Chapter 9  Ingredients of Multivariable Change:
Models, Graphs, Rates

9.1 Cross-Sectional Models and Multivariable Functions

For a multivariable function with two input variables, obtain a cross-sectional model by entering the data in c1 and c2 and then fitting the appropriate function as indicated in previous chapters of this Guide. Unless you are told otherwise, we assume that the data are given in a table with the values of the first input variable listed horizontally across the top of the table and the values of the second input variable listed vertically down the left side of the table.

9.1.1 FINDING A CROSS-SECTIONAL MODEL FROM DATA (HOLDING THE FIRST INPUT VARIABLE CONSTANT)  Using the elevation data in Table 9.2 of Section 9.1 in Calculus Concepts, find the cross-sectional model $E(0.8, n)$ as described below. Remember that “rows” go from left to right horizontally and “columns” go from top to bottom vertically.

<table>
<thead>
<tr>
<th>n (miles)</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
<th>1.2</th>
<th>1.1</th>
<th>1.0</th>
<th>0.9</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation (feet above sea level)</td>
<td>797.6</td>
<td>798.1</td>
<td>798.5</td>
<td>798.9</td>
<td>799.2</td>
<td>799.5</td>
<td>799.7</td>
<td>799.9</td>
</tr>
<tr>
<td>n (miles)</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Elevation (feet above sea level)</td>
<td>800.0</td>
<td>800.1</td>
<td>800.1</td>
<td>800.1</td>
<td>800.0</td>
<td>799.9</td>
<td>799.7</td>
<td>799.5</td>
</tr>
</tbody>
</table>

So, refer for a moment to Table 9.1. When you are asked to find $E(0.8, n)$, $e$ is constant at 0.8 and $n$ varies. Thus, enter the values for $n$ appearing on the left side of the table (vertically) in c1 and the elevations $E$ obtained in the $e=0.8$ column of Table 9.1 in c2. The outputs that you enter in c2 will always be in the main body of any multivariable data table given in this text.

Note: A shortcut to entering the input values is \( \text{seq}(x, x, 1.5, 0, -0.1) \). (See Section 2.1.1.)

After entering the data, clear the Y= list and turn on Plot1. Draw a scatter plot of the data with F2 [Zoom] 9 [ZoomData]. The data look quadratic.

Fit a quadratic function and copy it to the Y= list. (Refer to Section 2.4.2 of this Guide.) Overdraw the graph of the function on the scatter plot with \( \text{F3} \) (GRAPH).

CAUTION: Because you will often be asked to find several different cross-sectional models using the same data table, calling different variables by the same names $x$ and $y$ would be very confusing. It is very important that you call the variables by the names that have been assigned in the problem. Remember that when finding or graphing a function, the TI-89 always calls the input variable $x$ and the output variable $y$. When working with multivariable functions, you must translate the calculator’s equation $y1 \approx -2.5x^2 + 2.497x + 799.490$ into the symbols that are used in the application. You should write the cross-sectional function as $E(0.8, n) = -2.5n^2 + 2.497n + 799.490$. Don’t forget to completely describe (including units) all of the variables.
9.1.2 FINDING A CROSS-SECTIONAL MODEL FROM DATA (HOLDING THE SECOND INPUT VARIABLE CONSTANT)
The only difference in this and the previous section of this Guide is that the second input, instead of the first, is held constant. We illustrate using the situation in Example 1 of Section 9.1 in Calculus Concepts. Refer to Table 9.1. Because we are asked to find the cross-sectional model \( E(e, 0.6) \), \( n = 0.6 \) and the inputs are the values of \( e \) that are across the top of the table. Enter these values in \( c1 \). (See below for a shortcut.) The outputs are the elevations \( E \) obtained in the \( n = 0.6 \) mile row in Table 9.1. Enter these in \( c2 \).

**NOTE:** You may find it helpful to place a piece of paper or a ruler under the row (or to the right of the column) in which the data appear to help avoid typing an incorrect value.

