CHAPTER 11

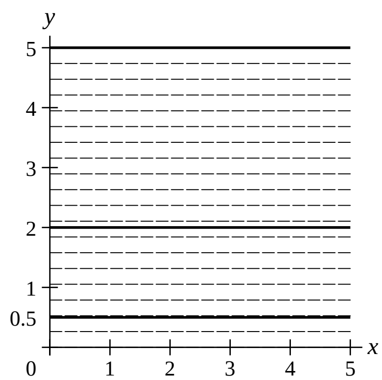
Section 11.1

1. $c = kg$

3. $v = kt$, where v = velocity and t = time (in seconds)

5. $\frac{dp}{da} = ka$, so $p(a) = \frac{ka^2}{2} + C$

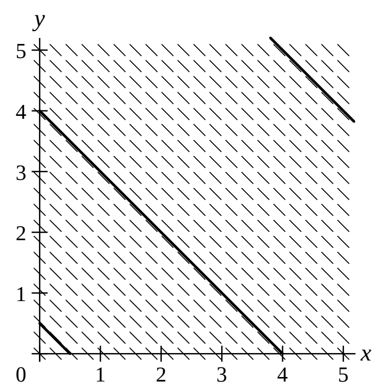
7. a. The displayed particular solutions go through the points (0, 0.5), (2, 2), and (4, 5). Answers will vary.



b. All particular solutions are horizontal lines, each passing through the chosen initial condition.

c. $y(x) = C$, where C is a constant

9. a. The displayed particular solutions go through the points (0, 0.5), (2, 2), and (4, 5). Answers will vary.



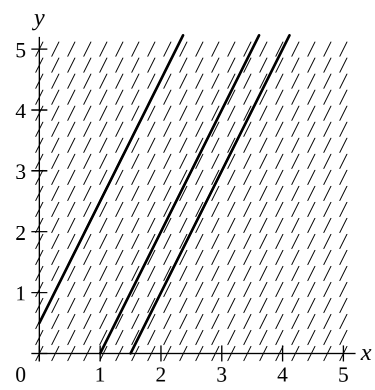
b. All particular solutions are parallel lines with negative slopes.

c. $y(x) = -x + C$, where C is a constant

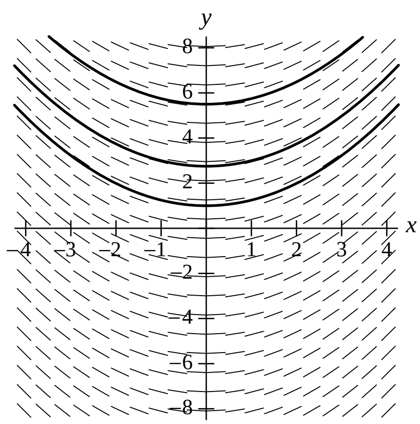
11. a. The displayed particular solutions go through the points (0, 0.5), (2, 2), and (4, 5). Answers will vary.

b. All particular solutions are parallel lines with positive slopes.

c. $y(x) = 2x + C$, where C is a constant



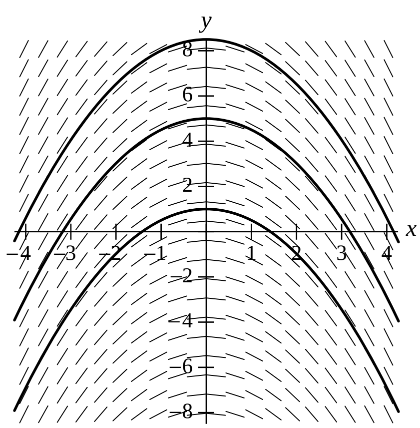
13. a. The displayed particular solutions go through the points (0, 1), (1, 3), and (2, 6.5). Answers will vary.



b. All particular solutions are concave-up parabolas with minimum points on the vertical axis.

c. $y(x) = 0.25x^2 + C$, where C is a constant

15. a. The displayed particular solutions go through the points (0, 1), (2, 3), and (2, 6.5). Answers will vary.



b. All particular solutions are concave-down parabolas with maximum points on the vertical axis.

c. $y(x) = -0.5x^2 + C$, where C is a constant

17. The particular solutions with initial condition (0, 0) all pass through the point (0, 0). Each particular solution has a constant term of zero, but the nonconstant terms all differ.

