

Section 3.3 Vectors in the Plane

Objective: In this lesson you learned how to write the component forms of vectors, perform basic vector operations, and find the direction angles of vectors.

Course Number

Instructor

Date

Important Vocabulary

Define each term or concept.

Vector \mathbf{v} in the plane

Standard position

Zero vector

Unit vector

Standard unit vectors

Direction angle

I. Introduction (Page 279)

A **directed line segment** has an _____ and a _____.

The **magnitude of the directed line segment** \overrightarrow{PQ} , denoted by _____, is its _____. The magnitude of a directed line segment can be found by . . .

What you should learn

How to represent vectors as directed line segments

II. Component Form of a Vector (Page 280)

A vector whose initial point is at the origin $(0, 0)$ can be uniquely represented by the coordinates of its terminal point (v_1, v_2) . This is the _____, written $\mathbf{v} = \langle v_1, v_2 \rangle$, where v_1 and v_2 are the _____ of \mathbf{v} .

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

$$\overrightarrow{PQ} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \mathbf{v}.$$

What you should learn

How to write the component forms of vectors

The **magnitude** (or length) of \mathbf{v} is:

$$\|\mathbf{v}\| = \sqrt{\quad} = \sqrt{\quad}$$

Example 1: Find the component form and magnitude of the vector \mathbf{v} that has $(1, 7)$ as its initial point and $(4, 3)$ as its terminal point.

III. Vector Operations (Pages 281–283)

In operations with vectors, numbers are usually referred to as _____. Geometrically, the product of a vector \mathbf{v} and a scalar k is . . .

What you should learn

How to perform basic vector operations and represent them graphically

If k is positive, $k\mathbf{v}$ has the _____ direction as \mathbf{v} , and if k is negative, $k\mathbf{v}$ has the _____ direction.

To add two vectors geometrically, . . .

This technique is called the _____ for vector addition because the vector $\mathbf{u} + \mathbf{v}$, often called the _____ of vector addition, is . . .

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number). Then the sum of \mathbf{u} and \mathbf{v} is the vector:

$$\mathbf{u} + \mathbf{v} = \underline{\hspace{2cm}}$$

and the scalar multiple of k times \mathbf{u} is the vector:

$$k\mathbf{u} = \underline{\hspace{2cm}}$$

Example 2: Let $\mathbf{u} = \langle 1, 6 \rangle$ and $\mathbf{v} = \langle -4, 2 \rangle$. Find:

- (a) $3\mathbf{u}$
- (b) $\mathbf{u} + \mathbf{v}$

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors and c and d be scalars. Complete the following properties of vector addition and scalar multiplication:

1. $\mathbf{u} + \mathbf{v} =$ _____
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$ _____
3. $\mathbf{u} + \mathbf{0} =$ _____
4. $\mathbf{u} + (-\mathbf{u}) =$ _____
5. $c(d\mathbf{u}) =$ _____
6. $(c + d)\mathbf{u} =$ _____
7. $c(\mathbf{u} + \mathbf{v}) =$ _____
8. $1(\mathbf{u}) =$ _____
9. $0(\mathbf{u}) =$ _____
10. $\|c\mathbf{v}\| =$ _____

IV. Unit Vectors (Pages 283–284)

To find a unit vector \mathbf{u} that has the same direction as a given nonzero vector \mathbf{v} , . . .

What you should learn
How to write vectors as linear combinations of unit vectors

In this case, the vector \mathbf{u} is called a _____.

Example 3: Find a unit vector in the direction of $\mathbf{v} = \langle -8, 6 \rangle$.

Let $\mathbf{v} = \langle v_1, v_2 \rangle$. Then the standard unit vectors can be used to represent \mathbf{v} as $\mathbf{v} =$ _____, where the scalar v_1 is called the _____ and the scalar v_2 is called the _____. The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a _____ of the vectors \mathbf{i} and \mathbf{j} .

Example 4: Let $\mathbf{v} = \langle -5, 3 \rangle$. Write \mathbf{v} as a linear combination of the standard unit vectors \mathbf{i} and \mathbf{j} .

Example 5: Let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 9\mathbf{j}$. Find $\mathbf{v} + \mathbf{w}$.

V. Direction Angles (Page 285)

If \mathbf{u} is a unit vector and \mathbf{q} is its direction angle, the terminal point of \mathbf{u} lies on the unit circle and

$$\mathbf{u} = \langle x, y \rangle = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Now, if \mathbf{v} is any vector that makes an angle \mathbf{q} with the positive x -axis, it has the same direction as \mathbf{u} and

$$\mathbf{v} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

If \mathbf{v} can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, then the direction angle \mathbf{q} for \mathbf{v} can be determined from $\tan \mathbf{q} = \underline{\hspace{2cm}}$.

Example 6: Let $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$. Find the direction angle for \mathbf{v} .

VI. Applications of Vectors (Pages 286–287)

Describe several real-life applications of vectors.

What you should learn
How to find the direction angles of vectors

What you should learn
How to use vectors to model and solve real-life problems

Homework Assignment

Page(s)

Exercises