Because the input values for this function are the same as the input values in the last section of this Guide (but in reverse order), enter the input in \( c1 \) using seq(\( x \), \( x \), 0, 1.5, 0.1). Then enter the outputs in \( c2 \).

After entering the data, clear any functions from the \( Y= \) list, and turn on Plot1. Draw a scatter plot of the data. There is an inflection point and no evidence of limiting values, so the data appear to be cubic.

Fit a cubic function and copy it to the \( Y= \) list. (Refer to Section 2.4.3 of this Guide.) Draw the cubic function on the scatter plot with \( \text{F3 (GRAPH)} \).

9.1.3 EVALUATING OUTPUTS OF MULTIVARIABLE FUNCTIONS
As is the case with single-variable functions, outputs of multivariable functions are found by evaluating the function at the given values of the input variables. The main difference is that you usually will not be using \( x \) as the input variable symbol. One way to find multivariable function outputs is to evaluate them on the home screen. We illustrate with the investment function in Example 2 of Section 9.1 in Calculus Concepts.

The answer to part \( a \) of Example 2, as derived from the compound interest formula, uses the formula for the accumulated amount of an investment of \( P \) dollars for \( t \) years in an account paying 6% interest compounded quarterly: \( A(P, t) = P \left(1 + \frac{0.06}{4}\right)^{4t} \) dollars. When 10 is substituted for \( t \), the cross-sectional function becomes \( A(P, 10) = 1.814018409P \).

Part \( b \) of Example 2 asks for a model for the data in the row of Table 9.4 that corresponds to 10 years. Thus, we want to find \( A(P, 10) \). Enter the data from the table in \( c1 \) and \( c2 \).

A scatter plot of the data appears to be linear. Program DIFF confirms that a linear function is appropriate because the first differences are constant. Fit a linear function and paste it in \( y1 \).
Notice that the cross-sectional functions in parts $a$ and $b$ are slightly different because the data in Table 9.4 have been rounded to the nearest cent. Part $c$ of Example 2 asks for $A(5300, 10)$. Even though it is simplest here to substitute 5300 for $P$ in $A(P, 10) = 1.814018409P$, we return to the original function to illustrate evaluating multivariable formulas on the calculator.

To find the output on the home screen, type the formula for $A(P, t)$, substituting $P = 5300$ and $t = 10$. Press ENTER.

Again, be warned that you must carefully use the correct placement of parentheses. Check your result with the values in Table 9.4 in the text to see if $9614.30$ is a reasonable amount.

Even though it is not necessary in this example, you may encounter activities in this section in which you need to evaluate a multivariable function at several different inputs. You could use what is shown above, but there are easier methods than entering each one individually. You will also use the techniques shown below in later sections of this chapter. When evaluating a multivariable function at several different input values, you may find it more convenient to enter the multivariable function in the graphing list.

Clear any previously-entered equations. Enter in $y_1$ the function $A(P, t) = P \left(1 + \frac{0.06}{4}\right)^{4t}$ with the keystrokes alpha STO $\left(\begin{array}{c} \text{P} \\ \text{X} \end{array}\right)$ alpha $\left(\begin{array}{c} 4 \\ 4 \end{array}\right)$ $+ \left(\begin{array}{c} 1 \\ \text{X} \end{array}\right)$ $\times \left(\begin{array}{c} 1 \\ \text{X} \end{array}\right)$ $\div \left(\begin{array}{c} 4 \\ 4 \end{array}\right)$ $\times \left(\begin{array}{c} 1 \\ \text{X} \end{array}\right)$ $\div \left(\begin{array}{c} 4 \\ 4 \end{array}\right)$ $\times \left(\begin{array}{c} 1 \\ \text{T} \end{array}\right)$ $\times \left(\begin{array}{c} 1 \\ \text{T} \end{array}\right)$.

