Guide for Texas Instruments TI-89 Graphics Calculator

This Guide is designed to offer step-by-step instruction for using your TI-89 graphing calculator with the text Calculus Concepts: An Informal Approach to the Mathematics of Change. A technology icon next to a particular example or discussion in the text directs you to a specific portion of this Guide. You should also utilize the table of contents in this Guide to find specific topics on which you need instruction.

Setup Instructions

Generally, the TI-89 will show the home screen when it is turned on. If not press 2nd ESC (QUIT) or press HOME. Before you begin, check the TI-89 setup and make sure the settings described below are chosen. Whenever you use this Guide, we assume (unless instructed otherwise) that your calculator settings are as shown in the figures below.

- Press MODE and choose the settings shown in Figures 1, 2, and 3 for the basic setup. Pressing F1, F2, and F3 open up the three pages of the MODE menu that are shown below.

  ![Figure 1](image1)
  ![Figure 2](image2)
  ![Figure 3](image3)

  TI-89 Basic Setup

- If you do not have the darkened choices shown in any figure, use the arrow keys to move the blinking cursor over the setting you want to choose and press ENTER. When “→” appears next to a word, press ▲ to see the available choices and, if necessary, use ▼ to select the one you need. Then press ENTER to choose that selection and return to the mode screen.

**WARNING:** To save all changes when exiting from all menu boxes (such as the MODE screen), you must press ENTER to leave that box. If you exit the box by using ESC or by pressing any other key, the changes you made will disappear.
• Check the graph format by pressing \texttt{APPS} 2 [Y= Editor] (or use \texttt{▼} to highlight the second choice and press \texttt{ENTER}). Press \texttt{F1} [Tools] 9 [Format] and choose the settings in Figure 4. Press \texttt{ENTER} and return to the home screen with \texttt{HOME}.

• Begin by pressing \texttt{2nd} \texttt{F1} [F6: Clean Up] 2 [NewProb] \texttt{ENTER} which clears single-character variable names, turns off functions, clears graphs, and so forth. You can also clear the history area (the main portion of the screen showing previous entries) with \texttt{F1} 8 [Clear Home] and clear the entry line at the bottom of the screen with \texttt{CLEAR}. You may need to press \texttt{CLEAR} more than once. If you clear the history area and entry line, your screen looks like the one in Figure 6.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures.pdf}
\caption{TI-89 Graph Format}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures.pdf}
\caption{Begin a New Problem}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures.pdf}
\caption{Cleared Screen}
\end{figure}

\section*{Basic Operation}

You should be familiar with the basic operation of your calculator. With your calculator in hand, go through each of the following. We illustrate several methods – choose the ones you prefer.

1. \textbf{CALCULATING} You can type in lengthy expressions; just make sure that you use parentheses when you are not sure of the calculator's order of operations. Always enclose in parentheses any numerators and denominators of fractions and powers that consist of more than one term.

\begin{equation}
\frac{1}{4 \times 15 + \frac{895}{7}}.
\end{equation}

Enclose the denominator in parentheses so that the addition is performed before the division into 1. It is not necessary to use parentheses around the fraction 895/7.

The TI-89 prints the same way you should have the expression written on your paper. Always check the left side of the screen and compare what you entered with what you have on paper.

To make a correction in the entry line, use \texttt{►} or \texttt{◄} to position the cursor to the right of what you want to delete and then press \texttt{Del}. The entry line is always in insert mode. You do not need to clear the entry line before beginning to type a new expression.

\begin{equation}
\frac{(-3)^4 - 5}{8 + 1.456}.
\end{equation}

Use \texttt{(-)} for the negative symbol and \texttt{+} for the subtraction sign. The numerator and denominator must be enclosed in parentheses and $-3^4 \neq (-3)^4$. 
Now, evaluate \( e^{3 \times 0.027} \) and \( e^{3 \times 0.027} \). The TI-89 prints the left parenthesis when you press \( \boxed{\text{X}} \) \( (e^x) \). If you do not include a right parenthesis or insert an extra parenthesis, the TI-89 gives an error message. Press \( \boxed{\text{ESC}} \), fix the error, and press \( \boxed{\text{ENTER}} \).

If you want to edit a previous expression to a new expression, have the cursor on the entry line and press \( \boxed{\uparrow} \). The cursor appears on the right-hand side of the entry line expression. If you press \( \boxed{\downarrow} \), the cursor appears on the left-hand side of the entry line expression. Try editing \( e^{3 \times 0.027} \) to type \( e^{3 \times 0.027} \).

- To recall an expression prior to the current one (before the history area is cleared), press 2nd \( \boxed{\text{ENTRY}} \) (ENTRY). You can also select any entry or answer from the history area and “autopaste” a duplicate of it on the entry line. This allows you to insert something previously typed or calculated into a new expression. First, place the cursor in the entry line where you want to insert the entry or answer. Second, move the cursor, repeatedly using \( \boxed{\uparrow} \), until what you are inserting is highlighted. Third, press \( \boxed{\text{ENTER}} \).

2. USING THE ANS MEMORY

Next we explore recalling previous expressions and answers to use in new calculations. We also see how to use the TI-89 answer memory. Instead of again typing an expression that was just evaluated, use the answer memory by pressing 2nd \( \boxed{(-)} \) (ANS).

Find \( \left( \frac{1}{4 \times 15 + \frac{895}{7}} \right)^{-1} \) using this shortcut.

Enter Ans with 2nd \( \boxed{(-)} \) (ANS).

NOTE: The last-calculated answer is referred to by the TI-89 on the entry line as \( \text{ans}(1) \) when you enter each expression. The expression for the answer is substituted when the new expression appears in the history area. You can also use auto-paste to enter the previous results.

3. ANSWER DISPLAY

We have MODE set to AUTO which means that an exact answer is given whenever possible. If you have a decimal in an expression, a decimal value is returned for the answer. If you want a decimal approximation for the expression that appears on the entry line, press \( \boxed{\text{ENTER}} \). We also illustrate how to retrieve a previous entry that is in the history area.

Type \( \frac{2}{5} + \frac{1}{3} \), press \( \boxed{\text{ENTER}} \), and then press \( \boxed{\text{ENTER}} \). Press \( \boxed{\uparrow} \) until \( \frac{7}{1315} \) is highlighted. Press \( \boxed{\text{ENTER}} \).

Use \( \boxed{\downarrow} \) and insert a decimal point after the 7. Press \( \boxed{\text{ENTER}} \).
The TI-89 can switch to scientific notation when the number is too large for the display area in a table or graph. The TI-89’s symbol for “times 10^{−6}” is E−6. So, 1.4675E−6 means 1.4675×10^{−6}, which is scientific notation for 0.0000014675. Enter “E” by pressing [EE].

The result 1.5E28 means 15,000,000,000,000,000,000,000,000 = 1.5(10^{28}). In the history area, an arrowhead (→) that points to the right at the end of a number means that the number or expression continues. Highlight the number or expression and press [►] to scroll and see the rest of it. Press [▼] to return to the entry line.

4. **STORING VALUES**  It often is beneficial to store numbers or expressions for later recall. To store a number, type the number, press [STO], type the letter(s) corresponding to the storage location, and then press [ENTER]. Join several short commands with a colon between the statements. Note that when you join statements with a colon by pressing [2nd] 4 (:), only the value of the last statement is shown as a result.

Store 5 in $a$ and 3 in $b$, and then calculate $4a − 2b$.

To recall a stored value, press [alpha], type the letter in which the expression or value is stored, and then press [ENTER].

- Storage location names on the TI-89 can be from one to eight characters long and use letters and numbers, but they must begin with a letter. You cannot name what you are storing with the same name that the TI-89 already uses for a built-in variable (such as LOG or ans).
- Whatever you store in a particular memory location stays there until it is replaced by something else either by you or by executing a program containing that name. It is advisable to use single-letter names so that the values will be cleared when you begin with NewProb.

**NOTE:** The TI-89 allows you to enter upper and lower case letters, but it does not distinguish between them. For instance, VOL, Vol, VOL, vol, vol., and so forth all name the same variable. To type a lower-case letter, press [alpha] before typing a letter key (note that a lower-case $a$ appears under the entry line). To type an uppercase-letter, press [→alpha] (note that an upper-case $A$ appears under the entry line) before typing the key corresponding to the letter.

If a variable is undefined (i.e., you have not stored a value in it), it is treated as an algebraic symbol. If a variable is defined, its value replaces the variable when you enter an expression containing that variable. It is best, as we see in later chapters, to leave the names $x$ and $y$ as undefined variables.

**WARNING:** You must be very careful when entering expressions containing variables. In the next-to-last expression shown on the screen above, the TI-89 assumes “$ax$” refers to the name of a single undefined variable. To tell the calculator that you want to multiply the variable $a$ by the variable $x$, you must use [×] between the letters. Also, because 5 has been stored to $a$, the TI-89...
substitutes that value when the expression is evaluated. All user-defined variables are stored in a folder\(^1\) called MAIN unless you create other folders to hold them. If you cannot remember the name of a variable, press \(\text{2nd} - (\text{VAR-LINK})\) and the names of the variables are listed. After using \(\text{\textdownarrow}\) to move the cursor to the name of the variable, you can delete, copy, rename, etc. using the \([\text{F1}]\) [Manage] key on the VAR-LINK screen. We illustrate by deleting the variable \(a\):

**Deleting a single variable:** Suppose we want to delete the variable \(a\). Go to the VAR-LINK screen with \(\text{2nd} - (\text{VAR-LINK})\). Delete \(a\) by first highlighting it and then pressing \([\text{F1}]\) [Manage] \([\text{1}]\) [Delete] \(\text{ENTER}\) and \(\text{ESC}\). (Note that your MAIN folder may contain more or different variables from what is shown to the right.)

The “with” operator, which prints as “\(\mid\)”, gives a way to temporarily store values into a variable in order to evaluate expressions so that you do not have to delete the variable when you finish. Access this operator by pressing the key directly under \(\text{\textminus}\).

Evaluate \(5.3r - 2.1h^2\) for \(r = 6\) and then for \(r = 2.9\) and \(h = 7\) by typing in the instructions shown to the right. Access “and” with \([\text{CATALOG} = (\text{A}) \text{\textdownarrow} \text{ENTER}].\) Press \(\text{2nd} - (\text{VAR-LINK})\) and see that \(r\) and \(h\) are not defined variables.

### 5. ERROR MESSAGES

When your input is incorrect, the TI-89 displays an error message.

