit does not exist. (See *improper integral*.)

Dominant term A term (one of the parts of a polynomial that is joined to another part by a plus or a minus sign) containing the highest power of the variable.

Effective rate of interest See *annual percentage yield*.

End behavior The behavior of the output of a graph as the input becomes infinitely large or infinitely small.

Equilibrium point The point at which the demand curve and the supply curve intersect. At this point, there is market equilibrium; that is, the supply of a product is equal to the demand for that product.

Euler's method A technique for numerically estimating a solution to a differential equation using an initial condition and slopes given by the differential equation to approximate an output value of the solution function. A graph of the Euler estimates approximates a graph of the particular solution for the given initial condition.

Event An outcome of some happening whose results are subject to chance.

Expected value For a trig function, the expected (or *average*) value of one cycle is the midpoint between the high and low values of the function. See *mean* for the probability definition.

Exponential function/model A function with an equation of the form $f(x) = ab^x$ or $f(x) = ae^{kx}$. Exponential models are characterized by constant percentage change (percentage differences) in output values when input values are evenly spaced. For the exponential function $f(x) = ab^x$, the constant percentage change over one unit of x is $(b - 1) \cdot 100\%$. In terms of the function $f(x) = ab^x$, exponential growth occurs when b is greater than 1, and exponential decline (decay) takes place when b is between 0 and 1.

Extrapolation The process of predicting an output value using an input value that is outside a given interval of input data. Extrapolation should always be viewed with caution.

Extreme point A point at which a maximum or minimum output occurs. At an extreme point on a graph, the slope of the line tangent to the curve at that point is zero or the slope does not exist at that point (but the function output exists at that point). Extreme points occur at an input value, but the extreme value is an output value. (Also see *absolute maximum/minimum* and *relative maximum/minimum*.) For multivariable functions, relative extreme points cannot be visually identified on the edges of tables or contour graphs.

First differences The changes in successive output values. It is helpful to calculate first differences for a data set only if all input data values are evenly spaced.

Fixed costs Also called *start-up costs*, these costs do not vary with the number of items produced or the amount of service performed.

Four-Step Method An algebraic method of finding the derivative of a function using the definition of the derivative.

Frequency The frequency of a sine or cosine function is the number of cycles of the graph on an interval of length 2π .

Function A function is a rule that assigns exactly one output to each input. Functions are represented verbally by word descriptions, numerically in tables, visually with graphs, or algebraically with equations. If x is the input symbol and f is the rule, then $f(x)$ symbolizes the output. Input/output diagrams display the input, how the input is measured (input units), the rule that relates the input and output, and the output, including how the output is measured (output units).

Fundamental Theorem of Calculus For any continuous function f with input t , the derivative with respect to x of an accumulation function of f is the function f in terms of x . Symbolically, the Fundamental Theorem of Calculus is expressed $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$. This theorem verifies that if

we begin with a function f with input t , find an accumulation formula or accumulation graph,

$\int_a^x f(t) dt$, and then write the derivative formula or draw the slope graph, we get f , the function with

which we began, but with input x . This theorem connects the two basic concepts of calculus: rate of change and accumulation of change.

Future value The value of an investment at some time in the future. Future value for discrete situations is calculated using the appropriate compound interest formula. (See *compound interest formulas*.) The future value of a continuous income stream is the total accumulated value of the income stream and its earned interest.

General antiderivative See *antiderivative*.

General solution See *differential equation*.

Graph One of the ways to represent a function or a real-life situation by plotting output and input points on a coordinate axes. Discrete graphs are scatter plots. Continuous graphs can be drawn without lifting the writing instrument from the page.

Half-life The amount of time it takes for half of a substance to decay under exponential decay.

Histogram A graph that is composed of rectangles and constructed so that the area of each rectangle is the percentage of outputs in the corresponding input interval. These histograms are also called *probability histograms* because the area of each rectangle is the probability that the value of the random variable under discussion is in the interval that forms the base of the rectangle.

Improper integral An integral of the form $\int_a^\infty f(x)dx$, $\int_{-\infty}^b f(x)dx$, or $\int_{-\infty}^\infty f(x)dx$ for constants a and b .