WARNING: You must use a times sign between $p$ and the rest of the formula or the TI-89 assumes you are trying to evaluate a function named $p$ at the input $(1 + 0.06/4)$.

Press HOME. Evaluate $y_1$ at these inputs by typing $y_1(x)$, pressing |, and then typing the values of $p$ and $t$ as shown to the right. Access and with CATALOG $\downarrow$ ENTER.

Note that we type $y_1(x)$ on the home screen because this is the TI-89’s name for whatever is entered into the $y_1$ location of the Y= list. To evaluate $y_1$ at other inputs, edit the entry line to the new values and press ENTER. Repeat the process as many times as necessary.

CAUTION: It is very important to note at this point that while we have previously used $x$ as the input variable when entering functions in the Y= list, we do not follow this rule when we evaluate functions with more than one input variable. Realize that we are using the Y= list only as a holding position for the multivariable function. If you prefer, enter the formula on the home screen in the entry line instead of “$y_1(x)$” or define the function. (See Section 1.1.1.)

You should not graph $y_1$ nor use the table. If you attempt to graph the current $y_1 = A(P, t)$ or use the table, you get an Undefined Variable message because the key | only temporarily assigns values to $p$ and $t$ for the evaluation of the formula. If you have $p$ and $t$ defined from some other problem, you will get false results. This is another reason to clear the variable list by pressing 2nd F1 [F6: Clean Up] 2 [NewProb] ENTER whenever you begin a new problem.
9.2 Contour Graphs

Because technologies are not consistent in their abilities to graph three-dimensional functions and/or contour curves, we do not graph these. Instead, we discover information about three-dimensional graphs (that are drawn in the text) using their associated contour curves.

9.2.1 Sketching Contour Curves

When given a multivariable function with two input variables, you can draw contour graphs using the three-step process described below. We illustrate with the function that gives the monthly payments required to pay off a loan of A thousand dollars over a period of t years at 7% interest. This function, $M$, appears in Example 1 of Section 9.2 of Calculus Concepts:

$$M(t, A) = \frac{5833333A}{1 - 0.932583^t} \text{ dollars}$$

Step 1: We are told that the monthly payment on the loan is $520. Set $M(t, A) = 520$. We use the solve instruction to find the value of A at various values of t.

Step 2: Choose values for t and solve for A to obtain points on the $520 constant-contour curve. Check to see if the resulting values of A are reasonable using the table of loan amounts in Figure 9.15 of the text.

Enter the formula for $M$ in y1. Be sure that you use the letters A and t and not x. (We are only using y1 as a “holding place” for the formula – if you prefer, use only the home screen.)

We want to solve $M(t, A) = 520$ for A given various values of t.

Use the solve instruction on the home screen to solve the equation $y1(x) = 520$.

Note: The solve syntax is solve(equation, input variable.) In this case, we are solving for the input A. We type $y1(x)$ for the function because this is the TI-89’s name for whatever we have in y1.

Enter another value for t, say t = 16 years by editing the entry line with $\uparrow$ $\leftarrow$. Find $A \approx 59.963$ thousand. Repeat the procedure for $t = 20$.

Again repeat the procedure for $t = 24$, $t = 28$, and $t = 30$. Make a table of the values of t and A as you find them.

Step 3: Plot the points obtained in Step 2 with pencil and paper. You need to find as many points as it takes to see the pattern the points are indicating when you plot them. Connect the points with a smooth curve.

You need a function to draw a contour graph using the above method. Even though there may be several functions that seem to fit the data points obtained in Step 2, their use would be misleading because the real best-fit function can only be determined by substituting the appro-

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1 The TI-89 draws 3-D graphs and contour plots of multivariable functions. While you are not asked to do this in the text, you may want to explore this topic by reading Chapter 10 of the TI-89 Guidebook.
appropriate values in a multivariable function. The focus of this section is to use contour graphs to study the relationships between input variables, not to find the equation of a function to fit a contour curve. Thus, we always sketch the contours on paper rather than with the TI-89.