If you have more than one command on a line without using a colon (\(\text{:}\)) to separate them, an error message results when you press \(\text{ENTER}\). Press \(\text{ESC}\) and correct the error.

If you try to store something to a particular memory location that is being used for a different type of object, an error results. (In the screen shown to the right, the variable \(abc\) was defined as a data list, so a number cannot be stored to it.) Consult either *Trouble-Shooting the TI-89* in this Guide or your *TI-89 Owner’s Guidebook*.

- The error message shown directly above usually occurs when you are executing a program that is trying to store a value to a variable that you have previously defined as some other type of object. Rename the other type of object and rerun the program. Remember, store only numbers to single-letter names to avoid problems such as these.

A common mistake is using the negative symbol \((\text{-})\) instead of the subtraction sign \((\text{-})\) or vice-versa. The TI-89 does *not* give an error message, but a wrong answer results. The negative sign is shorter and raised slightly more than the subtraction sign.

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\(^1\) We work only in the MAIN folder. If you wish to create different folders, see pages 88-90 of the *TI-89 Guidebook*.
Chapter 1  Ingredients of Change:  
Functions and Linear Models

1.1  Models, Functions, and Graphs

There are many uses for a function that is entered in the calculator. One of the ways we often use functions is to graph them in an appropriate viewing windows. Because you must enter a function on one line (that is, you cannot type fractions and exponents the same way you write them on paper even though the TI-89 displays them in this form after they are entered), it is very important to have a good understanding of the calculator’s order of operations and to correctly use parentheses whenever they are needed.

1.1.1  ENTERING AN EQUATION IN THE GRAPHING LIST

The graphing list contains space for 99 equations, and the output variables are called by the names \( y_1(x) = \), \( y_2(x) = \), ..., and \( y_{99}(x) = \). When you want to graph an equation, first enter in the graphing list. You must use \( x \) as the input variable if you intend to draw the graph of the equation or use the TI-89 table.

We illustrate graphing with the equation in Example 4 of Section 1.1: \( v(t) = 3.622(1.093^t) \).

Press either  \( \text{F1} \) (Y = ) or  \( \text{APPS} 2 \) [Y= Editor] to access the graphing list. If there are any previously entered equations that you will no longer use, delete them from the graphing list. For convenience, we use the first, or \( y_1 \), location in the graphing list.

To delete an equation from the Y = list, position the cursor on the line with the equation. Press CLEAR.

We intend to graph this equation, so the input variable must be called \( x \), not \( t \). Enter the right-hand side of the equation, \( 3.622(1.093^x) \), with 3.622 (1.093 ^x) ENTER.

Note that you must use the X key for \( x \) (under the HOME key), not the times sign key, \( \times \).

**CAUTION:** Press  \( \text{▲} \) and notice the names Plot 1, Plot 2, …, Plot 9 at the top of the Y= list. These are the data plots that we will not use until Section 1.5. None of these plots should have a check mark to the left of the name when you are graphing an equation and not graphing data points. If any of these has a check mark, use  \( \text{▲} \) until the cursor is on the plot name and press  \( \text{F4} \) [b] to make the check mark go away. Otherwise, you may receive an error message.

**DEFINING A FUNCTION ON THE HOME SCREEN**

Functions can be entered in the graphing list or defined on the home screen. We illustrate both of these methods, and you can use the one you prefer. Press  \( \text{2nd} \) [ESC] (QUIT) or  \( \text{HOME} \) to return to the home screen.

Type in \( v(t) = 3.622(1.093^t) \) with \( \text{F4} \) [Other]  \( 1 \) [Define] alpha 0 (V) [T] ] \( \Rightarrow \) 3.622 [6 2 2 [1 [0 9 3 [T] ] ] ENTER.

On the home screen, any letter can be used for the variable. Note that if you prefer, you can enter the equation with \( x \) instead of \( t \) as the input variable. (The TI-89 now knows this function by two names: \( y_1 \) and \( v \).)
1.1.2 DRAWING A GRAPH  As is the case with most applied problems in Calculus Concepts, the problem description indicates the valid input interval. Consider Example 4 of Section 1.1:

The value of a piece of property between 1980 and 2000 is given by \( v(t) = 3.622(1.093^t) \) thousand dollars where \( t \) is the number of years since the end of 1980.

The input interval is 1980 (\( t = 0 \)) to 2000 (\( t = 20 \)). Before drawing the graph of \( v \) on this interval, have the function \( v \) entered in the \( Y= \) list using \( x \) as the input variable. (See Section 1.1.1 of this Guide.) We now draw the graph of \( v \) for \( x \) between 0 and 20.

Press \( \boxed{\text{F2}} \) (WINDOW) to set the view for the graph. Enter 0 for xmin and 20 for xmax. (For 10 tick marks between 0 and 20, enter 2 for xscl. If you want 20 tick marks, enter 1 for xscl, etc.; xscl does not affect the shape of the graph. Ignore the other numbers — we set their values in the next set of instructions.)

The numbers xmin and xmax are, respectively, the settings for the left and right edges of the viewing screen, and ymin and ymax are, respectively, the settings for the lower and upper edges of the viewing screen. xscl and yscl set the spacing between the tick marks on the x- and y-axes. (Leave xres set to the default value of 2 for all applications in this Guide.)

To have the TI-89 determine the view for the output, press \( \boxed{\text{F2}} \) [Zoom] \( \alpha \) (A) [ZoomFit] or use \( \boxed{\text{▼}} \) to highlight ZoomFit and press \( \boxed{\text{ENTER}} \).

Note that any vertical line drawn on this graph intersects it in only one point, so the graph does represent a function.

Press \( \boxed{\text{F2}} \) (WINDOW) to see the view set by ZoomFit.

The view has \( 0 \leq x \leq 20 \) and \( 3.622 \leq y \leq 21.446... \).

(Leave xres set to the default value of 2 for all applications in this Guide.)

Let’s now explore how to graph with the function defined on the home screen as \( v(t) \).

Press \( \boxed{\text{HOME}} \) and type \( v(t) \) \( \boxed{\text{ENTER}} \) to be sure you are using the correct function. Press \( \boxed{\text{F1}} \) (Y=) and enter \( v(x) \) in the \( y_2 \) location. Press \( \boxed{\text{▲}} \) and \( \boxed{\text{F4}} \) [b] to turn off \( y_1 \).

Draw the graph of \( v = y_2 \) using exactly the same procedure as that for drawing the graph of \( y_1 \). If you need to edit a function, press \( \boxed{\text{ENTER}} \) or \( \boxed{\text{F3}} \) [Edit] to move the cursor to the entry line from the graphing list. Press \( \boxed{\text{F4}} \) [b] to deactivate \( y_2 \) after you draw its graph.

WARNING: You must use \( x \) as the input variable when in the \( Y= \) list. If you use \( t \) or another letter inside the parentheses following \( v \), you will likely get an “undefined variable” message.
1.1.3 MANUALLY CHANGING THE VIEW OF A GRAPH  We just saw how to have the TI-89 set the view for the output variable. Whenever you draw a graph, you can also manually set or change the view for the output variable. If for some reason you do not have an acceptable view of a graph or if you do not see a graph, change the view for the output variable with one of the zoom options or manually set the WINDOW until you see a good graph. (We will later discuss other zoom options.) We continue using the function \( v \) in Example 4 of Section 1.1, but assume here that you have not yet drawn the graph of \( v \).

Press \( \boxed{\mathbf{F}2} \) (WINDOW), enter 0 for xmin, 20 for xmax, and (assuming we do not know what to use for the vertical view), enter some arbitrary values for ymin and ymax. (This graph was drawn with ymin = -5 and ymax = 7). Press \( \boxed{\mathbf{F}3} \) (GRAPH).

**NOTE:** If you see nothing on the screen, press \( \boxed{\mathbf{F}1} \) (Y = ) and highlight \( y_1 \). If there is no check mark next to \( y_1 \), press \( \boxed{\mathbf{F}4} \) [ ] and \( \boxed{\mathbf{F}3} \) (GRAPH).

### Evaluating Outputs on the Graphics Screen:
Press \( \boxed{\mathbf{F}3} \) [Trace].

* Recall we are given in this application that the input variable is between 0 and 20. If you now type the number that you want to substitute into the function whose graph is drawn, say 0, you see the screen to the right. A 1 appears at the top right of the screen because the equation of the function whose graph you are drawing is in \( y_1 \).

Press ENTER and the input value is substituted in the function.

The input and output values are shown at the bottom of the screen. (This method works even if you do not see any of the graph on the screen.)

Substitute the right endpoint of the input interval into the function by pressing 20 ENTER. We see that two points on this function are approximately (0, 3.622) and (20, 21.446).

Press \( \boxed{\mathbf{F}2} \) (WINDOW), enter 3.5 for ymin and 22 for ymax, and press \( \boxed{\mathbf{F}3} \) (GRAPH). If the graph you obtain is not a good view of the function, repeat the above process using \( x \)-values in between the two endpoints to see if the output range should be extended in either direction. (Note that the choice of the values 3.5 and 22 was arbitrary. Any values close to the outputs in the points you find are also acceptable.)

*Instead of using TRACE, you could use the TI-89 TABLE or evaluate the function at 0 and 20 on the home screen to find the range of values for the output value. We later discuss using these features.

1.1.4 TRACING TO ESTIMATE OUTPUTS  You can display the coordinates of certain points on the graph by tracing. Unlike the substitution feature of TRACE that was just discussed, the \( x \)-values that you see when tracing the graph depend on the horizontal view that you choose. The output values that are displayed at the bottom of the screen are calculated by substituting the \( x \)-
values into the equation that is being graphed. We again assume that you have the function \( v(x) = 3.622(1.093^x) \) entered in the \( y_1 \) location of the \( Y= \) list.

With the graph on the screen, press \( \text{F3} \) [Trace], press and hold \( \text{►} \) to move the trace cursor to the right, and press and hold \( \text{◄} \) to move the trace cursor to the left. Again, note that the number corresponding to the location of the equation (in the \( Y= \) list) that you are tracing appears at the top right of the graphics screen.

Trace past one edge of the screen and notice that even though you cannot see the trace cursor, the \( x \)- and \( y \)-values of points on the line are still displayed at the bottom of the screen. Also note that the graph scrolls to the left or right as you move the cursor past the edge of the current viewing screen.

Use either \( \text{►} \) or \( \text{◄} \) to move the cursor near \( x = 15 \). We estimate that \( y \) is approximately 13.7 when \( x \) is about 15.

It is important to realize that trace outputs should never be given as answers to a problem unless the displayed \( x \)-value is identically the same as the value of the input variable.