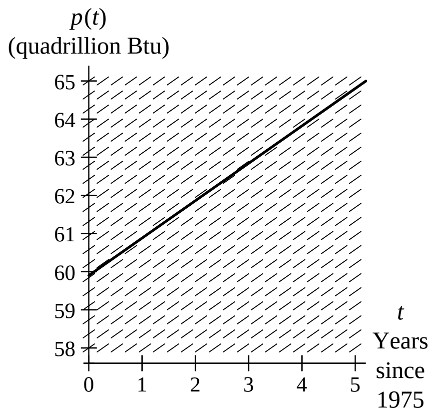
19. a. $\frac{dp}{dt} = 0.98$ quadrillion Btu per year t years after 1975

b. $p(t) = 0.98t + C$ quadrillion Btu t years after 1975

c. $p(t) = 0.98t + 59.9$ quadrillion Btu t years after 1975

d. In 1975 production was 59.9 quadrillion Btu and was increasing by 0.98 quadrillion Btu per year.

e.



$p(0) \approx 59.9$ quadrillion Btu

21. a. $\frac{dc}{dt} = 0.0342$ million square kilometers per year t years after 1970

b. $c(t) = 0.0342t + C$ million square kilometers t years after 1970

c. $c(t) = 0.0342t + 13.828$ million square kilometers t years after 1970

d. Cropland was increasing by 0.0342 million square kilometers per year in both 1970 and 1990. In 1970 there were 13.828 million square kilometers of cropland, and in 1990 there were 14.512 million square kilometers.

23. a. $v(t) = -32t$ feet per second t seconds after the object is dropped

b. $\frac{ds}{dt} = -32t$ feet per second t seconds after the object is dropped

c. $s(t) = -16t^2 + C$ feet t seconds after the object is dropped

d. Approximately 1.5 seconds after the object is dropped, it will hit the ground at a velocity of about −47.3 feet per second. (The negative sign on the velocity indicates downward motion.)

25. a. $\frac{df}{dx} = kx$

b. $f(x) = \frac{k}{2}x^2 + C$

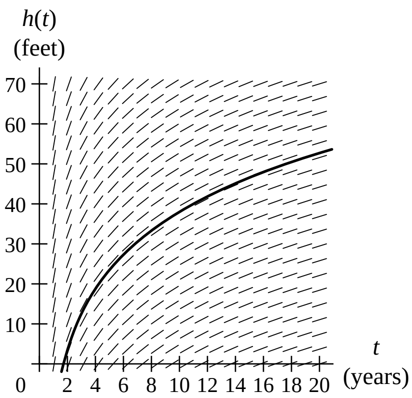
c. $\frac{d}{dx}\left(\frac{k}{2}x^2 + C\right) = kx$ leads to the identity $kx = kx$.

27. a. $\frac{dh}{dt} = \frac{k}{t}$ feet per year after t years

b. $h(t) = 20.75 \ln t - 10.39$ feet after t years

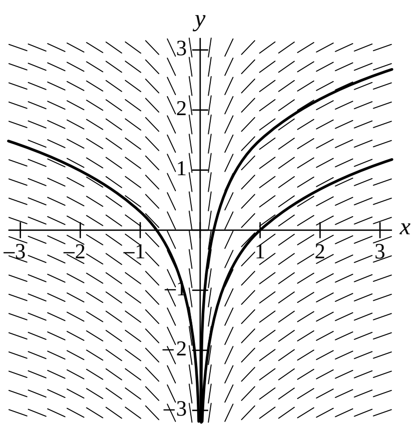
c. $h(15) \approx 45.8$ feet. Over time, the tree will continue to grow, but the rate of increase will be smaller and smaller.