9.3 Partial Rates of Change

When you hold all but one of the input variables in a multivariable function constant, you are actually looking at a function of one input variable. Thus, every technique we discussed previously can be used. In particular, the calculator’s derivative can be used to find partial rates of change at specific values of the varying input variable.

9.3.1 CHECKING PARTIAL DERIVATIVE FORMULAS

As mentioned in Chapter 4, the TI-89 gives the algebraic formula for derivatives. So checking your partial derivative formula (or finding the formula) simply involves being able to correctly use the calculator syntax to tell the TI-89 what you are doing. You must be aware of these three major problem areas:

- Unless you use a times sign between two variables, the TI-89 considers what you type to be a single new variable.

- When you use parentheses after a variable, the TI-89 thinks that you are evaluating a function at the input enclosed in the parentheses unless you use a times sign between the variable and the parentheses.

- If the variable is defined (i.e., a number has been stored to that location), the TI-89 automatically evaluates the derivative at that stored value.

We briefly illustrate the above ideas with the function \( f(t, y) = 2ty - t(5y^2 - 2) \). Begin by clearing the single-letter variable locations with 2nd F1 [F6: Clean Up] 2 [NewProb] ENTER or 2nd F1 [F6: Clean Up] 1 [Clear a-z...].

On the home screen, (correctly) take the partial derivative of \( f \) with respect to \( t \) as shown to the right. Note that a times sign is used between \( t \) and \( y \) in the first term and after the parentheses in the second term. The TI-89 rearranges the terms when it prints the answer.

Now, edit the entry line expression to remove the first times sign that is between the \( t \) and \( y \). An incorrect partial derivative results because the TI-89 has considered “\( ty \)” as a new variable that is constant when \( t \) is varying. The calculator is not wrong – this is what is has been programmed to do!

Next, edit the entry line by inserting a times sign between \( t \) and \( y \) in the first term and removing the times sign after \( t \) in the second term. Take the partial derivative of \( f \) with respect to either \( t \) or \( y \). Depending on the function, the TI-89 either does not recognize the second expression or will give an error message.

We now find \( \frac{df}{dy} = 2t - t(10y) = -2(5y - 1)t \). Don’t forget to insert a times sign before the parentheses and have \( y \) at the end of the instruction.
Find \( \frac{\partial f}{\partial y} \) at \( y = 2 \) by editing the entry line to that shown on the first screen to the right. Next, remove "|y=2" and see that \( \frac{\partial f}{\partial y} \) is returned. In other words, \( y \) is still an undefined variable.

Now suppose that \( y \) was defined in an earlier problem to be 6.3. (Store 6.3 in \( y \).) Press \( \uparrow \) four times and \( \text{ENTER} \) to place the instruction to find \( \frac{\partial f}{\partial y} \) on the entry line. Press \( \text{ENTER} \). The answer is not \( \frac{\partial f}{\partial y} \), but \( \frac{\partial f}{\partial y} \approx 6.3 \). Unless you check this answer by finding the partial derivative with algebraic formulas or clear the variables, your answer is incorrect and you won’t know it!

Before proceeding, press \( \text{2nd} \ F1 \) \( \text{ [F6: Clean Up] 2 [NewProb] ENTER} \) or \( \text{2nd} \ F1 \) \( \text{ [F6: Clean Up] 1 [Clear a-z…] ENTER} \). We continue with finding the answers for the partial derivative formulas found in parts \( b \) and \( d \) of Example 1 for this investment function:

The accumulated value of an investment of \( P \) dollars over \( t \) years at an APR of 6% compounded quarterly is \( A(P, t) = P(1.061363551^t) \).

Part \( b \) of Example 1 asks for a formula for \( \frac{\partial A}{\partial t} \), so enter the instruction shown to the right. Whether you enter the function \( A \) in the \( Y= \) list or the home screen, use the letters \( P \) and \( t \) that appear in the formula, and don’t forget to type the times sign after typing \( P \).