1.1.5 EVALUATING OUTPUTS ON THE HOME SCREEN

The input values used in the evaluation process are actual function values, not estimated values such as those generally obtained by tracing near a certain value. Actual values are also obtained when you evaluate outputs from the graphing screen using the process that was discussed in Section 1.1.3 of this Guide.

We again consider the function \( v(t) = 3.622(1.093^t) \) that is in Example 4 of Section 1.1.

Using \( x \) as the input variable, have \( 3.622(1.093)^x \) entered in \( y_1 \).

Return to the home screen by pressing \( \text{HOME} \). Substitute 15 into the function with \( Y_1(15) \) and find the function value by pressing \( \text{ENTER} \).

NOTE: We choose \( y_1 \) as the function location most of the time, but you can use any of the available locations. If you do, replace \( y_1 \) in the instructions with the function you choose.

It is now a simple matter to evaluate the function at other inputs. For instance, substitute \( x = 20 \) into the equation by editing the entry line using \( \text{►} \), changing 15 to 20 by pressing \( \leftarrow \leftarrow \leftarrow \) and typing 20 \( \leftarrow \), and then pressing \( \text{ENTER} \). Evaluate \( y_1 \) at 0 using the same procedure.

If you defined the function \( v \) with input \( t \) (see the bottom of page C-6), it can easily be used to find the function outputs.
1.1.6 Evaluating Outputs Using the Table

Function outputs can be determined by evaluating on the graphics screen, as discussed in Section 1.1.3, or by evaluating on the home screen as discussed in Section 1.1.5 of this Guide. You can also evaluate functions using the TI-89 TABLE. When you use the table, you can either enter specific input values and find the outputs or generate a list of input and output values in which the inputs begin with tblStart and differ by Δtbl.

Let’s use the TABLE to evaluate the function \( v(t) = 3.622(1.093)^t \) at the input \( t = 15 \). Even though you can use any of the function locations, we again choose to use \( y_1 \). Press \( \blacklozenge \) F1 (Y =), clear the function locations, and enter \( 3.622(1.093)^x \) in location \( y_1 \).

Choose TblSet with \( \blacklozenge \) F4 (TblSet). To generate a list of values beginning with 13 such that the table values differ by 1, enter 13 in the tblStart location, 1 in the Δtbl location, have Graph \( \Rightarrow \) Table set to OFF, and choose AUTO in the Indpnt: location. (Remember that you “choose” a certain setting by having it highlighted and pressing \( \text{ENTER} \).) Exit the setup with \( \text{ENTER} \).

Press \( \blacklozenge \) F5 (TABLE) and observe the list of input and output values. Notice that you can scroll through the table with \( \downarrow \), \( \uparrow \), \( \leftarrow \), and/or \( \rightarrow \). The table values may be rounded in the table display. You can see more of the output by highlighting a particular value and viewing the bottom of the screen.

**NOTE:** If you wish, while in the table, you can change the cell width using the F1 [Tools] menu and option 9 [Format]. The cell widths can vary between 3 and 12, which result in as few as 2 columns and as many as 7 columns. All functions that are checked in the Y= list display in the table, so we suggest leaving the width at its default setting.

If you want to evaluate a function at inputs that are not evenly spaced and/or you only need a few outputs, you should use the ASK feature of the table instead of using AUTO. Note that when using ASK, the settings for tblStart and Δtbl do not matter and are dimmed on screen.

Choose TblSet with \( \blacklozenge \) F2 [Setup] from within the table or \( \blacklozenge \) F4 (TblSet). Choose ASK in the Indpnt: location. To enter the x-values 15, 0, and 20, type each value, press \( \text{ENTER} \), type the next value, and so forth.

**NOTE:** Unwanted entries or values in the table can be cleared by highlighting the value in the x column and pressing \( \leftarrow \) or by using \( \text{F1} \) [Tools] 8 [Clear Table] to delete all previous entries.

1.1.7 Finding Input Values Using the Solver

Your calculator solves for the input values of all the equations we use in this course. The expression can, but does not have to, use \( x \) as the input variable. The TI-89 offers several methods of solving for input variables. We first illustrate using the solve instruction. (Solving using graphical methods will be discussed after using the solve instruction and the TI-89 numeric solver are explored.) You can refer to an equa-
tion that you have already entered in the Y= list or you can enter the equation directly in the solver. After we finish exploring all the methods, you should choose the one you prefer.

Return to the home screen with [HOME]. (If you want to clear the history area, remember that [F1] 8 [Clear home] does it.) Enter solve( on the entry line by typing “solve(” using the keyboard, or by pressing [F2] 1 [solve()].

Let’s now use the solver to answer the question in part e of Example 4 in Section 1.1: “When did the land value reach $20,000?” Because the land value is given by \( v(t) = 3.622(1.093^t) \) thousand dollars where \( t \) is the number of years after the end of 1980, we are asked to solve the equation \( 3.622(1.093^t) = 20 \). That is, we are asked to find the input value \( t \) that makes this equation a true statement.

If you already have \( y_1 = 3.622(1.093^x) \) in the graphing list, you can refer to the function as \( y_1(x) \) in the entry line. If not, you can enter \( 3.622(1.093^x) \) instead of \( y_1 \). You must also tell the TI-89 the name of the input variable. Type either \( y_1(x) = 20 \) or \( "3.622(1.093^x) = 20 \) behind “solve(“ and press [ENTER].

WARNING: If you refer to an equation by its location in the Y= list, you must use the complete name of the equation (i.e., \( y_1(x) \) instead of \( y_1 \)) when you enter it in the solve instruction.

- If a solution continues beyond the edge of the calculator screen, you see “►” to the right of the value. Be certain that you press [▲] to highlight the solution and then press and hold ► to scroll to the end of the answer.

USING THE NUMERICAL SOLVER The solve instruction is very easy to use and offers a fairly quick method of finding an answer. However, if you have previously used other model TI calculators, you may be more familiar with the numeric equation solver.

Access the numeric solver with [APPs] 9 [Numeric Solver]. If there are no equations in the solver, you will see the screen displayed on the right. (If there is an equation entered after eqn:, press [CLEAR] to delete it.)

If you already have \( y_1 = 3.622(1.093^x) \) in the graphing list, you can refer to the function as \( y_1(x) \) in the numeric solver. If not, enter \( 3.622(1.093^x) = 20 \) instead of \( y_1(x) = 20 \) in the eqn: location. Press [ENTER]. Enter a guess* for \( x \) – say 19.

If you need to edit the equation, press [▲] until the previous screen reappears. Edit the equation and then return here. Next, be certain that the cursor is on the line corresponding to the input variable for which you are solving (in this example, \( x \)). Solve for the input by pressing [F2] [Solve].

*More information on entering a guess appears in the next section of this Guide.
CAUTION: You should not change anything in the “bound” location of the SOLVER. The values in that location are the ones between which the TI-89 searches for a solution. If you should accidentally change anything in this location, exit the solver, and begin the entire process again. (The bound is automatically reset when you exit the numeric solver.)

The answer to the original question is that the land value was $20,000 about 19.2 years after 1980 – i.e., in the year 2000.

- Notice the black dot that appears next to $x$ and next to the last line on the above screen. This is the TI-89’s way of telling you that a solution has been found. When the bottom line on the screen that states left – rt $\approx 0$, the value found for $x$ is an exact solution.

1.1.8 HOW TO DETERMINE A GUESS TO USE IN THE NUMERIC SOLVER This section of the Guide applies only if you are using the numeric solver described directly above. If you have decided to use only the solve instruction in the entry line, skip this discussion.

What you use in the solver as a guess tells the TI-89 where to start looking for the answer. How close your guess is to the actual answer is not very important unless there is more than one solution to the equation. If the equation has more than one answer, the numeric solver will return the solution that is closest to the guess you supply. In such cases, you need to know how many answers you should search for and their approximate locations.

Three of the methods that you can use to estimate the value of a guess for an answer from the numeric solver follow. We illustrate these methods using the land value function from Example 4 of Section 1.1 and the equation $v(t) = 3.622(1.093^t) = 20$.

1. Enter the function in some location of the graphing list – say $y_1 = 3.622(1.093^x)$ and draw a graph of the function. Press [F3] [Trace] and hold down either ► or ◄ until you have an estimate of where the output is 20. Use this x-value, 19 or 19.4 or 19.36, as your guess in the SOLVER.

2. Enter the left- and right-hand sides of the equation in two different locations of the Y= list – say $y_1 = 3.622(1.093^x)$ and $y_2 = 20$. With the graph on the screen, press [F3] [Trace] and hold down either ► or ◄ until you get an estimate of the x-value where the curve crosses the horizontal line representing 20.

3. Use the AUTO setting in the Table, and with ▲ or ▼ scroll through the table until a value near the estimated output is found. Use this x-value or a number near it as your guess in the numeric solver. (Refer to Section 1.1.6 of this Guide to review the instructions for using the table)

- You may find it helpful to employ a combination of the above methods by using the split screen feature of the TI-89. Using a split screen, you can view the graph and table or the numeric solver and a graph, and so forth, at the same time. See the TI-89 Guidebook for details.

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2 It is possible to change the bound if the calculator has trouble finding a solution to a particular equation. This, however, should happen rarely. Refer to the TI-89 Guidebook for details.
1.1.9 **GRAPHICALLY FINDING INTERCEPTS** Finding the input value at which the graph of a function crosses the vertical and/or horizontal axis can be found graphically, by using the solve instruction on the entry line, or by using the numeric solver. Remember the process by which we find intercepts:

- To find the y-intercept of a function \( y = f(x) \), set \( x = 0 \) and solve the resulting equation.
- To find the x-intercept of a function \( y = f(x) \), set \( y = 0 \) and solve the resulting equation.

An intercept is the point where the graph crosses or touches an axis. Also remember that the x-intercept of the function \( y = f(x) \) has the same value as the root or solution of the equation \( f(x) = 0 \). Thus, finding the x-intercept of the graph of \( f(x) = c \) is the same as solving the equation \( f(x) = c \).

We illustrate this method with a problem similar to the one in Activity 40 in Section 1.1 of *Calculus Concepts*. You should practice by solving the equation \( 3.622(1.093^x) = 20 \) using this graphical method and by solving the equation that follows using one of the solvers.

Suppose we are asked to find the input value of \( f(x) = 3x - 0.8x^2 + 4 \) that corresponds to the output \( f(x) = 2.3 \). That is, we are asked to find \( x \) such that \( 3x - 0.8x^2 + 4 = 2.3 \). Because this function is not given in a context, we have no indication of an interval of input values to use when drawing the graph. We will use the zoom features to set an initial view and then manually set the WINDOW until we see a graph that shows the important points of the function (in this case, the intercept or intercepts.) You can solve this equation graphically using either the x-intercept method or the intersection method. We present both, and you should use the one you prefer.