29. a.



b. $h(15) \approx 46$ feet

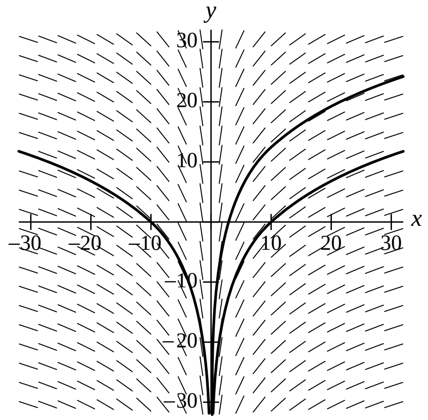
31. a. i. The particular solutions that are sketched go through (1, 1.5), (−2, 1), and (1, 0). Answers will vary.



ii. When $x > 0$, the graph of the particular solution rises as x gets larger. When $x < 0$, the particular solution graph rises as x gets smaller. The particular solution graphs are concave down.

iii. The family of solutions appears to increase rapidly as x moves away from the origin (in both directions), and then the increase slows down. The line $x = 0$ (lying along the y-axis) appears to be a vertical asymptote for the family.

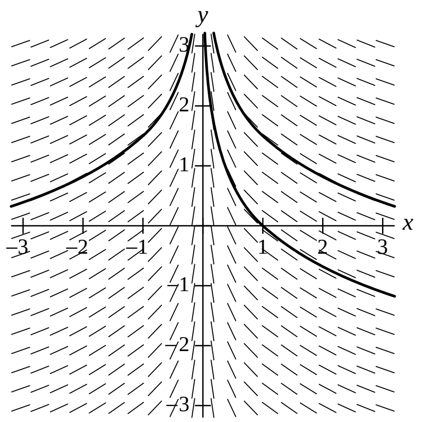
b. i. The particular solutions that are sketched go through (10, 0), (−10, 0), and (5, 5). Answers will vary.



ii. When $x > 0$, the graph of the particular solution rises as x gets larger. When $x < 0$, the particular solution graph rises as x gets smaller. The particular solution graphs are concave down.

iii. The family of solutions appears to behave the same as that in part *a*, but the slope at each point on a particular solution graph is 10 times the slope at the corresponding point on a particular solution graph in part *a*. Again, the line $x = 0$ appears to be a vertical asymptote for the family.

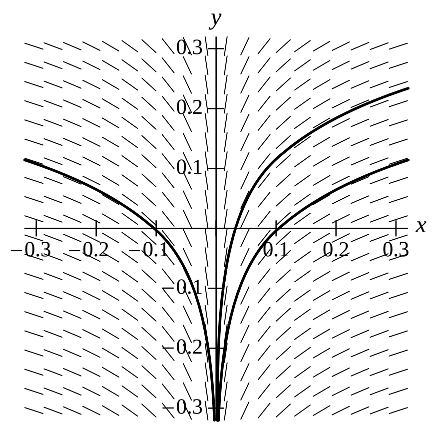
c. i. The particular solutions that are sketched go through (1, 1.5), (−2, 1), and (1, 0). Answers will vary.



ii. When $x > 0$, the graph of the particular solution falls as x gets larger. When $x < 0$, the particular solution graph falls as x gets smaller. The particular solution graphs are concave up.

iii. The slope at each point on a particular solution graph is the negative of the slope at a corresponding point on a particular solution graph in part *a*. The family of solutions appears to decrease rapidly as x moves away from the origin (in both directions), and then the decrease levels off. Again, the line $x = 0$ appears to be a vertical asymptote for the family.

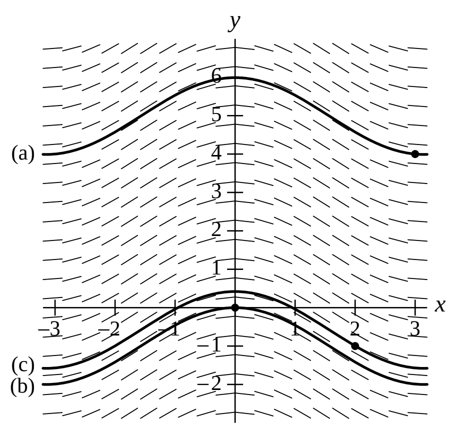
d. i. The particular solutions that are sketched go through (0.1, 0), (−0.1, 0), and (0.05, 0.05). Answers will vary.



