Section 8.5 Applications of Matrices and Determinants

Objective: In this lesson you learned how to use Cramer’s Rule to solve systems of linear equations and how to use determinants and matrices to model and solve problems.

I. Cramer’s Rule (Pages 619–621)

Cramer’s Rule states that if a system of \( n \) linear equations in \( n \) variables has a coefficient matrix \( A \) with a nonzero determinant \( |A| \), the solution of the system is

\[
x_1 = \frac{A_1}{|A|}, \quad x_2 = \frac{A_2}{|A|}, \ldots, x_n = \frac{A_n}{|A|}
\]

where the \( i \)th column of \( A_i \) is the column of constants in the system of equations.

Cramer’s Rule does not apply if the determinant of the coefficient matrix is zero, in which case the system has either no solution or infinitely many solutions.

Example 1: Use Cramer’s Rule to solve the system of linear equations.

\[
\begin{align*}
2x + y + z &= 6 \\
-x - y + 3z &= 1 \\
y - 2z &= -3
\end{align*}
\]

The solution is \((3, -1, 1)\).

II. Area of a Triangle (Page 622)

The area of a triangle with vertices \((x_1, y_1)\), \((x_2, y_2)\), and \((x_3, y_3)\) is

\[
\text{Area} = \pm \frac{1}{2} \begin{vmatrix}
x_1 & y_1 & 1 \\
x_2 & y_2 & 1 \\
x_3 & y_3 & 1 \\
\end{vmatrix}
\]

where the symbol \( \pm \) indicates that the appropriate sign should be chosen to yield a positive area.

Example 2: Find the area of a triangle whose vertices are \((-3, 1)\), \((2, 4)\), and \((5, -3)\).

22 square units
III. **Lines in a Plane** (Pages 623–624)

Three points \((x_1, y_1), (x_2, y_2),\) and \((x_3, y_3)\) are collinear (lie on the same line) if and only if

\[
\begin{vmatrix}
  x_1 & y_1 & 1 \\
  x_2 & y_2 & 1 \\
  x_3 & y_3 & 1 \\
\end{vmatrix} = 0.
\]

**Example 3:** Determine whether the points \((-2, 4), (0, 3),\) and \((8, -1)\) are collinear.

Yes, they are collinear.

An equation of the line passing through the distinct points \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[
\begin{vmatrix}
  x & y & 1 \\
  x_1 & y_1 & 1 \\
  x_2 & y_2 & 1 \\
\end{vmatrix} = 0.
\]

**Example 4:** Find an equation of the line passing through the points \((-2, 9)\) and \((3, -1)\).

\[y = -2x + 5\]

IV. **Cryptography** (Pages 625–627)

A cryptogram is . . . a message written according to a secret code.

To use matrix multiplication to encode and decode messages, . . . begin by assigning a number to each letter in the alphabet and then converting the message to numbers and partitioning them into uncoded row matrices, each having \(n\) entries. To encode a message, choose an \(n \times n\) invertible matrix \(A\) and multiply the uncoded row matrices by \(A\) on the right to obtain coded row matrices. An authorized receiver of the message then need only multiply the coded row matrices by \(A^{-1}\) on the right to retrieve the coded row matrices.

**Homework Assignment**

Page(s)

Exercises

**What you should learn**

How to use a determinant to test for collinear points and find an equation of a line passing through two points

How to use matrices to encode and decode messages