The value of an improper integral, if it exists, is found by replacing the infinity symbol with a constant, performing the integration, and then determining the limit of the resulting expression as the constant increases (or decreases) without bound.

Income stream A flow of money into an interest-bearing account over a period of time. When the money flows continuously into the account, the flow is called a *continuous income stream*. A *discrete income stream* is a one into which money flows at specific intervals of time (quarterly, monthly, daily, and so on.)

Increasing without bound A term applied to the output of a function that infinitely increases in height. Increasing without bound may describe either the end behavior of a function or the limiting value of a function as the input approaches a certain value.

Inflection point A point where the concavity of a graph changes. Cubic and logistic functions have one point. Sine and cosine functions have two inflection points in each cycle and an infinite number of inflection points over all real number inputs. In real-life applications, the inflection point is interpreted as the point of most rapid change or least rapid change in an area near the inflection point.

Initial condition A known point on the graph of a particular solution for a differential equation.

Input See *function*.

Input/output diagram See *function*.

Instantaneous rate of change The instantaneous rate of change at a point on a curve is the slope of the curve at that point and the slope of the line tangent to the curve at that point. Instantaneous rates of change have labels of output units per input unit.

Integration The process of evaluating a definite integral to determine the accumulation of change or the process of recovering a quantity function from a rate-of-change function.

Interpretation of a result A simple non-technical sentence explaining the real-life meaning of a result.

Intercept The input value where the graph crosses or touches the horizontal axis or the output value where the graph touches or crosses the vertical axis.

Interpolation The process of predicting an output value using an input value that is within a given interval of input data.

Inverse function If a rule obtained by reversing the input and output of a function is also a function, then it is called an *inverse function*.

Inverse proportionality For variables x and y , y is inversely proportional to x if there exists some constant k such that $y = \frac{k}{x}$. The constant k is the *constant of proportionality*.

Joint proportionality For variables x , y , and z , y is jointly proportional to x and z if there exists some constant k such that $y = kxz$. The constant k is the *constant of proportionality*.

Lagrange multiplier The rate of change of an optimal value with respect to a change in the constraint in a constrained optimization problem. Denoted by λ , the Lagrange multiplier is often referred to by economists as the *marginal productivity of money*.

Least squares method A procedure to determine the line that best fits a set of data using the criterion that the sum of the squares of the deviations of all the data points from the fitted line (*i.e.*, SSE) is at a minimum.

Least squares line The linear function that best fits a set of data, where *best fit* is defined according to the least squares method.

Left-rectangle approximation A method of approximating the accumulated change in a quantity between two specified inputs. The area between the rate-of-change graph of the quantity and the input axis between the two inputs is approximated with the sums of the areas of rectangles whose heights are output values of the rate-of-change function evaluated at the left endpoint of each subinterval on the input axis.

Limit A number to which the output of a function becomes closer and closer as the input becomes closer and closer to a stated value.

Limit of sums A method of evaluating a definite integral as the trend in the sum of the areas of approximating midpoint rectangles when the number of approximating rectangles becomes infinitely large.

Linear function/model A function that repeatedly and at even intervals adds the same value to the output. A linear model is a function of the form $f(x) = ax + b$ representing a situation in which incremental change is constant. In the linear function, a is the constant rate of change of the output (*i.e.*, the slope of the graph of the linear function) and b is the output corresponding to an input of zero (*i.e.*, the vertical axis intercept). When input values in a set of data are evenly spaced and the first differences of the output values are constant, the data should be modeled by a linear function.

Linear system of equations Two or more equations in which all the variables occur to the first power and there are no terms in which two different variables are multiplied or divided.

Local linearity The principle that if we graph a smooth, continuous function over a small enough interval around a point, then the graph looks like the line tangent to the curve at that point. That is, the tangent line and the curve are basically indistinguishable over the interval.

Local maximum or minimum See *relative maximum/minimum*.

Logarithmic (log) function/model A function with an equation of the form $f(x) = a + b \ln x$. This function is the inverse of the exponential function $y = AB^x$, where $A = e^{-a/b}$ and $B = e^{1/b}$.