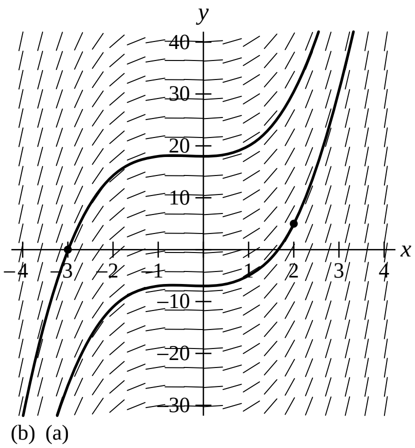
ii. When $x > 0$, the graph of the particular solution rises as x gets larger. When $x < 0$, the particular solution graph rises as x gets smaller. The particular solution graphs are concave down.

iii. The family of solutions appears to behave the same as that in part *a*, but the slope at each point on a particular solution graph is $\frac{1}{10}$ times the slope at the corresponding point on a particular solution graph in part *a*. Again, the line $x = 0$ appears to be a vertical asymptote for the family.

33.



35.



Section 11.2

1. $\dfrac{dT}{dt} = \dfrac{k}{T}$, so $T = \sqrt{2kt + C}$

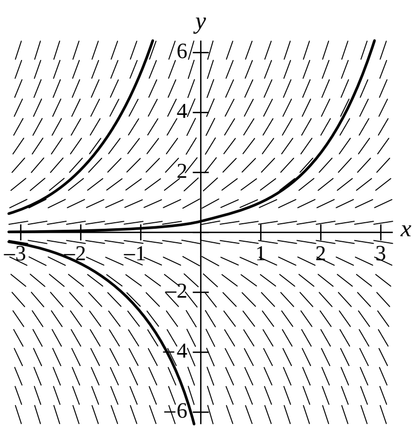
(We use the positive square root because thickness can't be negative.)

3. $\dfrac{dA}{dt} = kA$, so $A(t) = ae^{kt}$

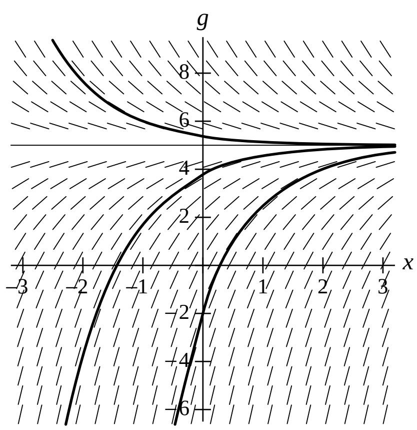
5. $\dfrac{dx}{dt} = kx(N - x)$, so $x(t) = \dfrac{N}{1 + Ae^{-Nkt}}$

7. $\dfrac{dD}{dt} = \dfrac{k}{\sqrt{D}} - hD$, where k and h are both constants

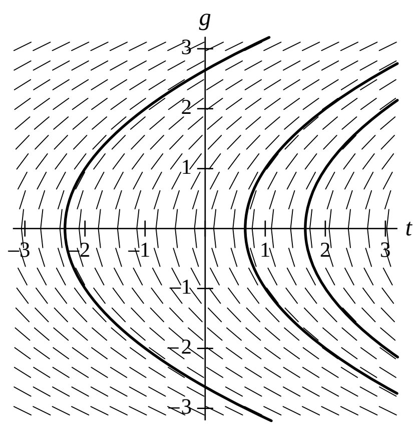
9. The particular solutions that are sketched go through (−2, 2), (−2, −1), and (1, 1). Answers will vary.



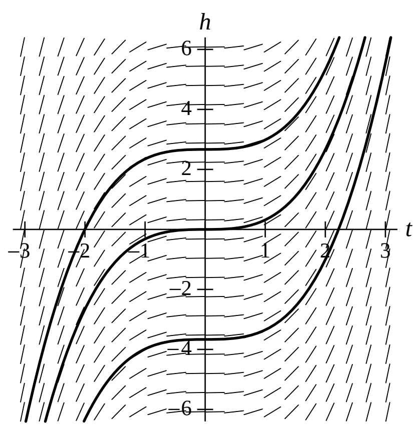
11. The particular solutions that are sketched go through (0, −2), (−2, −4), and (−1, 6). Answers will vary.



