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Chapter 8 Matrices and Determinants

Section 8.1 Matrices and Systems of Equations

Objective: In this lesson you learned how to use matrices, Gaussian elimination, and Gauss-Jordan elimination to solve systems of linear equations.

Important Vocabulary	Define each term or concept.
Entry of a matrix	
Order of a matrix	
Square matrix	
Main diagonal	
Row matrix	
Column matrix	
Elementary row operations	
Gauss-Jordan elimination	

I. Matrices (Pages 572–573)

If m and n are positive integers, an $m \times n$ **matrix** is . . .

What you should learn
How to write matrices
and identify their orders

An $m \times n$ matrix has _____ rows and _____ columns.

An **augmented matrix** is . . .

A **coefficient matrix** is . . .

Example 1: Consider the following system of equations.

$$\begin{cases} 2x + y - z = 5 \\ x - 3y + 2z = 9 \\ 3x + 2y = 1 \end{cases}$$

- Write the augmented matrix for this system.
- What is the order of the augmented matrix?
- Write the coefficient matrix for this system.
- What is the order of the coefficient matrix?

II. Elementary Row Operations (Pages 574–576)

The **elementary row operations** on a matrix are:

What you should learn
How to perform elementary row operations on matrices

Two matrices are **row-equivalent** if . . .

A matrix in **row-echelon form** has the following three properties:

-
-
-

A matrix in row-echelon form is in **reduced row-echelon form**
if . . .

III. Gaussian Elimination with Back-Substitution (Pages 577–578)

To solve a system of linear equations using Gaussian
Elimination with Back-Substitution, . . .

What you should learn
How to use matrices and
Gaussian elimination to
solve systems of linear
equations

If, during the elimination process, you obtain a row with zeros
except for the last entry, you can conclude that the system has
_____.

Example 2: Solve the following system using Gaussian
Elimination with Back-Substitution.

$$\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 1 \\ x - 3y + 5z = -11 \end{cases}$$

IV. Gauss-Jordan Elimination (Pages 579–581)

Example 3: Apply Gauss-Jordan elimination to the following matrix to obtain the unique reduced row-echelon form of the matrix.

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

What you should learn
How to use matrices and Gauss-Jordan elimination to solve systems of linear equations

Example 4: Solve the following system using Gauss-Jordan elimination.

$$\begin{cases} 2x - y + 3z = 1 \\ x + 2y - 4z = -6 \\ -2x + 3y - z = 13 \end{cases}$$

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Section 8.2 Operations with Matrices

Objective: In this lesson you learned how to add and subtract matrices, multiply matrices by scalars, and multiply two matrices.

Important Vocabulary	Define each term or concept.
Scalars	
Scalar multiple	
Zero matrix	
Matrix multiplication	
Identity matrix of order n	

I. Equality of Matrices (Page 587)

Name three ways that a matrix may be represented.

- 1)
- 2)
- 3)

Two matrices are equal if they have the same order and _____ are equal.

What you should learn

How to decide whether two matrices are equal

II. Matrix Addition and Scalar Multiplication

(Pages 588–591)

To add two matrices of the same order, . . .

To multiply a matrix A by a scalar c , . . .***What you should learn***

How to add and subtract matrices and multiply matrices by scalars

Example 1: Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 2 & -5 \end{bmatrix}$.

Find (a) $A + B$ and (b) $-2B$

Let A , B , and C be $m \times n$ matrices and let c and d be scalars. Give an example of each of the following properties of matrix addition and scalar multiplication:

- 1) Commutative Property of Matrix Addition: _____
- 2) Associative Property of Matrix Addition: _____
- 3) Associative Property of Scalar Multiplication: _____
- 4) Scalar Identity Property: _____
- 5) Distributive Property (two forms): _____

If A is an $m \times n$ matrix and O is the $m \times n$ zero matrix consisting entirely of zeros, then $A + O =$ _____.

The additive identity for the set of all $m \times n$ matrices is the $m \times n$ matrix _____.

III. Matrix Multiplication (Pages 592–594)

When multiplying an $m \times n$ matrix A by an $n \times p$ matrix B , to obtain the entry in the i th row and j th column of AB , . . .

What you should learn
How to multiply two matrices

Example 2: If A is a 3×5 matrix and B is a 6×3 matrix, find the order, if possible, of the product (a) AB , and (b) BA .

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Example 3: Find the product AB , if

$$A = \begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

List four properties of Matrix Multiplication:

If A is an $n \times n$ matrix, the identity matrix I of order n has the property that _____ and _____.

IV. Applications of Matrix Operations (Pages 595–596)

Matrix multiplication can be used to represent a system of linear equations. The system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

can be written as the matrix equation _____, where A is the coefficient matrix of the system and X and B are column matrices.

What you should learn
How to use matrix operations to model and solve real-life problems

Example 4: Consider the following system of linear equations.

$$\begin{cases} 2x_1 - x_2 + 3x_3 = -11 \\ x_1 - 3x_3 = -1 \\ -x_1 + 4x_2 + 2x_3 = 2 \end{cases}$$

Write this system as a matrix equation $AX = B$, and then use Gauss-Jordan elimination on the augmented matrix $[A : B]$ to solve for the matrix X .