We would probably write \( \frac{\partial A}{\partial t} = P(\ln 1.061363551)(1.061363551^t) \). Because \( \ln 1.061363551 \approx 0.059554450328 \), the answers are equivalent except that the TI-89 has rounded the log value.

Part \( d \) of Example 1 says to find \( \frac{\partial A}{\partial P} \) and to find and interpret \( \frac{\partial A}{\partial P} \bigg|_{(P, t) = (7500, 10)} \).

Because we use the function in the \( Y= \) list on the next page, enter \( A \), with inputs \( P \) and \( t \), in \( y1 \). Press \( \text{HOME} \) and find \( \frac{\partial A}{\partial P} \) and \( \frac{\partial A}{\partial P} \bigg|_{(P, t) = (7500, 10)} \).

(Access and with the \( \text{CATALOG} \).)

Geometrically, \( \frac{\partial A}{\partial P} \bigg|_{(P, t) = (7500, 10)} \approx 1.81 \) per dollar invested is the slope of the tangent line to the cross section \( A(P, 10) \) when \( P = 7500 \) is invested for 10 years at 6% compounded quarterly. Let’s view this statement geometrically.

NOTE: If you need to draw a graph or use the table with a multivariable function, remember that when using either of these, the TI-89 considers \( x \) as the variable that is changing. When
finding a partial derivative formula, all other variables are held constant except the one that is
changing. So, to draw a graph or use the table, just store values in all constants and call the
changing variable \( x \). Then, proceed according to the directions given in Chapter 4.

Press \( \bullet \) [F1] (Y=) and replace * \( t \) by
10. and because \( P \) is the variable that is changing in \( \Delta P \), replace \( P \) with \( x \) in
\( y_1 \). Find a suitable viewing window such as the one shown to the right.

*You could instead store 10 in \( t \) on the home screen. However, if you do this, you need to delete the
variable \( t \) before finding any partial derivative formulas.

Draw the graph of \( A \) with \( \bullet \) F3
(GRAPH). Let’s now draw the line
tangent to the graph at \( P = x = 7500 \).
Press [F5] [Math] alpha \( \equiv \) (A) [Tangent]. At the prompt, type 7500.

Why do we not see the tangent line on the last graph? It is because the function in \( y_1 \) is a linear
function with a constant slope equal to \( 1.061363551 \times 10^{-1} \approx 1.81 \). Note that the equation of the
tangent line that the TI-89 prints at the bottom of the last screen is essentially the same as \( y_1 \).

9.3.2 **VISUALIZING AND ESTIMATING PARTIAL RATES OF CHANGE**

As we just saw, the partial rate of change of a multivariable function (evaluated at a specific point) is the slope
of the line tangent to the graph of a cross-sectional model at a given location. We illustrate this
important concept in this section and again in Section 9.3.3 using the Missouri farmland cross-
sectional equations for elevation: \( E(0.8, n) \) and \( E(e, 0.6) \). It would be best to use the unrounded
functions that we found in Sections 9.1.1 and 9.1.2 of this Guide. However, for convenience,
we use the rounded functions rather than re-enter all the data.

Enter \( -2.5n^2 + 2.497n + 799.490 = E(0.8, n) \) in \( y_1 \). Because
we are going to graph this function, use \( x \), not \( n \), as the input
variable. Press \( \bullet \) F2 (WINDOW) and set values such as
those shown to the right.

**NOTE:** The window settings can be obtained by drawing a scatter plot of the data used to
obtain the function \( E(0.8, n) \) or by looking at the \( e = 0.8 \) column in Table 9.1 in the text.

You can draw the line tangent to the graph of the cross section \( E(0.8, n) \) at \( n = x = 0.6 \)
from either the home screen or the graphics screen. (See Section 3.2.2 of this Guide for an
explanation of all the methods.) We use the home screen in this section and the graphing
screen in the next section.