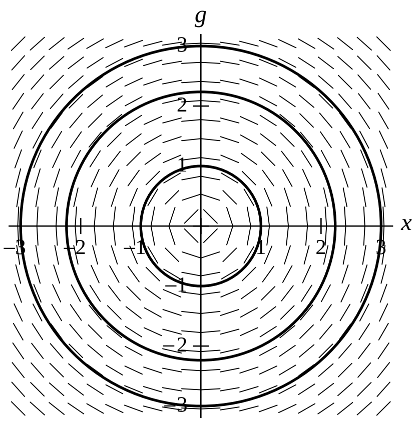
13. The particular solutions that are sketched go through (−2, 1), (2, 1), and (1, −1). Answers will vary.



15. The particular solutions that are sketched go through (0, 0), (−1, −4), and (1, 3). Answers will vary.



17. The particular solutions that are sketched go through (0, 3), (1, 0), and (2, 1). Answers will vary.



19. Solve by separation of variables; $y(x) = \pm ae^{kx}$

21. Solve by antidifferentiation; $y(x) = k \ln|x| + C$

23. Solve by separation of variables; $y(x) = \pm ax^k$

25. a. $\frac{dq}{dt} = kq$ milligrams per hour

b. $q(t) \approx 200e^{-0.346574t}$ milligrams after t hours

c. Approximately 50 milligrams after 4 hours and 12.5 milligrams after 8 hours.

27. a. $\frac{da}{dt} \approx ka$ units per day

b. $a(t) \approx ce^{-0.181262t}$ units after t days

c. Approximately 0.9 gram after 12 hours, 0.5 gram after 4 days, 0.2 gram after 9 days, and 0.004 gram after 30 days

29. a. $\frac{dN}{dt} = 0.0049N(37 - N)$ countries per year

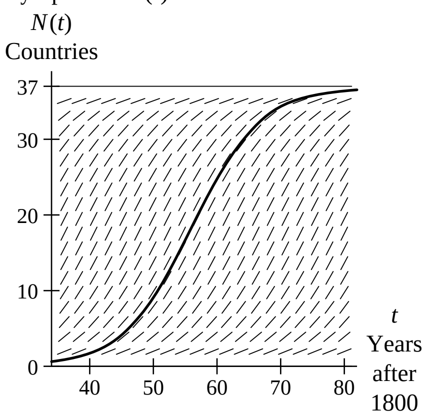
b. $N(t) = \frac{37}{1 + Ae^{-0.1813t}}$ countries t years after 1800

c. $N(t) = \frac{37}{1 + 28{,}097.439e^{-0.1813t}}$ countries t years after 1800

d. 2 countries in 1840 and 24 countries in 1860

e. The upper asymptote is $N(t) = 37$, and the lower asymptote is $N(t) = 0$.

f.



31. a. $\frac{df}{dx} = kf(L - f)$

b. $f(x) = \frac{L}{1 + Ae^{-Lkx}}$

c. $\frac{d}{dx}\left(\frac{L}{1 + Ae^{-Lkx}}\right) = k\left(\frac{L}{1 + Ae^{-Lkx}}\right)\left(L - \frac{L}{1 + Ae^{-Lkx}}\right)$

Each side of this equation can be simplified to $\frac{L^2Ake^{-Lkx}}{(1 + Ae^{-Lkx})^2}$.