**X-INTERCEPT METHOD** for solving the equation \( f(x) - c = 0 \):

1. Press \( \bullet \) [F1] (Y=) and clear all locations with [CLEAR]. Enter the function \( 3x - 0.8x^2 + 4 - 2.3 \) in \( y_1 \). (Type \( x^2 \) with \( X^2 \). Remember to use \( - \), not \( ( - ) \), for the subtraction signs.)

2. Draw the graph of \( y_1 \) with [F2] [Zoom] 4 [ZoomDec] or [F2] [Zoom] 6 [ZoomStd]. If you use the former, press \( \bullet \) [F2] (WINDOW) and reset \( y_{\text{max}} \) to 5.5 to get a better view of the top of the graph. (If you reset the window, press \( \bullet \) [F3] (GRAPH) to redraw the graph.)

3. To graphically find an x-intercept, i.e., a value of \( x \) at which the graph crosses the horizontal axis, press [F5] [Math] 2 [Zero]. Press and hold [◄] until you are near, but to the left of, the leftmost x-intercept. Press \( \bullet \) [ENTER] to mark the location of the lower bound for the x-intercept.

4. Notice the small arrowhead (\( \bullet \)) that appears above the location to mark the left bound. Now press and hold [►] until you are to the right of this x-intercept. Press [ENTER] to mark the location of the upper bound for the x-intercept and to find it.
The value of the leftmost x-intercept has the x-coordinate \( x = -0.5 \).

Repeat the above procedure to find the rightmost x-intercept. Confirm that it is where \( x = 4.25 \).

**NOTE:** If this process does not return the correct value for the intercept that you are trying to find, you have probably not included the place where the graph crosses the axis between the two bounds (i.e., between the \( \uparrow \) and \( \downarrow \) marks on the graph.)

**INTERSECTION METHOD** for solving the equation \( f(x) = c \):

Press \( \text{FNC} \) \( \text{F1} \) (Y=) and clear all locations with \( \text{CLEAR} \). Enter one side of the equation, \( 3x - 0.8x^2 + 4 \), in \( y_1 \) and the other side of the equation, 2.3, in \( y_2 \).

Draw the graphs with \( \text{F2} \) [Zoom] 4 [ZoomDec] or \( \text{F2} \) [Zoom] 6 [ZoomStd]. If you use the former, press \( \text{F2} \) (WINDOW) and reset ymax to 8 to get a better view of the top of the graph. (If you reset the window, \( \text{F3} \) (GRAPH) redraws the graph.)

To locate where \( y_1 = y_2 \), press \( \text{F5} \) [Math] 5 [Intersection]. Press \( \text{ENTER} \) to mark the first curve. The cursor jumps to the other function – here, the line. Next, press \( \text{ENTER} \) to mark the second curve.

Note that the number corresponding to the location of each function appears on the top left of the screen as the cursor moves from the curve to the line. Next, press \( \text{◄} \) to move the cursor to the left of the intersection point you want to find – in this case, the leftmost point. Press \( \text{ENTER} \).

Use \( \text{►} \) to move the cursor to the right of the intersection point you are finding to supply the upper bound for that point. The point of intersection must lie between the two markers for the lower and upper bounds. Press \( \text{ENTER} \).

The value of the leftmost x-intercept has the x-coordinate \( x = -0.5 \).
Repeat the above procedure to find the rightmost x-intercept. Confirm that it is where \( x = 4.25 \).
1.1.10 SUMMARY OF ESTIMATING AND SOLVING METHODS  Use the method you prefer.
When you are asked to estimate or approximate an output or an input value, you can:
• Trace a graph  (Section 1.1.4)
• Use close values obtained from the TI-89 table  (Sections 1.1.3, 1.1.5, 1.1.7)
When you are asked to find or determine an output or an input value, you should:
• Evaluate an output on the graphics screen  (Section 1.1.3)
• Evaluate an output on the home screen  (Section 1.1.5)
• Evaluate an output value using the table  (Section 1.1.6)
• Find an input using the solve instruction or the numeric solver  (Sections 1.1.7, 1.1.8)
• Find an input value from the graphic screen (using the $x$-intercept method or the intersection method)  (Section 1.1.9)

1.3 Constructed Functions
Your calculator can find output values of and graph combinations of functions in the same way that you do these things for a single function. The only additional information you need is how to enter constructed functions in the graphing list or on the home screen.

1.3.1 FINDING THE SUM, DIFFERENCE, PRODUCT, QUOTIENT OR COMPOSITE FUNCTION  
Suppose that a function $f$ has been entered in $y1$ in the Y= list or defined as $y1(x)$ and that a function $g$ has been entered in $y2$ in the Y= list or defined as $y2(x)$. Your TI-89 will evaluate, graph, and actually find the symbolic form of these constructed functions:

Enter $y1(x) + y2(x)$ to obtain the sum function $(f + g)(x) = f(x) + g(x)$.
Enter $y1(x) - y2(x)$ to obtain the difference function $(f - g)(x) = f(x) - g(x)$.
Enter $y1(x) \cdot y2(x)$ to obtain the product function $(f \cdot g)(x) = f(x) \cdot g(x)$.
Enter $y1(x)/y2(x)$ to obtain the quotient function $(f \div g)(x) = \frac{f(x)}{g(x)}$.
Enter $y1(y2(x))$ to obtain the composite function $(f \circ g)(x) = f(g(x))$.

1.3.2 FINDING A DIFFERENCE FUNCTION  
We illustrate this technique with the functions that are given on page 31 of Section 1.3 of Calculus Concepts: Sales = $S(t) = 3.570(1.105^t)$ million dollars and costs = $C(t) = -39.2t^2 + 540.1t + 1061.0$ thousand dollars $t$ years after 1996. We use the functions on the home screen, but you can also use the Y= list locations. However, if you are in the Y= list, the symbolic form of the constructed function is not displayed.

NOTE: Before you start a problem in which you want a symbolic result (i.e., a formula rather than a number), clear all individual letter variable names with 2nd F1 F6 2 [NewProb] ENTER or 2nd F1 F6 1 [Clear a-z] ENTER. (See the warning message on the next page.)

- As we previously mentioned, you can use any input variable on the home screen. We choose to use $t$ because it is given as the input variable in the text illustration. However, if you prefer $x$, replace every $t$ by $x$ in the instructions.
Type on the entry line the difference function, which is the profit function \( S(t) - 0.001C(t) \). Press ENTER. Press ▲ and press and hold ► to scroll to the right to see the entire expression.

**WARNING:** If you do not get a symbolic result, it is because the variable you are using as the input variable has a number stored in it. That is, the variable is a defined rather than an undefined variable. The variable you use for the input variable must be an undefined variable in order to obtain a symbolic result. Refer to the information at the top of page C-5 of this Guide for more information.

To find the profit in 1998, evaluate the profit function at \( t = 2 \).

You can edit the entry line and replace \( t \) by 2 or you can enter \( s(x) - 0.001c(x) \) in the Y= list and use the table. We find that the profit in 1998 was \( P(2) \approx 2.375 \) million dollars.

If you need to use the profit function for other calculations, you may find it easier to define a new function, \( p(t) = s(t) - 0.001c(t) \) and then find \( p(2) \). We illustrate the copy and paste feature of the TI-89 to do this task. Press ▲ until \( s(t) - 0.001c(t) \) in the history area is darkened.

Press F1 [Tools] 5 [Copy]. Use ▼ to move the cursor to the entry line and press F1 [Tools] 6 [Paste]. Use◄ to move the cursor to the far left position in the entry line. Type F4 [Other] 1 [Define] alpha STO /triangle45right (P) (T) = and press ENTER.

You can now find \( p(2) \) and/or enter \( p(x) \) in the Y= list so that you can use the table to find other values, graph the function, and so forth.

### 1.3.3 FINDING A PRODUCT FUNCTION

We illustrate this technique with the functions that are given on page 32 of Section 1.3 of Calculus Concepts: Milk price = \( S(x) = 0.007x + 1.492 \) dollars per gallon on the \( x \)th day of last month and milk sales = \( G(x) = 31 - 6.332(0.921^x) \) gallons of milk sold on the \( x \)th day of last month.

You can, if you wish, define the functions \( s \) and \( g \). (Note that the new definition for \( s \) will replace the one defined in Section 1.3.3.) However, because we are only finding the product function and only one output of it, we choose to name only the product function.

NOTE: You do not have to, but you can, use the times sign \( \times \) between \( S \) and \( G \) to indicate the product function. We enclosed each of \( S \) and \( G \) in parentheses to indicate a product when they are written next to each other, so the times sign is not necessary.

To find milk sales on the 5th day of last month, evaluate \( t(x) \) at \( x = 5 \). On the entry line, type in \( T(5) \) and press \( \text{ENTER} \).

We find that milk sales were \( T(5) \approx \$40.93 \).

If you want to find the product function, copy the “Define \( t(x) \)” statement and paste it into the entry line, press \( \leftarrow \) and use \( \rightarrow \) until the cursor is just to the right of the = sign, and press \( /b2\text{left} \) to delete “Define \( t(x) = \)”. Press \( \text{ENTER} \).

1.3.4 CHECKING YOUR ANSWER FOR A COMPOSITE FUNCTION

We illustrate finding a composite function with those functions given on page 33 of Section 1.3 of Calculus Concepts: altitude = \( F(t) = -222.22t^3 + 1755.95t^2 + 1680.56t + 4416.67 \) feet above sea level where \( t \) is the time into flight in minutes and air temperature = \( A(F) = 277.897(0.99984^F - 66 \) degrees Fahrenheit where \( F \) is the number of feet above sea level. While the same technique that we used to find the product function also can be used to find the composite function, for variety we use a slightly different procedure.

Press \( \bullet \) \( F1 \) (Y=) and clear any previously entered functions.

Enter \( F \) in \( y1 \) by pressing \( (-) 222 \times 22 \times 3 + 1755 \times 95 \)

Enter \( A \) in \( y2 \) by pressing \( 277 \times \times 897 \times 99984 \times 66 \times \times \)

Enter the composite function \( (A\circ F)(x) = A(F(x)) = y2(y1(x)) \) in \( y3 \).

Press \( \text{HOME} \). Enter \( y3(x) \) on the entry line to display the composite function.