Logistic function/model A function of the form $f(x) = \frac{L}{1 + Ae^{-Bx}}$. The logistic function increases if $B > 0$ and decreases if $B < 0$. The graph of a logistic function is bounded by the horizontal axis and the line $f(x) = L$. We refer to L as the *limiting value* (or *carrying capacity* or *saturation level*) of the function.

Marginal analysis A type of approximation of change used in economics. The rates of change of cost, revenue, and profit with respect to the number of units produced or sold are called *marginal cost*, *marginal revenue*, and *marginal profit*. These rates are often used to approximate the actual change in cost, revenue, or profit when the number of units produced or sold is increased by one.

Market equilibrium See *equilibrium point*.

Market price The actual price that a consumer pays for one unit of goods or services.

Mathematical modeling The process of translating a real-world problem into a useable mathematical equation.

Matrix A rectangular arrangement of numbers in rows and columns. Matrices are useful in solving systems of linear equations. (Also see *second-partials matrix*.)

Mean One of the measures of the center of a probability distribution, the mean (also called the *expected value* or *average*) is the input value of the “balance point” of the region between the density function and the horizontal axis.

Midpoint-rectangle approximation A method of approximating the accumulated change in a quantity between two specified inputs. The area between the rate-of-change graph of that quantity and the input axis is approximated with the sums of the areas of rectangles whose heights are output values of the rate-of-change function evaluated at the midpoint of each subinterval on the input axis. A midpoint-rectangle approximation usually gives the least absolute error when compared with the errors involved with corresponding left- or right-rectangle approximations.

Model A mathematical model is an equation, along with descriptions and units of the variables, that describes a real-life situation. There are four important elements to every model: an equation, a label denoting the units on the output, a description (including units) of what the input variable represents, and an indication of the interval of input values over which the model is valid.

Multivariable function A function that has two or more input variables and one output variable.

Multivariable functions with two input variables can be illustrated with graphs of three-dimensional surfaces, tables of data, and/or contour graphs.

Nominal rate See *annual percentage rate*.

Optimal point See *extreme point*.

Optimization The process of finding relative or absolute extreme points.

Output See *function*.

Partial derivative A derivative of a multivariable function found by taking the derivative of the function with respect to one of the input variables while all the other input variables are held constant.

Partial rate of change The rate of change of a cross-sectional function that is computed as a partial derivative.

Particular solution See *differential equation*.

Percentage change A quantity calculated from data with increasing input values by dividing each first difference by the output value of the lesser input value and multiplying by 100%. That is, if a quantity changes from a value of m to a value of n over a certain interval, then the percentage change equals $\frac{n-m}{m} \cdot 100\%$. For the exponential function $f(x) = ab^x$, the constant percentage change over an interval of width one unit is $(b - 1) \cdot 100\%$.

Percentage rate of change A quantity that is useful in describing the relative magnitude of a rate of change. A percentage rate of change at a point is found by dividing the rate of change at the point by the function value at that same point and multiplying the result by 100%. Percentage rates of change have labels of percent per input unit.

Periodic function A function that repeats itself with the length between repetitions being constant. The shortest interval over which the function repeats itself is called the *period*.

Piecewise continuous function A function formed by combining two or more pieces of continuous functions. A piecewise continuous function is not necessarily a continuous function.

Point of diminishing returns An inflection point on the graph of a function beyond which the function output increases at a decreasing rate.

Point of tangency The point at which the line tangent to a curve touches the curve and at which the slope of the line tangent to the curve is the instantaneous rate of change of the curve.

Polynomial function A function that has the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where a_0, a_1, \dots, a_n are constants and n is a positive integer called the *degree of the polynomial*. If $n = 1$, then the polynomial function is a *linear* function; if $n = 2$, it is a *quadratic* function; and if $n = 3$, it is a *cubic* function.

Present value The amount of money that would have to be invested now in an interest-bearing account in order for the amount to grow to a given future value.

Probability A measure of how likely an event is to happen. The probability of an event is always a number between 0 (for an *impossible event*) and 1 (for a *sure event*). One of the methods for computing probabilities is to find areas under probability density functions.

Probability density function A continuous or piecewise continuous function with input consisting of some interval of real numbers and output satisfying the conditions that each output value is greater than or equal to 0 and the area of the region between the density function and the horizontal axis is 1.