Section 11.3

1. a. $y(4) \approx 2$

x	Estimate of *y*(*x*)	Slope at *x*
0	0	0.5
2	1	0.5
4	2	

b. $y(7) \approx 34$

x	Estimate of *y*(*x*)	Slope at *x*
1	4	2
4	10	8
7	34	

3. a. $y(5) \approx 11.91$

x	Estimate of *y*(*x*)	Slope at *x*
1	1	5
3	11	0.45
5	11.91	

b. $y(8) \approx 12.5$

x	Estimate of *y*(*x*)	Slope at *x*
2	2	2.5
5	9.5	1
8	12.5	

5. a. $\frac{dw}{dt} = \frac{33.67885}{t}$ pounds per month after t months

b. $w(3) \approx 46.0$ pounds; $w(6) \approx 70.0$ pounds

t (months)	Estimate of *w*(*t*) (pounds)	Slope at *t* (pounds per month)
1	6	33.679
1.25	14.420	26.943
1.5	21.155	22.453
1.75	26.769	19.245
2	31.580	16.839
2.25	35.790	14.968
2.5	39.532	13.472
2.75	42.900	12.247
3	45.961	11.226
3.25	48.768	10.363
3.5	51.359	9.623
3.75	53.764	8.981
4	56.010	8.420
4.25	58.115	7.924
4.5	60.100	7.484
4.75	61.967	7.090
5	63.739	6.736
5.5	67.027	6.123
5.75	68.558	5.857
6	70.022	

c. $w(3) \approx 56.5$ pounds: $w(6) \approx 82.9$ pounds

t (months)	Estimate of *w*(*t*) (pounds)	Slope at *t* (pounds per month)
1	6	33.679
2	39.679	16.839
3	56.518	11.226
4	67.745	8.420
5	76.164	6.736
6	82.90	

d. The answer to part *b* should give a more accurate approximation of the true weight at each input because it uses a smaller step size.

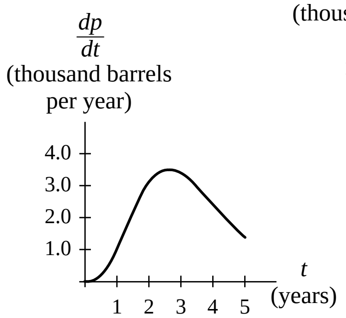
7. a. $\frac{dp}{dt} = 3.935t^{3.55}e^{-1.35135t}$ thousand barrels per year *t* years after production begins, where $p(t)$ is the total amount of oil produced after *t* years.

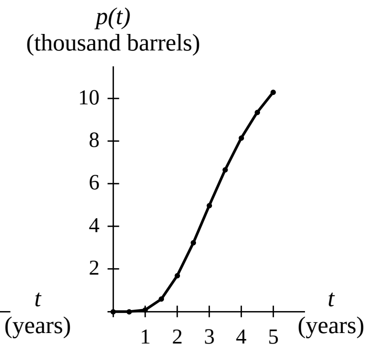
b. See table at top right.

During the first 5 years, the oil well will produce approximately 10.3 thousand barrels.

c. The graph of the differential equation is the slope graph for the graph of the Euler estimates. Similarly, the graph of the Euler estimates is an approximation to the accumulation graph of the differential equation graph.

t	Estimate of *p*(*t*)	*p*′(*t*)
0	0	0
0.5	0	0.17094
1	0.0855	1.0187
1.5	0.5948	2.1865
2	1.6881	3.0891
2.5	3.2326	3.4707
3	4.9680	3.3734
3.5	6.6547	2.9668
4	8.1381	2.4251
4.5	9.3507	1.8744
5	10.2879	





9. a. $\frac{dT}{dt} = k(T - A)$ °F per minute after *t* minutes

b. $k \approx -0.064$

c.

t	Estimate of *T*(*t*)	*T*′(*t*)
0	98	-1.8
1	96.2	-1.684
2	94.516	-1.576
3	92.940	-1.475
4	91.465	-1.380
5	90.085	-1.291
6	88.794	-1.208
7	87.586	-1.131
8	86.455	-1.058
9	85.397	-0.990
10	84.408	-0.926
11	83.481	-0.867
12	82.615	-0.811
13	81.804	-0.759
14	81.044	-0.710
15	80.335	

The temperature of the object is approximately 80.3°F after 15 minutes.

11. Answers will vary.

Section 11.4

1. $\frac{d^2S}{dt^2} = \frac{k}{S^2}$

3. $\frac{d^2P}{dy^2} = k$, so $P(y) = \frac{k}{2}y^2 + Cy + D$

5. a. $\frac{d^2R}{dt^2} = 6.14$

b. $R(t) = 3.07t^2 - 7.01t + 15.94$ jobs in the tth month of the year

c. 156 jobs in August and 310 jobs in November

7. a. $\frac{d^2A}{dt^2} = -2099$ cases per year^2

b. $A(t) = -1049.5t^2 + 5988.7t + 33{,}590$ cases t years after 1988

c. −308.3 cases per year and 42,111 cases

9. a. $\frac{d^2f}{dx^2} = k$

b. $f(x) = \frac{k}{2}x^2 + Cx + D$

c. $\frac{d^2}{dx^2}\left(\frac{k}{2}x^2 + Cx + D\right) = k$ leads to the identity $k = k$.