WARNING: The TI-89 requires that you give the output symbol, not the function symbol, when you refer to the formula that computes the output of a function. If you do not type \( x \) in parentheses following \( y3 \), an argument error results.

- Note that the symbolic form of the composite function contains \( e \) raised to a power, not \( 0.99984 \) raised to a power (as in \( y2 \)). Converting between these two exponential forms is discussed in Section 2.2 of Calculus Concepts.

1.3.5 TURNING FUNCTIONS IN THE GRAPHING LIST OFF AND ON

There are times when you would like to keep the equations of certain functions in the Y= list but you do not want them to graph or to be shown in the table. Any function that is turned on will graph and will show in the TI-89 table. A function is turned on when a check mark appears next to it in the Y= list and is turned off when there is no check mark next to it in the Y= list.
On the screen shown to the right, $y_1$ and $y_3$ are turned on (i.e., activated) and $y_2$ is turned off (i.e., deactivated).

With the function highlighted, use $[F_4]$ [ ] to toggle the check (and graph) on and off. $[F_5]$ [All] turns all functions on or off.

1.3.6 GRAPHING AN INVERSE FUNCTION  The TI-89 can draw the graph of the inverse of a function. Using the calculator graph, you can check your algebraic answer. We illustrate this idea using the function in Example 4 of Section 1.3 of *Calculus Concepts*:

The fares for a cab company are determined by the function $F(d) = 1.8d + 2.5$ dollars where $d$ is the distance traveled in miles.

Press $[\Diamond] F_1$ (Y = ), clear any previously entered functions, and enter the function $F$ in $y_1$. Remember to use $x$ as the input variable. (Refer to Sections 1.1.2 and 1.1.3 of this Guide for hints on how to set the window to graph this function.)

This problem does not state an interval of values for which the input variable is defined, so we begin by guessing one that makes sense in context. Because $d$ is the distance traveled, we know that $d \geq 0$. We choose a maximum value of 50. (Your guess is as good as the one that is given below.)

Press $[\Diamond] F_2$ (WINDOW) and enter appropriate values for the input and output. Draw the graph of the function for the view you choose with $[\Diamond] F_3$ (GRAPH).

Press $[\Diamond] F_1$ (Y = ) and enter in $y_2$ your answer to part a of Example 4 – the inverse function for $F$. (Your answer to part a may or may not be the same as what is shown to the right.)

NOTE: Remember that the TI-89 requires that you use $x$ as the input variable and that you use parentheses around any numerator and/or denominator that consists of more than one symbol in a fraction.


Press ENTER. If you do see a third graph, your inverse function formula is not correct.
CAUTION: If you do not see a third graph, as is the case in the graph in the last picture, you may have computed the formula correctly. However, you might have an incorrect inverse function formula whose graph does not show in the current viewing window. We suggest that if you do not see a third graph that you turn off the graphing location where you have entered your inverse function formula – here \( y_2 \). (Remember that you turn off a function by having the check mark to the left of the name not showing.) Then, on the home screen, press \( \text{ENTER} \) to redraw the inverse function graph and visually check that it is the same graph as the one for \( y_2 \). Realize that these methods provide only a visual check on your answer and are not exact.

The answers to parts \( b \) and \( c \) of Example 4 are found using the distance (inverse) function in \( y_2 \) and the fare (original) function in \( y_1 \), respectively.

Don’t forget to include units of measure with the answers.

1.3.7 COMPOSITION PROPERTY OF INVERSE FUNCTIONS This concept, involved in part \( d \) of Example 4, provides another check on your answer for the inverse function.

Form the compositions of the inverse function and the original function according to the Composition Property of Inverse Functions and enter them in \( y_3 \) and \( y_4 \). Turn off the functions in \( y_1 \) and \( y_2 \). (Section 1.3.3 of this Guide.)

Have the TABLE set to ASK mode (See Section 1.1.6) and press \( \blacktriangle \text{F5} \) (TABLE). Enter several values for \( x \) to see if both composite functions return that value of \( x \). If so, your answer for the inverse function is very likely correct.

1.3.8 GRAPHING A PIECEWISE CONTINUOUS FUNCTION Piecewise continuous functions are used throughout the text. You will need to use your calculator to graph and evaluate outputs of piecewise continuous functions. Several methods can be used to draw the graph of a piecewise function. One of these is presented below using the function that appears in Example 6 of Section 1.3 in Calculus Concepts:

The population of West Virginia from 1985 through 1993 can be modeled by

\[
P(t) = \begin{cases} 
-23.514t + 3903.667 & \text{thousand people when } 85 \leq t < 90 \\
7.7t + 1098.7 & \text{thousand people when } 90 \leq t \leq 93 
\end{cases}
\]

where \( t \) is the number of years since 1900.

The TI-89 syntax for drawing a piecewise function consisting of two pieces is

\[
\text{when}(\text{condition, true expression, false expression})
\]

The TI-89 CATALOG contains all the TI-89 commands. If you press the first letter of the word you are trying to find in the catalog, it automatically scrolls to the first word that begins with that letter. Then use \( \blacktriangledown \) to find what you are looking for and enter it in what you are typing.

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3 Instructions for drawing a piecewise function consisting of three pieces and more than three pieces are given on pages 194-195 of the TI-89 Guidebook.
Be on the home screen and press \textbf{CATALOG}. Because \textit{when} begins with \textit{w}, press \textbf{(W)}. \textbf{ENTER} copies the instruction to the entry line. Also notice that the general syntax for each instruction is printed at the bottom of the screen in the catalog.

We intend to graph the population function \(P\), so we need to enter it in the Y= list with \(x\) as the input variable. Because the break point is at \(x = 90\), the piecewise function syntax becomes

\[
\text{when}(x < 90, -23.514x + 3903.667, 7.7x + 1098.7)
\]

Clear any functions that are in the Y= list. Type the above \textit{when} statement in the \(y_1\) location. The \(<\) symbol is accessed with \textbf{2nd} \textbf{0} (\(<\)).

\begin{itemize}
  \item Note that \(x < 90\) prints at the end of the first piece of the function in the \(y_1\) position. The \textit{else} that appears with the second piece means that this part of the function should be used everywhere else; that is, when \(x \geq 90\).
\end{itemize}

Your calculator draws graphs by connecting function outputs wherever the function is defined. However, this piecewise function breaks at \(x = 90\). The TI-89 will connect the two pieces of \(P\) unless you tell it not to do. Whenever you draw graphs of piecewise functions, you should set your calculator to Dot mode as described below so that it will not connect the different pieces of the function. (When a particular style is chosen, a check mark appears by it in the style list.)

Highlight \(y_1\) and press \textbf{2nd} \textbf{F1} \textbf{[F6: Style\textsuperscript{4}]} \textbf{2} \textbf{[Dot]}. Next, set the window. The function \(P\) is defined only when the input is between 85 and 93. So, on the home screen we evaluate \(P(85)\), \(P(93)\), and \(P(90)\) to help when setting the vertical view.

\textbf{NOTE:} Instead of finding the outputs as shown above, you can set the window with ZoomFit as described in Section 1.1.2 of this \textit{Guide}. If you do this, reset \(y_{\text{min}}\) to a smaller value so that you can better view the break point.

Press \textbf{F2} (WINDOW), set \(\text{xmin} = 85\), \(\text{xmax} = 93\), \(\text{ymin} = 1780\), and \(\text{ymax} = 1905\). Press \textbf{F3} (GRAPH).

Take a closer look at the break point with the window given below.

Set \(\text{xmin} = 89\), \(\text{xmax} = 91\), \(\text{ymin} = 1780\), and \(\text{ymax} = 1810\). Use the entry line on the home screen to evaluate the other outputs needed in this example.

\textsuperscript{4} The different graph styles you can draw from this location are described in more detail on page 100 in your \textit{TI-89 Guidebook}. 
1.4 Limits: Describing Function Behavior

The TI-89 table is an essential tool when you are estimating limits numerically. Even though rounded values are shown in the table due to space limitations, the TI-89 displays at the bottom of the screen many more decimal places for a particular output when you highlight that output. Your calculator also finds limits algebraically, but you should use the numerical and graphical methods to gain insight as to the meaning of the algebraic answers.

1.4.1 Estimating Limits Numerically

Whenever you use the TI-89 to estimate limits numerically, you will find it easiest to set the TABLE to ASK mode. We illustrate using the function \( u \) that appears in Example 2 of Section 1.4 in Calculus Concepts:

Press \( \text{F1} \) (Y=) and use CLEAR to delete all previously-entered functions. Enter \( u(x) = \frac{3x^2 + 3x}{9x^2 + 11x + 2} \). Be certain to enclose the numerator and denominator of the fraction in parentheses.

Press \( \text{F4} \) (TblSet). If ASK is not selected, choose ASK by moving the cursor to the in the Independent: location, press \( \text{►} \), choose ASK, and press ENTER. Press ENTER again to exit the screen.

Press \( \text{F5} \) (TABLE). Delete any values that are there with \( \text{F1} \) [Tools] 8 [Clear Table]. To numerically estimate \( \lim_{x \to -1^-} u(x) \), enter values to the left of, and becoming closer and closer to, -1.

**NOTE:** The values you enter do not have to be those shown in the text or these shown in this table. The cell width in the table has been increased in order to show you the values entered. If you want to do this, use \( \text{F1} \) [Tools] 9 [Format] and choose a larger cell width.

**CAUTION:** Your instructor will very likely have you write the table you construct on paper. Even if you increase the cell width, the TI-89 eventually displays rounded values (such as the one that is highlighted in the table above) because of space limitations. When this happens, be certain to highlight the value and look on the bottom of the screen to see what the value actually is. The inputs that were entered in the table above are \(-1.5, -1.05, -1.005, -1.0005, -1.00005, \) and \(-1.000005\). The outputs also have been rounded off.

**ROUNDING OFF:** Recall that rounded off means that if one digit past the digit of interest if less than 5, other digits past the digit of interest are dropped. If one digit past the one of interest is 5 or over, the digit of interest is increased by 1 and the remaining digits are dropped. (For instance, the highlighted number in the last screen shown above, \(-1.000005\), has been rounded off by the TI-89 to 5 decimal places. Because the 5 in the sixth decimal place location is in the “5 or over” category, the 0 before this 5 is increased by 1 with the digits past that point dropped to give \(-1.00001\).)

**Rule of Thumb for Determining Limits From Tables:** Suppose that we want \( \lim_{x \to -1^-} u(x) \) accurate to 3 decimal places. Watch the table until you see that the output is the same.
value to one more decimal place (here, to 4 decimal places) for three consecutive outputs. Then, round that common value off to the requested 3 places for the desired limit. Your instructor may establish a different rule from this one, so be sure to ask.