Press \( \bullet \) F3 (GRAPH) to draw the
graph of \( y_1 \). Use [HOME] [CATALOG]
4 to scroll down to LineTan. Press
[ENTER]. Next enter \( y_1(x) \) \( \square \) 0.6.
You can obtain an *estimate* of the tangent line slope by pressing [F5] [Math] 6 [Derivatives] 1 [dy/dx]. Use ► or ◄ to move the cursor as close as possible to \( n = x = 0.6 \). Press ENTER.

We could have obtained a much better estimate than the one given above if we had found the slope of the tangent line at \( x = 0.6 \) rather than a value close to that number. One way to do this is to type 0.6 at the \( dy/dx \) at ? prompt rather than using the arrow keys in the previous discussion. Another way is to use the tangent line instruction on the graphing screen.

Have \( E(0.8, n) \), using \( x \) as the input variable, in \( y_1 \). Draw the tangent line to \( E(0.8, n) \) at \( n = 0.6 \) by first drawing the graph of the function with ◀ [F3] (GRAPH). If the tangent line is still on the screen from the previous section, press [F4] [ReGraph]. On the graph screen, press [F5] [Math] alpha (A) [Tangent]. At the prompt, type 0.6 and press ENTER.

Because this method gives the equation of the line tangent to the graph at the chosen input, you can see that the slope of the tangent, \( \frac{dE(0.8, n)}{dn} \) at \( n = 0.6 \), is about \(-0.503\) foot/mile.

### 9.3.3 Finding Partial Rates of Change Using Cross-Sectional Models

The procedures described in the last section required that you use \( x \) as the input variable to find partial rates of change. The process of replacing other variables with \( x \) can be confusing. You can avoid having to do this replacement when you use the TI-89 derivative on the home screen to evaluate partial rates of change at specific input values as shown in Section 9.3.1.

We use the cross-sectional function \( E(0.8, n) \) that was discussed in the last section of this Guide and the cross-sectional function \( E(e, 0.6) = -10.124e^3 + 21.347e^2 - 13.972e + 802.809 \) feet above sea level that is given in Example 2 of Section 9.3 in *Calculus Concepts* to again illustrate this method.

Have \( E(0.8, n) = -2.5n^2 + 2.497n + 799.490 \), using \( n \) as the input variable, in \( y_1 \). Enter \( E(e, 0.6) \), using \( e \) as the input variable, in \( y_2 \). Type \( e \) with alpha \( \div \) (E).

(Remember that because we are not using \( x \) as the input variable, we should not draw a graph or use the table.)

**NOTE:** These functions could be entered on the home screen instead of in the \( Y= \) list. We use \( y_1 \) and \( y_2 \) in the \( Y= \) list because they are convenient locations to hold the equations.

On the home screen, find \( \frac{dE(0.8, n)}{dn} \) evaluated at \( n = 0.6 \). Note that because each of these functions involves only one variable, it is not necessary to specify both coordinates of the point in the derivative notation.

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Find \( \frac{dE(e, 0.6)}{de} \) evaluated at \( e = 0.8 \) to be about 0.745 foot per mile. (Always remember to attach units of measure to the numerical values when writing your answer.)

If you want to see the line tangent to the graph of \( E(e, 0.6) \) at \( e = 0.8 \), first change every \( e \) in \( y_2 \) to \( x \) and turn off \( y_1 \). Draw the graph of \( y_2 \) and use the LineTan instruction in the CATALOG or Tangent in the MATH menu.

### 9.4 Compensating for Change

The TI-89 derivative function is very useful and can help you eliminate many potential calculation mistakes when you find the rate of change of one input variable with respect to another input variable (that is, the slope of the tangent line) at a point on a contour curve.