11. a. $\frac{d^2E}{dt^2} = -0.212531E$

b. $E(t) = -4.713 \sin(0.461011t + 2.248008)$ mm per day, where t is the month of the year

c. $E(t) = -4.713 \sin(0.461011t + 2.248008) + 12.5$ mm per day, where t is the month of the year.

d. The model estimates 14.7 mm per day in March and 12.0 mm per day in September.

Chapter 11 Review Test

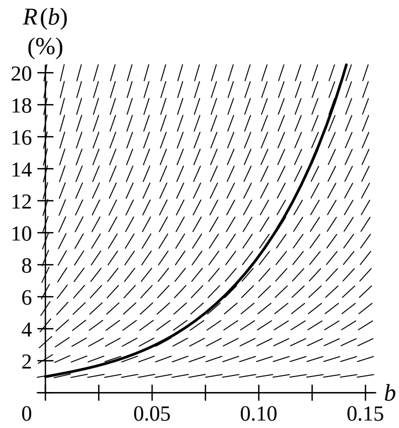
1. a. The relative risk of having a car accident is changing with respect to the blood alcohol level at a rate that is proportional to the risk of having a car accident at a certain blood alcohol level.

b. $R(b) = ae^{kb}$ percent, where b is the proportion of alcohol in the blood stream

c. $R(b) = e^{21.398b}$ percent, where b is the proportion of alcohol in the blood stream

d. A crash is certain to occur (that is, it has a 100% probability of occurring) when the blood alcohol level is 21.5%.

e.



2. a. $\frac{dP}{dx} = 0.001175P(16.396 - P)$ million people per year, where x is the number of years since 1800

b. $P(x) = \frac{16.396}{1 + Ae^{-0.019265x}}$ million people, where x is the number of years since 1800

c. $P(x) = \frac{16.396}{1 + 2.108e^{-0.019265x}}$ million people, where x is the number of years since 1800

d. 8.3 million people in 1840 and 9.1 million people in 1850

3. a. $P(40) \approx 8.18$ million people; $P(50) \approx 8.97$ million people

x (years after 1800)	Estimate of $P(x)$ (million people)	Slope at x (million people per year)
−20	4	0.058
−10	4.583	0.064
0	5.219	0.069
10	5.904	0.073
20	6.632	0.076
30	7.393	0.078
40	8.175	0.079
50	8.965	

b. $P(40) \approx 8.24$ million people; $P(50) \approx 9.03$ million people

c. These estimates are slightly smaller than those found in part d of Question 2.

4. a. $\frac{dQ}{dx} = -0.008307Q(7.154 - Q)$ million people per year, where the population is $P(x) = Q(x) + 4.4$ million people and x represents the number of years since 1800

x (years after 1800)	Estimate of $P(x)$ (million people)	Slope at x (million people per year)
−20	4	0.058
−15	4.291	0.061
−10	4.596	0.064
−5	4.915	0.066
0	5.247	0.069
5	5.590	0.071
10	5.945	0.073
15	6.310	0.075
20	6.684	0.076
25	7.066	0.077
30	7.453	0.078
35	7.844	0.079
40	8.239	0.079
45	8.633	0.079
50	9.027	

b. $Q(x) = \dfrac{7.154}{1 + 0.185e^{0.059428x}}$ million people, where x is the number of years since 1800

c.

$$P(x) = \begin{cases} \dfrac{16.396}{1 + 2.108e^{-0.019265x}} \text{ million people} & \text{when } x \leq 40 \\ \dfrac{7.154}{1 + 0.185e^{0.059428x}} + 4.4 \text{ million people} & \text{when } x \geq 50 \end{cases}$$

where x is the number of years since 1800

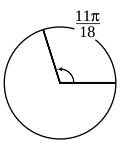
d. 6.0 million people in 1850; This answer is significantly smaller than the one found in part *d* of Question 2.