Using this Rule of Thumb and the results that are shown on the last calculator screen, we estimate that \( \lim_{x \to -1^-} u(x) = 0.429 \). We now need to estimate the limit from the right of \(-1\).

Delete the values currently in the table. To numerically estimate \( \lim_{x \to -1^+} u(x) \), enter values to the right of, and becoming closer and closer to, \(-1\). (Note: Again, the values that you enter do not have to be those shown in the text or these shown to the right.)

Because the output 0.4285... appears three times in a row, we estimate that \( \lim_{x \to -1^+} u(x) = 0.429 \). Then, because \( \lim_{x \to -1^-} u(x) = \lim_{x \to -1^+} u(x) = 0.429 \), we estimate that \( \lim_{x \to -1} u(x) = 0.429 \).

We now illustrate finding the limit in part b of Example 2 in Section 1.4 of Calculus Concepts:

Delete the values currently in the table. To numerically estimate \( \lim_{x \to -2/9^-} u(x) \), enter values to the left of, and becoming closer and closer to, \(-2/9\). Because the output values appear to become larger and larger, we estimate that \( \lim_{x \to -2/9^-} u(x) \to \infty \). Thus, \( \lim_{x \to -2/9^-} u(x) \) does not exist.

1.4.2 CONFIRMING LIMITS GRAPHICALLY – ZOOMING IN AND OUT A graph can be used to confirm a limit that you estimated numerically. You also can zoom in or zoom out on the graph to obtain a better view of the limit you are estimating. We again illustrate using the function \( u \) that appears in Example 2 of Section 1.4 in Calculus Concepts.

Have the function \( u(x) = \frac{3x^2 + 3x}{9x^2 + 11x + 2} \) entered in the y1 location of the Y= list. A graph drawn with \([\text{F2}]\) [Zoom] 4 [ZoomDec] or \([\text{F2}]\) [Zoom] 6 [ZoomStd] is not very helpful.

To confirm that \( \lim_{x \to 1} u(x) = 0.429 \), we are only interested in values of \( u \) that are near \(-1\). So, choose values very near to \(-1\) for the x-view and evaluate the function at those x-values to help determine the y-view. We manually set the window to values such as those shown to the right and draw the graph.
If you look closely, you can actually see the “hole” in the graph at \( x = -1 \). Press \( \text{F3} \) [Trace], use \( \uparrow \) to trace to where \( x = -1 \), and the TI-89 confirms that \( u \) is not defined at \( x = -1 \). Now press \( \uparrow \) and observe the \( y \)-value. Press \( \downarrow \) several times to go to the left of \( x = -1 \) and confirm from the \( y \)-values that the limit is approximately what we had determined numerically.

Note that we confirmed that the limit exists by seeing that the two parts of the graph (to the left and right of \( x = -1 \)) move toward the same point. Tracing around \( x = -1 \) provides a check on the numerical value of the limit.

The previous instructions show how to zoom in by manually setting the window. You can also zoom in with the zoom menu of the calculator. We next describe this method.

Return to the graphing screen set with \( \text{F2} \) [Zoom] 4 [ZoomDec] or \( \text{F2} \) [Zoom] 6 [ZoomStd] or any screen where you can see the portion of the graph around \( x = -1 \). Press \( \text{F2} \) [Zoom] 2 [Zoom In] and use \( \leftarrow \) and \( \uparrow \) to move the blinking cursor until you are near the point on the graph where \( x = -1 \). Press \( \text{ENTER} \).

Depending on the horizontal view, you may or may not be able to see the hole in the graph at \( x = -1 \). If your view is not magnified enough to see what is happening around \( x = -1 \), repeat the zoom-in process. To check your numerical estimate of the limit, press \( \text{F3} \) [Trace], use the arrow keys to move to either side of \( x = -1 \), and observe the \( y \)-values.

Now let’s consider the limit in part \( b \) of Example 2 of Section 1.4. We want to confirm with a graph what was found numerically; that is, we wish to confirm that \( \lim_{x \to -2/9} u(x) \) does not exist.

We want to zoom out on the graph to confirm that the limit of \( u(x) \) as \( x \) approaches \(-2/9\) does not exist. To do this, we set a small \( x \)-view and a larger \( y \)-view. (Note that these values are arbitrary – any small \( x \)-view that includes \(-2/9\) and any \( y \)-view in which the graph can be seen clearly will do.)

Draw the graph with \( \bullet \) \( \text{F3} \) (GRAPH). Before continuing with the limit investigation, we need to eliminate the “extra” vertical line that appears on the above graph at \( x = -2/9 \). The line appears because we are graphing in Line mode, which tells the TI-89 to connect points on the graph. (You may or may not have the line on your graph.)

Set \( y1 \) set to draw in Dot mode by highlighting \( y1 \) and pressing \( \text{2nd} \) \( \text{F1} \) [F6: Style] 2 [Dot]. Redraw the graph with \( \bullet \) \( \text{F3} \) (GRAPH).
1.4.3 INVESTIGATING END BEHAVIOR OR NUMERICALLY

Investigating end behavior using the TI-89 table is very similar to numerically estimating the limit at a point. We illustrate this using the function that appears in Example 4 of Section 1.4 in Calculus Concepts.

Have \( u(x) = \frac{3x^2 + x}{3x + 11x^2 + 2} \) in the \( y1 \) location of the \( Y= \) list. Be certain that you remember to enclose both the numerator and denominator of the fraction in parentheses.

Have TblSet set to ASK and press \( \boxed{F5} \) (TABLE). Delete any values that are in the table. In order to numerically estimate \( \lim_{x \to \infty} u(x) \), enter values of \( x \) that get larger and larger. (Note: The values that you enter do not have to be those shown in the text or these shown to the right.)

We assume that we want the limit accurate to 3 decimal places. According to our Rule of Thumb for Determining Limits from Tables (page C-21), once we see the same value to 4 decimal positions 3 times in a row, we can estimate the limit by rounding off the answer to 3 decimal places.

We estimate \( \lim_{x \to \infty} u(x) = 0.273 \).

Part b of Example 4 asks for \( \lim_{x \to -\infty} u(x) \). Delete or type over the values that are currently in the table. Then, enter values of \( x \) that get smaller and smaller. We estimate \( \lim_{x \to -\infty} u(x) = 0.273 \).

CAUTION: It is not the final value, but a sequence of several values, that is important when determining limits. If you enter a very large or very small value, you may exceed the limits of the TI-89’s capability and obtain an incorrect number. Always look at the sequence of values obtained to make sure that all values found make sense.

1.4.4 INVESTIGATING END BEHAVIOR OR GRAPHICALLY

As was the case with limits at a point, a graph of the function can be used to confirm a numerically-estimated limit. We again illustrate with the function that appears in Example 4 of Section 1.4 in Calculus Concepts.

Have \( u(x) = \frac{3x^2 + x}{3x + 11x^2 + 2} \) in the \( y1 \) location of the \( Y= \) list. (Be certain that you remember to enclose both the numerator and denominator of the fraction in parentheses.) A graph drawn with \( \boxed{F2} \) [Zoom] 4 [ZoomDec] is a starting point.
We estimated the limit as \( x \) gets very large or very small to be 0.273. Now, \( u(0) = 0 \), and it does appears from the graph that \( u \) is never negative. Set a window with values such as \( x_{\text{min}} = -10 \), \( x_{\text{max}} = 10 \), \( y_{\text{min}} = 0 \), and \( y_{\text{max}} = 0.35 \). Press [MENU] [F3] (GRAPH).

To examine the limit as \( x \) gets larger and larger, change the window so that \( x_{\text{max}} = 100 \), view the graph with [MENU] [F3] (GRAPH), change the window so that \( x_{\text{max}} = 1000 \), view the graph with [MENU] [F3] (GRAPH), and so forth. The graph to the right was drawn with \( x_{\text{max}} = 10,000 \). Use [F3] [Trace] with each graph.

Repeat the process as \( x \) gets smaller and smaller, but change \( x_{\text{min}} \) rather than \( x_{\text{max}} \) after drawing each graph. The graph to the right was drawn with \( x_{\text{min}} = -9000 \), \( x_{\text{max}} = 10 \), \( y_{\text{min}} = 0 \), and \( y_{\text{max}} = 0.35 \). Again, press [F3] [Trace] on each graph screen to view some of the outputs and confirm the numerical estimates.

1.4.5 FINDING LIMITS ALGEBRAICALLY As previously mentioned, the TI-89 finds limits algebraically. You can use this feature to confirm numerical and graphical estimates or use it as a method of finding limits. The TI-89 syntax is limit (function, input variable, point, direction).

For a limit from the left, direction = any negative number; for a limit from the right, direction = any positive number; for a limit from both sides, direction = 0 or direction is omitted from the instruction. We illustrate using the function \( u(x) \) that appears in Example 2 of Section 1.4.

Press HOME [F4] 1 [Define] and type

\[
u(x) = \frac{3x^2 + 3x}{9x^2 + 11x + 2}
\]

Find \( \lim_{x \to -1^-} u(x) \).

Access the limit instruction with [F3] [Calc] 3 [limit(].

NOTE: We used \(-3\) to indicate the direction in the limit statement. The TI-89 considers only the negative sign, so any negative number gives the same result – try it! Also remember that if you want a decimal approximation for the answer, [ENTER] gives it.

Find \( \lim_{x \to -1^+} u(x) \), \( \lim_{x \to -1^-} u(x) \), \( \lim_{x \to -2/9^+} u(x) \), \( \lim_{x \to -2/9^-} u(x) \), \( \lim_{x \to 2/9^+} u(x) \), and \( \lim_{x \to 2/9^-} u(x) \).

(See the note below for a shortcut.)

NOTE: Instead of retyping the limit statement each time to find the above limits, remember that when the entry line is highlighted, you can press [►] to put the cursor on the right-hand side of the statement and press [◄] to put the cursor on the left-hand side of the statement. The statement is ready for editing using the arrow keys to move to the correct location and using [/] to delete unwanted symbols.
The results of our numerical and graphical investigations are confirmed by the algebraic answers. Note that the TI-89 prints undef to mean undefined or does not exist.

1.5 Linear Functions and Models

This portion of the Guide gives instructions for entering real-world data into the calculator and finding familiar function curves to fit that data. You will use the beginning material in this section throughout all the chapters in Calculus Concepts.