#### 9.4.1 Evaluating Partial Derivatives of Multivariable Functions

The last few sections of this Guide indicate how to estimate and evaluate partial derivatives using cross-sectional models and multivariable function formulas. The most important thing to remember is that you must supply the name of the input variable that is changing and the values at which the partial derivative is evaluated. We illustrate using the body-mass index function that is in Example 1 of Section 9.4 of Calculus Concepts:

A person’s body-mass index is given by \( B(h, w) = \frac{0.4536w}{0.0064516h^2} \), where \( h \) is the person’s height in inches and \( w \) is the person’s weight in pounds. We first find \( B_h \) and \( B_w \) at a specific height and weight and then use those values in the next section of this Guide to find the value of the derivative \( \frac{dw}{dh} \) at that particular height and weight. The person in this example is 5 feet 7 inches tall and weighs 129 pounds: \( 67h \times 129w \)

First we find the value of \( B_h \) at \( h = 67 \) and \( w = 129 \). The symbol \( B_h \) (or \( \partial B/\partial h \)) tells you that \( h \) is varying and \( w \) is constant, so \( h \) is the variable you type in the derivative notation.

Type \( B \) in the derivative instruction, using the letters \( h \) and \( w \) for the input variables. Type the name of the changing variable and the values of \( h \) and \( w \) at the given point. (Remember that and is accessed in the CATALOG.)

Store this value in num for use in the next section of this Guide.

Find the value of \( B_w \) at \( h = 67 \) and \( w = 129 \). The easiest way to do this is to press \( \blacktriangle \) until the previous derivative expression is highlighted and press [ENTER] to copy it to the entry line. Then edit the expression by replacing the last \( h \) by \( w \).
Store this value in den for use in the next section of this Guide.

Note: Have you noticed that if you just press \texttt{STO} that \texttt{ans(1)} appears automatically on the screen?

**CAUTION:** Forgetting to specify the values of all variables at the point at which the derivative is to be evaluated is a common mistake made when using this method – you must tell the calculator the values of all inputs.

### 9.4.2 Finding the Slope of a Line Tangent to a Contour Curve

We continue the previous illustration with the body-mass index function in Example 1 of Section 9.4 of *Calculus Concepts*. Part a of Example 1 asks for $\frac{dw}{dh}$ at the point (67, 129) on the contour curve corresponding to the person’s current body-mass index. The formula is $\frac{dw}{dh} = -\frac{B_h}{B_w}$.

An easy way to remember this formula is that whatever variable is in the numerator of the derivative (in this case, $w$) is the same variable that appears as the changing variable in the denominator of the slope formula. This is why we stored $B_w$ as \texttt{den} (for denominator) and $B_h$ as \texttt{num} (for numerator). Don’t forget to put a minus sign in front of the numerator.

In the previous section, we stored $B_h$ as \texttt{num} and $B_w$ as \texttt{den}. So, $\frac{dw}{dh} = -\frac{B_h}{B_w} = -\text{num} \div \text{den}$. (Storing these values also avoids round-off error.) The rate of change is about 3.85 pounds per inch.

### 9.4.3 Compensating for Change

When one input of a two-variable multivariable function changes by a small amount, the value of the function is no longer the same as it was before the change. The methods illustrated below show how to determine the amount by which the other input must change so that the output of the function remains at the value it was before any changes were made. We again continue the previous illustration with the body-mass index function and part b of Example 1 of Section 9.4 of *Calculus Concepts*.

To estimate the change in weight needed to compensate for growths of 0.5 inch, 1 inch, and 2 inches if the person’s body-mass index is to remain constant, you need to find $\Delta w \approx \frac{dw}{dh} (\Delta h)$ at the given values of $\Delta h$.

Again, to avoid rounding error, it is easiest to store the slope of the tangent line to some location, say \texttt{tn}, and use the unrounded value to calculate.

**NOTE:** You probably have noticed that when we store results we use names containing more than one letter (for instance: \texttt{den}, \texttt{num}, and \texttt{tn}). The reason we do this is because defining these variables will not interfere with symbolic results in later problems when single-character letters are used for variable names.