APPENDIX

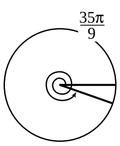
1. Rotation (in turns): $\frac{1}{8}, \frac{1}{6}, \frac{5}{8}, \frac{3}{4}, 2, \frac{21}{2}$

Angle measure (in degrees): 30, 90, 315, 360, 1080, 21,600

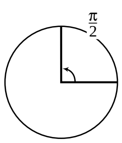
3. a. $\frac{11\pi}{18}$ radians



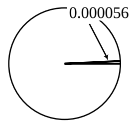
b. $\frac{35\pi}{9}$ radians



c. $\frac{\pi}{2}$ radians



d. 0.000056 radian



5. The front wheel makes approximately 853.86 turns in 1 mile; the rear wheel makes approximately 731.79 turns in 1 mile.

7. a. 12° b. $\frac{\pi}{15}$ radian c. Approximately 10.47 feet

9. a. Approximately 5.508°

b. Approximately 35.2047°

11. a. i. 1 rotation ii. $\frac{3}{4}$ rotation iii. $\frac{1}{2}$ rotation

iv. $\frac{1}{4}$ rotation v. $\frac{1}{8}$ rotation vi. $\frac{1}{16}$ rotation

b. i. 2π radians ii. 1.5π radians iii. π radians

iv. $\frac{\pi}{2}$ radians v. $\frac{\pi}{4}$ radians vi. $\frac{\pi}{8}$ radians

c. i. 1 ii. 0 iii. −0.707 iv. 0.707

13. $\cos\theta$

15. $\tan\theta$

17. $\cos\theta$

19. a. 13.737 inches

b. 14.619 inches

21. a. 11.580 centimeters

b. 3.146 centimeters

23. a. 18.703 miles

b. 18.643 miles

25. a. 9.326 feet

b. 0.233

c. 1765.4 square feet

27. a. 10.38 inches

b. Four steps

29. a. Bearing 270°

b. Northwest

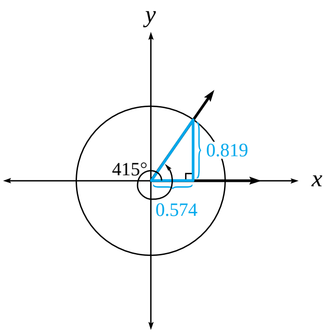
c. 4.415 miles north and 2.347 miles west

31.

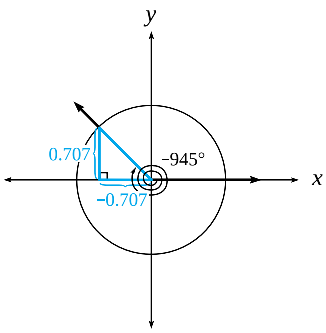
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
a. 315°	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	−1
b. −135°	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	1
c. −225°	$\frac{\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	−1

	θ	$\sin\theta$	$\cos\theta$
33.	415°	0.819	0.574
35.	310°	−0.766	0.643
37.	−945°	0.707	−0.707
39.	−37°	−0.602	0.799

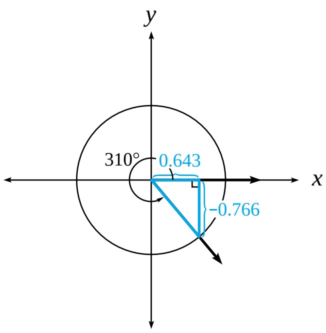
33.


y
x
0.819
415°
0.574

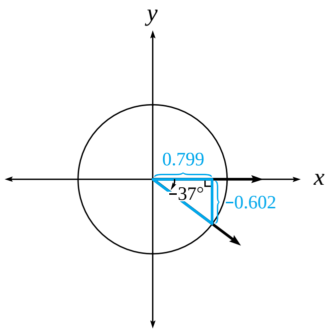
37.


y
x
0.707
−945°
−0.707

35.


y
x
310°
0.643
−0.766

39.


y
x
0.799
−37°
−0.602