Probability distribution When all outcomes of a particular situation are considered, the pattern indicated by the variability in the data is called the *distribution* of the quantity being studied. (See *probability density function*.)

Probability histogram See *histogram*.

Producers' revenue The actual amount of money that producers receive for supplying a certain quantity of goods or services. The producers' revenue equals the market price times the quantity supplied.

Producers' surplus The amount of money that producers receive above the minimum amount they are willing and able to accept for a certain quantity of goods or services.

Producers' willingness and ability to receive The minimum amount of money that producers are willing and able to receive for supplying a certain quantity of goods or services.

Profit Revenue minus total cost.

Quadratic function/model A function of the form $f(x) = ax^2 + bx + c$ where a , b , and c are constants and $a \neq 0$. In this function, a is called the *leading coefficient*. The graph of a quadratic model is called a *parabola*. If $a > 0$, the parabola is concave up and if $a < 0$, the parabola is concave down. When input values of a set of data are evenly spaced and the second differences of the output values are constant, the data should be modeled by a quadratic function.

Radian One of the units of angle measure that is a real number. The radian measure of an angle is the ratio of the length of the arc cut out of the circumference of a circle by an angle to the radius of the circle. A full revolution has radian measure of 2π . All angles are understood to be measured in radians unless the degree symbol is used to specify degree measure for an angle.

Random variable A variable whose numerical values are determined by the results of a situation involving chance.

Rate of change See *derivative* and *slope*.

Rate-of-change graph See *slope graph*.

Recovering a function The process of beginning with a rate-of-change function for a quantity and finding its antiderivative to obtain a function for the quantity.

Related-rates equation An equation showing how the rates of change of two or more *dependent variables* with respect to an *independent variable* (often time) are interconnected and can be used to answer questions about those rates.

Relative maximum An output value that is ~~that is~~ larger than all other output values in some interval around the maximum. A graph increases to the relative maximum and decreases after it. For a multivariable function, a relative maximum is an output value that is greater than all values around it.

Relative minimum An output value that is ~~that~~ smaller than all other output values in some interval around the minimum. A graph decreases to the relative minimum and increases after it. For a multivariable function, a relative minimum is an output value that is smaller than all values around it.

Revenue Quantity sold times price per unit.

Right-rectangle approximation A method of approximating the accumulated change in a quantity between two specified inputs. The area between the rate-of-change graph of the quantity and the input axis between the two inputs is approximated with the sums of the areas of rectangles whose heights are output values of the rate-of-change function evaluated at the right endpoint of each subinterval on the input axis.

Saddle point For a multivariable function, a saddle point appears to be a maximum point when approached from one direction and a minimum point when approached from another direction. Saddle points cannot be visually identified on the edges of tables or contour graphs.

Scatter plot A discrete graph showing points in isolation from one another on a rectangular grid. A graph of data is a scatter plot.

Secant line A line through two points on a scatter plot or a function graph. The slope of the secant line through two points is the average rate of change of the quantity between the input values of those two points.

Second derivative The derivative of the derivative (provided it exists). At a point of inflection on a graph, the second derivative is zero or does not exist. Where the second derivative of a function is positive, the function graph is concave up, and where the second derivative is negative, the function graph is concave down.

Second differences The differences between the first differences.

Second-partials matrix A matrix with rows that are labeled with the input variables and columns that are labeled with the input variables in the same order containing the indicated second partial derivatives of a multivariable function. The *determinant* of the second-partials matrix for a multivariable function $f(x, y)$ is $D = f_{xx}f_{yy} - f_{xy}f_{yx}$. This determinant is used in the *Determinant Test* to classify optimal points for multivariable functions.

Separation of variables A technique for solving certain differential equations in which all terms containing one variable are put on one side of the equation, all terms involving the other variable are put on to the other side of the equation, and then antiderivatives of both sides are taken.

Shifting data A process by which data that appears to be exponential or logistic but approaches a horizontal asymptote other than $y = 0$ is shifted vertically so that the shifted data approach the asymptote $y = 0$. The constant by which the data are shifted is added (when the data are shifted down) or subtracted (when the data are shifted up) to the function fit to the shifted data to obtain the function that models the original data. Horizontally shifting input data (remembering that log functions cannot have negative or zero input) can sometimes produce a better-fitting log equation.

