Chapter 4 Trigonometry

Section 4.1 Radian and Degree Measure

Objective: In this lesson you learned how to describe an angle and to convert between radian and degree measure.

<table>
<thead>
<tr>
<th>Important Vocabulary</th>
<th>Define each term or concept.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigonometry</td>
<td>The Greek word for “measurement of triangles.”</td>
</tr>
<tr>
<td>Central angle of a circle</td>
<td>An angle whose vertex is the center of the circle.</td>
</tr>
<tr>
<td>Complementary angles</td>
<td>Two positive angles whose sum is $\pi/2$ radians or $90^\circ$.</td>
</tr>
<tr>
<td>Supplementary angles</td>
<td>Two positive angles whose sum is $\pi$ radians or $180^\circ$.</td>
</tr>
<tr>
<td>Degree</td>
<td>The most common unit of angle measure, denoted by the symbol $^\circ$. A measure of one degree ($1^\circ$) is equivalent to a rotation of $1/360$ of a complete revolution about the vertex of an angle.</td>
</tr>
</tbody>
</table>

I. Angles (Page 282)

An angle is determined by . . . rotating a ray (half-line) about its endpoint.

The initial side of an angle is . . . the starting position of the rotated ray in the formation of an angle.

The terminal side of an angle is . . . the position of the ray after the rotation when an angle is formed.

The vertex of an angle is . . . the endpoint of the ray used in the formation of an angle.

An angle is in standard position when . . . the angle’s vertex is at the origin of a coordinate system and its initial side coincides with the positive $x$-axis.

A positive angle is generated by a _______ counterclockwise _______ rotation; whereas a negative angle is generated by a _______ clockwise _______ rotation.

If two angles are coterminal, then they have . . . the same initial side and the same terminal side.
II. Radian Measure (Pages 283–285)

The measure of an angle is determined by the amount of rotation from the initial side to the terminal side.

One radian is the measure of a central angle $\theta$ that intercepts an arc $s$ equal in length to the radius $r$ of the circle.

A central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$.

A full revolution around a circle of radius $r$ corresponds to an angle of $2\pi$ radians. A half revolution around a circle of radius $r$ corresponds to an angle of $\pi$ radians.

Angles with measures between $0$ and $\pi/2$ radians are acute angles. Angles with measures between $\pi/2$ and $\pi$ radians are obtuse angles.

To find an angle that is coterminal to a given angle $\theta$, add or subtract $2\pi$ (or integer multiples of $2\pi$) to the measure of $\theta$.

Example 1: Find an angle that is coterminal with $\theta = -\pi/8$.

15$\pi$/8

Example 2: Find the supplement of $\theta = \pi/4$.

3$\pi$/4

III. Degree Measure (Pages 285–286)

A full revolution (counterclockwise) around a circle corresponds to 360 degrees. A half revolution around a circle corresponds to 180 degrees.
To convert degrees to radians, multiply degrees by $(\pi \text{ rad})/180^\circ$.

To convert radians to degrees, multiply radians by $180^\circ/\pi \text{ rad}$.

Example 3: Convert $120^\circ$ to radians.

$$2\pi/3$$

Example 4: Convert $9\pi/8$ radians to degrees.

$$202.5^\circ$$

Example 5: Complete the following table of equivalent degree and radian measures for common angles.

<table>
<thead>
<tr>
<th>$\theta$ (degrees)</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (radians)</td>
<td>0</td>
<td>$\pi/6$</td>
<td>$\pi/4$</td>
<td>$\pi/3$</td>
<td>$\pi/2$</td>
<td>$\pi$</td>
<td>$3\pi/2$</td>
</tr>
</tbody>
</table>

IV. Applications of Angles (Pages 287–289)

For a circle of radius $r$, a central angle $\theta$ intercepts an arc length $s$ given by $s = r\theta$ where $\theta$ is measured in radians.

Note that if $r = 1$, then $s = \theta$, and the radian measure of $\theta$ equals the arc length.

Consider a particle moving at constant speed along a circular arc of radius $r$. If $s$ is the length of the arc traveled in time $t$, then the linear speed of the particle is

$$\text{linear speed} = \frac{(\text{arc length})}{(\text{time})} = \frac{s}{t}$$

If $\theta$ is the angle (in radian measure) corresponding to the arc length $s$, then the angular speed of the particle is

$$\text{angular speed} = \frac{(\text{central angle})}{(\text{time})} = \frac{\theta}{t}$$

Example 6: A 6-inch-diameter gear makes 2.5 revolutions per second. Find the angular speed of the gear in radians per second.

$$5\pi \text{ radians per second}$$
A sector of a circle is . . . the region bounded by two radii of the circle and their intercepted arc.

For a circle of radius $r$, the area $A$ of a sector of the circle with central angle $\theta$ is given by $A = \frac{1}{2} r^2 \theta$, where $\theta$ is measured in radians.