CAUTION: You need to name the data variable in the TI-89, and you are instructed below to use the name calc. Do not change or vary this name. If you do, the programs that you use later will not properly execute. Also, be careful when you enter data in the TI-89 because the model and all results depend on the values that are entered! Always check your entries.

1.5.1 ENTERING DATA

We illustrate data entry using the values in Table 1.46 in Section 1.5.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax (in dollars)</td>
<td>2532</td>
<td>3073</td>
<td>3614</td>
<td>4155</td>
<td>4696</td>
<td>5237</td>
</tr>
</tbody>
</table>

Before beginning, clear out previous work with HOME 2nd F1 [F6] 2 [NewProb] ENTER.

Press APPS 6 [Data/Matrix Editor] and choose 3 [New]. Choose the settings shown to the right for the first two positions. Name the variable by pressing 2nd alpha (a-lock) and type calc. (We use this variable name for all data in this Guide. See the CAUTION note above.)

Press ENTER until you see the screen shown to the right. These are the lists that hold data. You can access many other lists by highlighting the list name and using ►.

In this Guide, we usually use list c1 for the input data and list c2 for the output data. If there are any data values already in your lists, see Section 1.5.3 of this Guide and first delete any “old” data. To enter data in the lists, do the following:

Position the cursor in the first location in list c1. Enter the input data into list c1 by typing the years from top to bottom in the c1 column, pressing ENTER or ▼ after each entry. (If a letter rather than a number prints, alpha-lock is still on. Press alpha to release it.) Use the shortcut in the NOTE below to enter the tax values in c2.

NOTE: After typing the sixth input value, 2001, use ◀ ◀ to have the cursor go to the top of list c1 and then press ► to be in the first row of the c2 column. Enter the output data into list c2 by typing the entries from top to bottom in the c2 column, pressing ENTER or ▼ after each tax value. (◀ ◀ causes the cursor to jump to the bottom of the current column. See page 232 of the TI-89 Guidebook for other key combinations that scroll the data lists.)
1.5.2 EDITING DATA  If you incorrectly type a data value, use the cursor keys (i.e., the arrow keys ►, ◄, ▲, and/or ◄) to highlight the value you wish to correct and then type the correct value. Press ENTER or ▼ to enter the corrected value.

- To insert a data value, put the cursor over the value that will be directly below the one you will insert, and press 2nd F1 [F6: Util] 1 [Insert] 1 [cell]. The values in the list below the insertion point move down one location and undef is filled in at the insertion point. Type the data value to be inserted and press ENTER. The “undef” is replaced with the inserted data value.

- To delete a single data value, highlight the value you wish to delete, and press ◄ or press 2nd F1 [F6: Util] 2 [Delete] 1 [cell]. The values in the list below the deleted value move up one location.

1.5.3 DELETING OLD DATA  Whenever you enter new data in your calculator, you should first delete any previously-entered data. There are several ways to do this, and the most convenient method is illustrated below.

Access the data with APPS 6 [Data/Matrix Editor] and choose 1 [Current]. (You probably have different values in your lists if you are deleting “old” data.)

Put the cursor on any entry in the column to be cleared.

Press 2nd F1 [F6: Util] 5 [Clear column]. Use ► to move the cursor to any entry in the next column and repeat this procedure to clear old data from any other lists you will use.

1.5.4 FINDING FIRST DIFFERENCES  When the input values are evenly spaced, you can use program DIFF to compute first differences in the output values. Program DIFF is given in the TI-89 Program Appendix at the Calculus Concepts Website. Consult the Programs category in TroubleShooting the TI-89 in this Guide if you have questions about obtaining the programs.

Have the data given in Table 1.46 in Section 1.5 of Calculus Concepts entered in your calculator. (See Section 1.5.1 of this Guide.)

Exit the data editor and go to the home screen with HOME.

To run the program, type the name of the program in the entry line and put “( )” behind the name. It is easier to type the program name if you lock alpha mode with 2nd alpha [a-lock]. If you prefer, get the program name with 2nd − (VAR-LINK), press ▼ until diff is highlighted, and press ENTER. Type 3.
Press \text{ENTER}. The message on the right appears on your screen. Press \text{ENTER} again. If you have not entered the data in the correct location, press 2 [no]. If you are ready to continue, press 1 [yes] or \text{ENTER}.

Press \text{F1} [Choice?] 1 [first differences]. (Options 2 and 3 are used in the next chapter of this \textit{Guide}.) The first differences are displayed.

Press \text{F1} [Choice?] 4 [Quit program]. To return to the home screen, press \text{F5} [	ext{PrgmIO}] or \text{HOME}.

If you need to scroll the list to see the rest of the first differences or recall them, enter d1 in the entry line.

- The first differences are constant at 541, so a linear function gives a perfect fit to these tax data.

\textbf{Note:} Program \text{DIFF} should not be used for data with input values (c1) that are not evenly spaced. First differences give no information about a possible linear fit to data with inputs that are not the same distance apart. If you run program \text{DIFF} with input data that are not evenly spaced, the message \text{INPUT VALUES NOT EVENLY SPACED} appears and the program stops.

\textbf{1.5.5 SCATTER PLOT SETUP} The first time that you draw a graph of data, you need to set the TI-89 to draw the type of graph you want to see. Once you do this, you never need to do this set up again (unless for some reason the settings are changed). If you always put input data in list c1 and output data in list c2, you can turn the scatter plots off and on from the Y= screen rather than the Plot Setup screen after you perform this initial setup.

Access the data with \text{APPS} 6 [Data/Matrix Editor] and choose 1 [Current]. Press \text{F2} [Plot Setup]. (Your screen may not look exactly like this one.)

On the Plot Setup screen, have Plot 1 highlighted and press \text{F1} [Define]. Choose the options shown on the right. (You can choose any of the 5 available marks.) Type in c1 for \textit{x} and c2 for \textit{y}.

Press \text{ENTER} to save the settings. Note that Plot 1 is turned on because there is a check mark to the left of the name. Press \text{ENTER} to return to the data lists.
Press \( \bullet \) F1 (Y=) and \( \triangle \). Notice that Plot1 at the top of the screen has a check mark to the left of the name. This tells you that Plot1 is turned on and ready to be graphed. You can also see the settings you chose for the plot.

- A scatter plot is turned on when there is a check mark to the left of its name on the Y= screen. From now on, you can turn the scatter plot off and on by having Plot1 either checked or not checked. To turn Plot1 off, press \( \text{F4} \) \( \text{[} \) to remove the check mark. Reverse the process to turn Plot1 back on.

- TI-89 data lists can be named and stored in the calculator’s memory for later recall and use. Refer to Section 1.5.13 and 1.5.14 of this Guide for instructions on storing data lists and later recalling them for use.

### 1.5.6 DRAWING A SCATTER PLOT OF DATA

Any functions that are turned on in the Y= list will graph when you plot data. Therefore, you should clear or turn them off before you draw a scatter plot. We illustrate how to graph data using the modified tax data that follows Example 2 in Section 1.5 of Calculus Concepts.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax (in dollars)</td>
<td>2541</td>
<td>3081</td>
<td>3615</td>
<td>4157</td>
<td>4703</td>
<td>5242</td>
</tr>
</tbody>
</table>

Access the Y= graphing list. If any entered function is no longer needed, clear it by pressing \( \text{CLEAR} \). If you want the function(s) to remain but not graph when you draw the scatter plot, remove the check mark next to that function with \( \text{F4} \) \( \text{[} \). Also be sure that Plot 1 at the top of the Y= screen is turned on (i.e., checked).

Access the data with APPS 6 [Data/Matrix Editor] and choose 1 [Current]. Using the table given above, enter the year data in c1 and the modified tax data in c2 according to the instructions given in Section 1.5.1 of this Guide. (You can either leave values in the other lists or clear them.)

Press \( \bullet \) F1 (Y=) and then F2 [Zoom] 9 [ZoomData] to have the calculator set an autoscaled view of the data and draw the scatter plot. (Note that ZoomData also resets the x- and y-axis tick marks.)

Recall that if the data are perfectly linear (that is, every data point falls on the graph of a line), the first differences in the output values are constant. The first differences for the original tax data were constant at $541, so a linear function fit the data perfectly. What information is given by the first differences for these modified tax data?

Run program DIFF. (See Section 1.5.4 of this Guide.) Recall that the program stores the first differences in list d1 if you want to recall them on the home screen.
These first differences are close to being constant. This information, together with the linear pattern shown by the scatter plot, are a good indication that a linear function is likely to give a good fit to the data.

### 1.5.7 FINDING A LINEAR FUNCTION TO MODEL DATA

You will often have your TI-89 find the linear function that best fits a set of data. Other functions are fit using the same steps.

Access the data with [APPS 6 [Data/Matrix Editor] and choose 1 [Current]. Press [F5] [Calc]. On the first line (Calculation Type…), press ► and 5 [LinReg]. Type in c1 for x and c2 for y.

On the fourth line (Store RegEQ to…), press ► and choose where you want to paste the linear function. Press ENTER.

**CAUTION:** The best-fit function found by the calculator is also called a regression function. The coefficients of the regression function *never should be rounded!* This is not a problem because the calculator pastes the entire equation it finds into the Y= list at the same time the function is found if you follow the instructions given above.

Use ▼ to move between the various options in the Calculate screen. When you have entered all the information, press ENTER to exit the screen. Press ◼ F1 (Y=) and see that the function has been pasted in the chosen location.

**CAUTION:** The number that is labeled corr on the STAT VARS screen is called the correlation coefficient. This value and the one labeled $R^2$, which is the coefficient of determination, are numbers that you will learn about in a statistics course. It is not appropriate to use these values in a calculus course.

**Graphing the Line of Best Fit:** After finding a best-fit equation, you should always draw the graph of the function on a scatter plot to verify that the function provides a good fit to the data.

Press ◼ [F5] (GRAPH) or [F2] [Zoom] 9 [ZoomData] to overdraw the function on the scatter plot of the data.

(As we suspected from looking at the scatter plot and the first differences, this function provides a very good fit to the data.)

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5Unfortunately, there is no single number that can be used to tell whether one function better fits data than another. The correlation coefficient only compares linear fits and should not be used to compare the fits of different types of functions. For the statistical reasoning behind this statement, read the references in footnote 8 on page C-35.
1.5.8 US I N G A M O D E L F O R P R E D I C T I O N S  You can use one of the methods described in Sections 1.1.5 or 1.1.6 of this Guide to evaluate the linear function at the indicated input value.

Predict the tax owed in 2002 where the tax is found using the linear model computed in the last section of this Guide: Tax = 540.371t − 1,076,042.467 dollars where t is the year. Remember that you should always use the full model, i.e., the function you pasted in y1, not the rounded equation given above, for all computations.

