Section 1.2  Trigonometric Functions: The Unit Circle

Objective: In this lesson you learned how to identify a unit circle and its relationship to real numbers.

I.  The Unit Circle  (Page 137)

As the real number line is wrapped around the unit circle, each real number $t$ corresponds to . . . a point $(x, y)$ on the circle.

The real number $2\pi$ corresponds to the point $(1, 0)$ on the unit circle.

Each real number $t$ also corresponds to a central angle $\theta$ (in standard position) whose radian measure is $t$. With this interpretation of $t$, the arc length formula $s = r\theta$ (with $r = 1$) indicates that . . . the real number $t$ is the length of the arc intercepted by the angle $\theta$, given in radians.

II.  The Trigonometric Functions  (Pages 138–140)

The coordinates $x$ and $y$ are two functions of the real variable $t$. These coordinates can be used to define six trigonometric functions of $t$. List the abbreviation for each trigonometric function.

Sine  \hspace{1em} \sin \hspace{1em} Cosecant  \hspace{1em} \csc

Cosine  \hspace{1em} \cos \hspace{1em} Secant  \hspace{1em} \sec

Tangent  \hspace{1em} \tan \hspace{1em} Cotangent  \hspace{1em} \cot
Let \( t \) be a real number and let \((x, y)\) be the point on the unit circle corresponding to \( t \). Complete the following definitions of the trigonometric functions:

\[
\begin{align*}
\sin t &= y \\
\cos t &= x \\
\tan t &= \frac{y}{x}, \quad x \neq 0 \\
\cot t &= \frac{x}{y}, \quad y \neq 0 \\
\sec t &= \frac{1}{x}, \quad x \neq 0 \\
\csc t &= \frac{1}{y}, \quad y \neq 0
\end{align*}
\]

The cosecant function is the reciprocal of the sine function. The cotangent function is the reciprocal of the tangent function. The secant function is the reciprocal of the cosine function.

Complete the following table showing the correspondence between the real number \( t \) and the point \((x, y)\) on the unit circle when the unit circle is divided into eight equal arcs.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>( \pi/4 )</th>
<th>( \pi/2 )</th>
<th>( 3\pi/4 )</th>
<th>( \pi )</th>
<th>( 5\pi/4 )</th>
<th>( 3\pi/2 )</th>
<th>( 7\pi/4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>1</td>
<td>( \sqrt{2}/2 )</td>
<td>0</td>
<td>( -\sqrt{2}/2 )</td>
<td>-1</td>
<td>( -\sqrt{2}/2 )</td>
<td>0</td>
<td>( \sqrt{2}/2 )</td>
</tr>
<tr>
<td>( y )</td>
<td>0</td>
<td>( \sqrt{2}/2 )</td>
<td>1</td>
<td>( \sqrt{2}/2 )</td>
<td>0</td>
<td>( -\sqrt{2}/2 )</td>
<td>-1</td>
<td>( -\sqrt{2}/2 )</td>
</tr>
</tbody>
</table>

Complete the following table showing the correspondence between the real number \( t \) and the point \((x, y)\) on the unit circle when the unit circle is divided into 12 equal arcs.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>( \pi/6 )</th>
<th>( \pi/3 )</th>
<th>( \pi/2 )</th>
<th>( 2\pi/3 )</th>
<th>( 5\pi/6 )</th>
<th>( \pi )</th>
<th>( 7\pi/6 )</th>
<th>( 4\pi/3 )</th>
<th>( 3\pi/2 )</th>
<th>( 5\pi/3 )</th>
<th>( 11\pi/6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>1</td>
<td>( \sqrt{3}/2 )</td>
<td>1/2</td>
<td>0</td>
<td>(-1/2)</td>
<td>(-\sqrt{3}/2)</td>
<td>-1</td>
<td>(-\sqrt{3}/2)</td>
<td>-1/2</td>
<td>0</td>
<td>( 1/2 )</td>
<td>( \sqrt{3}/2 )</td>
</tr>
<tr>
<td>( y )</td>
<td>0</td>
<td>1/2</td>
<td>( \sqrt{3}/2 )</td>
<td>1</td>
<td>( \sqrt{3}/2 )</td>
<td>1/2</td>
<td>0</td>
<td>(-1/2)</td>
<td>(-\sqrt{3}/2)</td>
<td>-1</td>
<td>(-\sqrt{3}/2)</td>
<td>-1/2</td>
</tr>
</tbody>
</table>

**Example 1:** Find the following:

\[
\begin{align*}
&a\cos\frac{\pi}{3} \quad (a) \quad 1/2 \\
&b\tan\frac{3\pi}{4} \quad (b) \quad -1 \\
&c\csc\frac{7\pi}{6} \quad (c) \quad -2
\end{align*}
\]

**III. Domain and Period of Sine and Cosine** (Pages 140–141)

The sine function’s domain is **the set of all real numbers**.

and its range is **\([-1, 1]\)**.
The cosine function’s domain is the set of all real numbers, and its range is \([-1, 1]\).

The period of the sine function is \(2\pi\). The period of the cosine function is also \(2\pi\).

Which trigonometric functions are even functions? 
- cosine and secant

Which trigonometric functions are odd functions? 
- Sine, cosecant, tangent, and cotangent

Example 2: Evaluate \(\sin \frac{3\pi}{6} - \frac{1}{2}\)

IV. Evaluating Trigonometric Functions with a Calculator
(Page 141)

To evaluate the secant function with a calculator, . . . evaluate its reciprocal function, cosine, and then use the \(x^{-1}\) key.

Example 3: Use a calculator to evaluate (a) \(\tan \frac{4\pi}{3}\), and (b) \(\cos 3\).
(a) \(1.732050808\)
(b) \(-0.9899924966\)