Section 2.3 Polynomial and Synthetic Division

Objective: In this lesson you learned how to use long division and synthetic division to divide polynomials by other polynomials.

I. Long Division of Polynomials (Pages 153–155)

Dividing polynomials is valuable when . . . factoring and finding zeros of polynomial functions.

When dividing a polynomial $f(x)$ by another polynomial $d(x)$, if the remainder $r(x)$ is zero, $d(x)$ divides evenly into $f(x)$.

Before applying the Division Algorithm, follow these steps:
1. Write the dividend and divisor in descending powers of the variable.
2. Insert placeholders with zero coefficients for missing powers of the variable.

Example 1: Divide $3x^3 + 4x - 2$ by $x^2 + 2x + 1$.

$3x - 6 + (13x + 4)/(x^2 + 2x + 1)$

II. Synthetic Division (Page 156)

Can synthetic division be used to divide a polynomial by $x^2 - 5$? Explain.

No, the divisor must be in the form $x - k$.

Can synthetic division be used to divide a polynomial by $x + 4$? Explain.

Yes, rewrite $x + 4$ as $x - (-4)$.
Example 2: Fill in the following synthetic division array to divide $2x^4 + 5x^2 - 3$ by $x - 5$. Then carry out the synthetic division and indicate which entry represents the remainder.

$$
\begin{array}{c|ccccc}
5 & 2 & 0 & 5 & 0 & -3 \\
\hline
& 10 & 50 & 275 & 1375 \\
\end{array}
$$

The remainder is $2\ 10\ 55\ 275\ 1372 \leftarrow$ remainder

III. The Remainder and Factor Theorems (Pages 157–158)

The Remainder Theorem states that . . . if a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

To use the Remainder Theorem to evaluate a polynomial function $f(x)$ at $x = k$, . . . use synthetic division to divide $f(x)$ by $x - k$. The remainder will be $f(k)$.

Example 3: Use the Remainder Theorem to evaluate the function $f(x) = 2x^4 + 5x^2 - 3$ at $x = 5$.

The Factor Theorem states that . . . a polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

To use the Factor Theorem to show that $(x - k)$ is a factor of a polynomial function $f(x)$, . . . use synthetic division on $f(x)$ with the factor $(x - k)$. If the remainder is 0, then $(x - k)$ is a factor. Or, alternatively, evaluate $f(x)$ at $x = k$. If the result is 0, then $(x - k)$ is a factor.

List three facts about the remainder $r$, obtained in the synthetic division of $f(x)$ by $x - k$:

1) The remainder $r$ gives the value of $f$ at $x = k$. That is, $r = f(k)$.
2) If $r = 0$, $(x - k)$ is a factor of $f(x)$.
3) If $r = 0$, $(k, 0)$ is an $x$-intercept of the graph of $f$.

Homework Assignment

Page(s)

Exercises