The 2002 tax, predicted by evaluating an output on the home screen, is about $5781. We now predict the tax in 2003 using the TI-89 table. As seen to the right, the predicted tax is approximately $6322.

1.5.9 C O P Y I N G A G R A P H T O P A P E R  Your instructor may ask you to copy what is on your graphics screen to paper. If so, use the following ideas to more accurately perform this task.

After using a ruler to place and label a scale (i.e., tick marks) on your paper, use the trace values (as shown below) or the data to graph a scatter plot on your paper.

Press \( F3 \) (GRAPH) to return the modified tax data graph found in Section 1.5.7 to the screen. Press \( F3 \) [Trace] and \( \blacksquare \). The symbol P1 in the upper right-hand corner of the screen indicates that you are tracing the scatter plot (Plot 1) of the data.

Press \( \blacktriangleleft \) to move the trace cursor to the linear function graph. The number in the top right of the screen tells you the location of the function that you are tracing (in this case, \( y_1 \)). Use \( \blacktriangledown \) and/or \( \blacktriangleright \) to locate values that are as “nice” as possible and mark those points on your paper. Use a ruler to connect these points and draw the line.

- If you are copying the graph of a continuous curve rather than a straight line, you need to trace as many points as necessary to see the shape of the curve while marking the points on your paper. Connect the points with a smooth curve.

1.5.10 A L I G N I N G D A T A  We return to the modified tax data entered in Section 1.5.6. If you want \( c_1 \) to contain the number of years after a certain year instead of the actual year, you need to align the input data. In this illustration, we shift all of the data points to three different positions to the left of where the original values are located.

Press \([\text{APPS}] 6 \) [Data/Matrix Editor] 1 [Current] to access the data lists. To align the input data as the number of years past 1996, first press the arrow keys (\( \blacktriangleleft \) and/or \( \blacktriangleright \)) so that \( c_1 \) is highlighted. Tell the TI-89 to subtract 1996 from each number in \( c_1 \) with \( \alpha \) (c) 1 \( \blacktriangleleft \) 1996 \( \text{ENTER} \). Instead of an actual year, the input now represents the number of years since 1996.
NOTE: Another way to tell the TI-89 the alignment formula is to have the cursor on any cell in c1 and press \textbf{F4} \textbf{[Header]}. Type \texttt{alpha j (c) 1 – 1996} and press \textbf{ENTER}.

- If you did \textbf{not} get the values 0, 1, 2, 3, 4, and 5 in c1, press \textbf{F1} \textbf{[Tools]} 9 \textbf{[Format]} and select \textbf{ON} in the Auto-calculate position. Press \textbf{ENTER} to return to the data editor.

Once the data have been aligned, clear the alignment definition from the column header. To do this, have c1 highlighted and press \textbf{ENTER} (or press \textbf{F4} \textbf{[Header]}) to highlight the alignment definition at the bottom of the screen. Press \textbf{CLEAR} \textbf{ENTER}.

WARNING: If you do not clear the alignment definition from the column header, the Auto-calculate feature will keep subtracting 1996 from the data every time you have c1 highlighted and \textbf{ENTER} is pressed.

Find the linear function by pressing \textbf{F5} \textbf{[Calc]}. On the first line (Calculation Type...), press \textbf{►} and 5 \textbf{[LinReg]}. Type in c1 for \textit{x} and c2 for \textit{y}. On the fourth line (Store RegEQ to...), press \textbf{►} and choose \textit{y2(x)}.

Press \textbf{ENTER}. Press \texttt{Y=} and \textbf{F1} \textbf{[Y=]} and see that the function has been pasted in the chosen location. (Recall that you can highlight the function location and press \textbf{►} in order to see the complete function.)

Highlight the \textit{y1} line and uncheck the function with \textbf{F4} \textbf{[▼]} to turn \textit{y1} off. Press \textbf{F2} \textbf{[Zoom]} 9 \textbf{[ZoomData]} to overdraw the function \textit{y2(x)} on the scatter plot of the data.

If you now want to find the linear function that best fits the modified tax data using the input data aligned another way, say as the number of years after 1900, first return to the data lists with \textbf{APPS} 6 \textbf{[Data/Matrix Editor]} 1 \textbf{[Current]} and highlight c1.

Add 96 to each number currently in c1 with \texttt{alpha l (c) 1 + 96}. Press \textbf{ENTER}. The input now represents the number of years since 1900. Press \textbf{F4} \textbf{[Header]} \textbf{CLEAR} \textbf{ENTER}.

- The last step above was to clear the alignment definition once the data have been aligned. Remember to do this or the input data may change without you realizing it when you perform other calculations in the lists.
Press [F5] [Calc]. On the first line, press ► and 5 [LinReg].
Type in c1 for x and c2 for y. On the fourth line, press ► and store the linear function to y3(x). Press ENTER.

Press ◼ ◼ F1 (Y=) and see that the function is pasted in y3. Uncheck y1 and y2 and press F2 [Zoom] 9 [Zoom-Data] to overdraw the function on the scatter plot.

- Remember, if you have aligned the data, the input value at which you evaluate the function may not be the value given in the question you are asked. You can use any of the equations to evaluate function values.

Using the function in y1 (the input is the year), in y2 (the input is the number of years after 1996), or in y3 (the input is the number of years after 1900), we predict that the tax owed in 2003 is approximately $6322.

1.5.11 NAMING AND STORING DATA  You can name data and store it in the TI-89 memory for later recall. You may or may not want to use this feature; it would be helpful if you plan to use a large data set several times and don’t want to reenter it each time you use it. To illustrate the procedure, let’s name the current data.

Be in the data editor ([APPS] 6 [Data/Matrix Editor] 1 [Current]).
Press F1 [Tools] 2 [Save Copy As …]. In the Variable box, enter a name for the data, say tax. Press ENTER until this screen disappears.

Return to the home screen with HOME. You can verify that the data have been stored to a data variable by pressing 2nd (VAR-LINK). (Note that your list of variables may not be exactly the same as that shown on the screen to the right.)

CAUTION: The list of variables is in alphabetical order, so you may need to scroll to find the name of the data variable you stored. It is very important that you not store data to a name that is routinely used by the TI-89. Such names are c1, c2, ..., c99, MATH, Log, MODE, A, B, and so forth. Also note that if you use a single letter as a name, this might cause one or more of the programs to not execute properly.

1.5.12 RECALLING STORED DATA  The data you have stored remains in the memory of the TI-89 until you delete it using the instructions given in Section 1.5.15 of this Guide. When you wish to use the stored data, recall it to the data editor as we next illustrate with the tax data.
Press [APPs] 6 [Data/Matrix Editor] 2. [Open]. Press ▼ and choose tax.
Press ENTER ENTER. The data editor now contains the tax data.

1.5.13 **DELETING STORED DATA**  You do not need to delete any data unless your memory is getting low, the name interferes with a program execution, or you just want to do it. To illustrate, we delete the tax data. Note that this procedure works to delete any variable.

From the home screen, press 2nd - (VAR-LINK). Use ▼ to scroll down to and highlight the name of the variable you want to delete. Press F1 [Manage] 1 [Delete]. Press ENTER.

**WARNING:** When the data editor is open to a particular data set, that data variable does not appear in the list of variables. You should be on the home screen before accessing the list of variables with VAR-LINK.

1.5.16 **WHAT IS “BEST FIT”?** It is important to understand the method of least squares and the conditions necessary for its use if you intend to find the equation that best-fits a set of data. You can explore6 the process of finding the line of best fit with program LSLINE. (Program LSLINE is given in the TI-89 Program Appendix.) For your investigations of the least-squares process with this program, it is better to use data that is not perfectly linear and data for which you do not know the best-fitting line.

Before using program LSLINE, clear all functions from the Y= list, turn on Plot1 by having its name checked on the Y= screen, and enter your data in lists cl and c2 in the calcC data variable. (If Plot 1 is not turned on and the data is not stored in calcC, the program will not run.) Next, draw a scatter plot with ◀ F1 (Y=) F2 [Zoom] 9 [ZoomData]. Press WINDOW and reset xscl and yscl so that you can use the tick marks to help identify points when you are asked to give the equation of a line to fit the data. Press ◀ F3 (GRAPH), view the scatter plot, and then return to the home screen.

To activate program LSLINE, type lsline() on the entry line of the home screen or press 2nd - (VAR-LINK) followed by the key of to first letter of the program name, here 4 (L), and ENTER. The program displays this message.

**NOTE:** While the program is calculating, the indicator in the bottom right-hand corner of the screen says BUSY. When the program pauses, this indicator says PAUSE. Program LSLINE pauses several times during execution for you to view the screen. Whenever this happens, you should press ENTER to resume execution after you have viewed the screen.

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6 This program works well with approximately 5 data points. Interesting data to use in this illustration are the height and weight, the arm span length and the distance from the floor to the navel, or the age of the oldest child and the number of years the children’s parents have been married for 5 randomly selected persons.
Press **ENTER** to continue the program and it next tells you what the tick mark settings are. After pressing **ENTER**, you see a graph of the scatter plot.

The program next asks you to find the $y$-intercept and slope of some line you estimate will go “through” the data. (You should not expect to guess the best-fit line on your first try!) Use the tick marks to estimate rise divided by run and note a possible $y$-intercept. Enter your guess for the slope and $y$-intercept.

After pressing **ENTER** again, your line is drawn and the errors are shown as vertical line segments on the graph. Next the sum of squared errors, SSE, is displayed for your line.

Decide whether you want to move the $y$-intercept of the line up or down or change the slope of the line to improve the fit to the data.

Press **ENTER** type 1 to choose the Try Again? option. You are then shown the scatter plot to view. After deciding on a new guess for the slope and/or $y$-intercept, press **ENTER**. Input the new values when requested.

The process of viewing your line, the errors, and display of SSE is repeated. If the new value of SSE is smaller than the SSE for your first guess (as is the case here), you have improved the fit.

When you are satisfied with your line, enter 2 [no] at the Try Again? prompt. The program shows the linear fit calculation. Press **ENTER** and the line of best fit and your last line are drawn.

Press **ENTER** and the line of best fit and its errors display. The SSE for the best-fit line is shown and then all the parameters of the best-fit line are given.

Press **ENTER** to end the program. Use program LSLINE to explore the method of least squares that the TI-89 uses to find the line of best fit.

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7 Program LSLINE is for illustration purposes only. Actually finding the line of best fit for a set of data should be done according to the instructions in Section 1.5.7 of this Guide.