Shutdown price The price below which producers are not willing or able to supply any quantity of particular goods or services to consumers. The point on the supply curve that corresponds to the shutdown price is called the *shutdown point*.

Signed area The negative of the area of a region.

Sine The trigonometric function denoted by $\sin \theta$ whose output is y , the second coordinate of the point where the terminal side of angle θ (centered at the origin) intercepts the unit circle. When θ is one of the angles in a right triangle, $\sin \theta$ defined as the length of the side opposite θ divided by the length of the hypotenuse.

Sine function/model A function of the form $f(x) = a \sin(bx + h) + k$ where $|a|$ is the amplitude, b is the frequency, $\frac{2\pi}{b}$ is the period, $\frac{|h|}{b}$ is the horizontal shift (right if $h < 0$ and left if $h > 0$), and k , the vertical shift (up if $k > 0$ and down if $k < 0$), is the average or expected value of one cycle.

Slope A measure of how steeply tilted a line or curve is. The rate of change of a linear function is its slope. The slope of the line tangent to a curve at a point is the limiting value of the slopes of nearby secant lines. The slope of a curve (graph) at a point is the slope of the line tangent to the curve at that point (provided the slope exists). The instantaneous rate of change (also called the *derivative* or *rate of change*) at a point on a curve is the slope of the curve at that point (provided that slope exists).

Slope field A graph of a differential equation that is constructed by placing a grid on a portion of the Cartesian plane and, at each point on the grid, drawing a short line segment whose slope is determined by the differential equation.

Slope graph Also called the *rate-of-change graph* or *derivative graph*, a slope graph depicts the changing nature of the slopes of lines tangent to a graph of a function. Where a function is increasing, its slope graph is positive; where a function is decreasing, its slope graph is negative. Where a function has a maximum, has a minimum, or levels off, the slope graph is zero. Where a function has an inflection point, the slope graph has a maximum or minimum point.

Smooth A continuous function is smooth if it has no sharp points; that is, no points where two pieces of a function have different slopes at the point where they meet.

Specific antiderivative See *antiderivative*.

Standard deviation A measure of how closely the values of a probability distribution cluster about its mean.

Standard position An angle drawn so that one side, called the *initial side*, is along the positive x -axis and the other side, called the *terminal side*, is in one of the four quadrants.

Sum of squared errors (SSE) A measure of best fit for linear functions. SSE is calculated as the sum of the squares of the deviations where the deviation for each data point is the data output minus the output of the fitted linear function $y = ax + b$.

Supply curve/function A graph or equation that expresses the quantity supplied relative to the price per unit.

Symmetric difference quotient A method of approximating instantaneous rates of change from data or an equation by using a close point on either side of the point of interest and the same horizontal distance away. The symmetric difference quotient is the difference between the outputs of the two close points divided by the corresponding difference in inputs of the two close points.

Tangent line A line that touches a graph at a point and is tilted exactly the way the graph is tilted at that point. The tangent line at a point on a smooth continuous graph is the limiting position of

the secant lines between nearby points and that point (if the limiting position exists). Provided the slope exists, the slope of the tangent line at a point is a measure of the slope of the graph at that point and gives the instantaneous rate of change of the graph at that point.

Terminal edge An edge beyond which a table or graph cannot extend. Terminal edges are determined by the context of the situation that the multivariable function describes.

Total cost The sum of the fixed costs and the variable costs.

Total social gain The benefit to society whenever consumers and/or producers have surplus funds. When the market price of a product is the equilibrium price for that product, the total social gain is the consumers' surplus plus the producers' surplus.

Trend As associated with limits, a trend is a value to which a quantity becomes closer and closer as the input becomes infinitely large or infinitely small.

Unit of measure A word or short phrase telling *how* a quantity is measured, not an entire description telling what the variable represents.

Variable costs Costs that change according to the number of items produced or the amount of service performed.

Vertical Line Test A method of visually determining whether a graph with inputs located along the horizontal axis and outputs located along the vertical axis is a function. If there is no input at which a vertical line crosses or touches the graph in two or more places, then the graph represents a function.