Section 1.8 Combinations of Functions: Composite Functions

Objective: In this lesson you learned how to find arithmetic combinations and compositions of functions.

I. Arithmetic Combinations of Functions (Pages 84–85)

Just as two real numbers can be combined with arithmetic operations, two functions can be combined by the operations of addition, subtraction, multiplication, and division to create new functions. A combined function like this is called an arithmetic combination of functions.

The domain of an arithmetic combination of functions $f$ and $g$ consists of all real numbers that are common to the domains of $f$ and $g$. In the case of the quotient $f(x)/g(x)$, there is the further restriction that $g(x) \neq 0$.

Let $f$ and $g$ be two functions with overlapping domains. Complete the following arithmetic combinations of $f$ and $g$ for all $x$ common to both domains:

1) Sum: $(f + g)(x) = f(x) + g(x)$
2) Difference: $(f - g)(x) = f(x) - g(x)$
3) Product: $(fg)(x) = f(x) \cdot g(x)$
4) Quotient: $\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example 1: Let $f(x) = 7x - 5$ and $g(x) = 3 - 2x$. Find $(f - g)(4)$.

II. Composition of Functions (Pages 86–87)

The composition of the function $f$ with the function $g$ is defined as $(f \circ g)(x) = f(g(x))$. 

What you should learn

- How to add, subtract, multiply, and divide functions
- How to find the composition of one function with another function
For the composition of the function $f$ with $g$, the domain of $(f \circ g)$ is . . . the set of all $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.

For two functions $f$ and $g$, to find $(f \circ g)(x)$, . . . replace each occurrence of $x$ in $f$ with the algebraic expression which defines $g(x)$. Then simplify.

**Example 2:** Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find
(a) $(f \circ g)(x)$ and (b) $(g \circ f)(x)$.
(a) $6x^2 + 1$ (b) $18x^2 + 48x + 31$

To “decompose” a composite function, . . . look for an “inner” function and an “outer” function.

**Example 3:** Write the function given by $h(x) = |2x - 5| + 1$ as a composition of two functions.

Answers may vary. One possibility is to let $g(x) = 2x - 5$ and let $f(x) = |x| + 1$. Then $h(x) = f(g(x))$.

### III. Applications of Combinations of Functions  (Page 88)

The function $f(x) = 0.06x$ represents the sales tax owed on a purchase with a price tag of $x$ dollars and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of $x$ dollars during a 25% off sale. Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of $x$ dollars during a 25% off sale.

$$(f \circ g)(x) = (g \circ f)(x) = 0.045x$$