Additional notes

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Section 8.3 The Inverse of a Square Matrix

Objective: In this lesson you learned how to find the inverses of matrices and use inverse matrices to solve systems of linear equations.

Important Vocabulary

Define each term or concept.

Inverse of a matrix**I. The Inverse of a Matrix** (Page 602)

To verify that a matrix B is the inverse of the matrix A , . . .

What you should learn

How to verify that two matrices are inverses of each other

II. Finding Inverse Matrices (Pages 603–605)

If a matrix A has an inverse, A is called _____ or nonsingular. Otherwise, A is called _____.

What you should learn

How to use Gauss-Jordan elimination to find the inverses of matrices

To have an inverse, a matrix must be _____. Not all square matrices have inverses. However, if a matrix does have an inverse, that inverse is _____.

To find the inverse of a square matrix A of order n , . . .

Example 1: Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 6 \end{bmatrix}$.

III. The Inverse of a 2×2 Matrix (Page 606)

If A is a 2×2 matrix given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is invertible if

and only if _____. Moreover, if this condition is true, the inverse of A is given by:

$$A^{-1} = \frac{\quad}{\quad} \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

The denominator is called the _____ of the 2×2 matrix A .

Example 2: Find the inverse of the matrix $B = \begin{bmatrix} 3 & 9 \\ -2 & -7 \end{bmatrix}$.

What you should learn

How to use a formula to find the inverses of 2×2 matrices

IV. Systems of Linear Equations (Page 607)

If A is an invertible matrix, the system of linear equations represented by $AX = B$ has a unique solution given by

_____.

Example 3: Use an inverse matrix to solve (if possible) the system of linear equations:

$$\begin{cases} 12x + 8y = 416 \\ 3x + 5y = 152 \end{cases}$$

What you should learn

How to use inverse matrices to solve systems of linear equations

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Section 8.4 The Determinant of a Square Matrix

Objective: In this lesson you learned how to find minors, cofactors, and determinants of square matrices.

Important Vocabulary	Define each term or concept.
Determinant	
Minors	
Cofactors	

I. The Determinant of a 2 × 2 Matrix (Pages 611–612)

The **determinant** of the 2 × 2 matrix $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ is given by

$$\det(A) = |A| = \begin{vmatrix} & \\ & \end{vmatrix} = \underline{\hspace{2cm}}$$

What you should learn
How to find the determinants of 2 × 2 matrices

The determinant of a matrix of order 1 × 1 is defined as . . .

Example 1: Find the determinant of the matrix $A = \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix}$.

II. Minors and Cofactors (Page 613)

Complete the sign patterns for cofactors of a 3 × 3 matrix, a 4 × 4 matrix, and a 5 × 5 matrix:

What you should learn
How to find minors and cofactors of square matrices

Sign Pattern for Cofactors

3 × 3 matrix	4 × 4 matrix	5 × 5 matrix
$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$	$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$	$\begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$

Example 2: Use the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ to find:

- (a) the minor M_{13} , and (b) the cofactor C_{21} .

III. The Determinant of a Square Matrix (Pages 614–615)

Applying the definition of the determinant of a square matrix to find a determinant is called _____.

What you should learn

How to find the determinants of square matrices

Example 3: Find the determinant of the matrix:

$$A = \begin{bmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Example 4: Describe a strategy for finding the determinant of the following matrix, and then find the determinant of the matrix.

$$B = \begin{bmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{bmatrix}$$

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Section 8.5 Applications of Matrices and Determinants

Objective: In this lesson you learned how to use Cramer's Rule to solve systems of linear equations and how to use determinants and matrices to model and solve problems.

I. Cramer's Rule (Pages 619–621)

Cramer's Rule states that if a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant $|A|$, the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \dots, x_n = \frac{|A_n|}{|A|}$$

where the i th column of A_i is _____
_____.

Cramer's Rule does not apply if the determinant of the coefficient matrix is _____, in which case the system has either no solution or _____.

Example 1: Use Cramer's Rule to solve the system of linear equations.

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

What you should learn

How to use Cramer's Rule to solve systems of linear equations

II. Area of a Triangle (Page 622)

The area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol \pm indicates that the appropriate sign should be chosen to yield a positive area.

Example 2: Find the area of a triangle whose vertices are $(-3, 1)$, $(2, 4)$, and $(5, -3)$.

What you should learn

How to use determinants to find the areas of triangles

III. Lines in a Plane (Pages 623–624)

Three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are collinear (lie on the same line) if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

Example 3: Determine whether the points $(-2, 4)$, $(0, 3)$, and $(8, -1)$ are collinear.

An equation of the line passing through the distinct points (x_1, y_1) and (x_2, y_2) is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

Example 4: Find an equation of the line passing through the points $(-2, 9)$ and $(3, -1)$.

What you should learn

How to use a determinant to test for collinear points and find an equation of a line passing through two points

IV. Cryptography (Pages 625–627)

A cryptogram is . . .

What you should learn

How to use matrices to encode and decode messages

To use matrix multiplication to encode and decode messages, . . .

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