Chapter 8 Matrices and Determinants

Course/Section Lesson Number Date

Section 8.3 The Inverse of a Square Matrix

Section Objectives: Students will know how to find the inverses of matrices and use inverse matrices to solve systems of linear equations.

I. The Inverse of a Matrix (p. 602) Pace: 5 minutes

State that we want to solve systems of equations by expressing them as matrix equations, and then solving as if we were solving $ax = b$--i.e., multiplying both sides by the inverse of a.

Define the inverse of an n n matrix A, *if it exists*, to be the n n matrix A^{-1} such that $AA^{-1} = A^{-1}A = I_n$.

Example 1. Show that B is the inverse of A, where

$$A \quad \begin{matrix} 5 & 6 \\ 1 & 1 \end{matrix} \quad \text{and } B = \begin{matrix} 1 & 6 \\ 1 & 5 \end{matrix}.$$

$$AB \quad \begin{matrix} 5 & 6 \\ 1 & 1 \end{matrix} \begin{matrix} 1 & 6 \\ 1 & 5 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$BA \quad \begin{matrix} 1 & 6 \\ 1 & 5 \end{matrix} \begin{matrix} 5 & 6 \\ 1 & 1 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

II. Finding Inverse Matrices (pp. 603–605) Pace: 15 minutes

To dispel any thoughts of magic, show students that the process we will state shortly comes from the following.

Example 2. Find the inverse of $A \quad \begin{matrix} 4 & 5 \\ 7 & 9 \end{matrix}$.

We need to find the matrix $B \quad \begin{matrix} a & b \\ c & d \end{matrix}$ such that $AB = I_2$. By multiplying, we see that we need to solve the following two systems of equations.

$$\begin{matrix} 4a & 5c & 1 \\ 7a & 9c & 0 \end{matrix} \qquad \begin{matrix} 4 & 5 & 1 \\ 7 & 9 & 0 \end{matrix}$$

$$\begin{matrix} 4b & 5d & 0 \\ 7b & 9d & 1 \end{matrix} \qquad \begin{matrix} 4 & 5 & 0 \\ 7 & 9 & 1 \end{matrix}$$

$$\left.\begin{matrix} 4 & 5 \\ 7 & 9 \end{matrix}\right| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \qquad \left.\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\right| \begin{matrix} 9 & 5 \\ 7 & 4 \end{matrix}$$

$$B \quad \begin{matrix} 9 & 5 \\ 7 & 4 \end{matrix}$$

State the following **steps for finding an inverse matrix.**

Let A be a square matrix of order n.

1. Form the n $2n$ matrix $[A \vdots I_n]$.
2. Transform this matrix into reduced row-echelon form.
3. If this new matrix is of the form $[I_n \vdots B]$, then A is invertible and $B = A^{-1}$.

Example 3. Find the inverse of

$$A \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & 14 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 3 & 7 & 14 & 0 & 1 & 0 \\ 1 & 4 & 3 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 & 1 & 0 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 7 & 2 & 0 \\ 0 & 1 & 1 & 3 & 1 & 0 \\ 0 & 0 & 8 & 19 & 6 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 7 & 2 & 0 \\ 0 & 1 & 1 & 3 & 1 & 0 \\ 0 & 0 & 1 & 19/8 & 3/4 & 1/8 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 77/8 & 13/4 & 7/8 \\ 0 & 1 & 1 & 5/8 & 1/4 & 1/8 \\ 0 & 0 & 1 & 19/8 & 3/4 & 1/8 \end{array}\right]$$

$$A^{1} \frac{1}{8}\begin{bmatrix} 77 & 26 & 7 \\ 5 & 2 & 1 \\ 19 & 6 & 1 \end{bmatrix}$$

III. The Inverse of a 2 2 Matrix (p. 606) Pace: 5 minutes

State the following **formula for finding the inverse of a 2 2 matrix.**

If $A \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad \quad bc \quad 0,$ then

$$A^{1} \frac{1}{ad \quad bc}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

Example 4. Find the inverse of

$$A \begin{bmatrix} 3 & 2 \\ 9 & 5 \end{bmatrix}$$

$$A^{1} \frac{1}{3}\begin{bmatrix} 5 & 2 \\ 9 & 3 \end{bmatrix}$$

IV. Systems of Linear Equations (p. 607) Pace: 5 minutes

State that if A is invertible, then the system of equations represented by $AX = B$ has a unique solution $X = A^{-1}B$.

Example 5. Solve the following system of equations.

$$\begin{aligned} 2x \quad 5y \quad 8 \\ 3x \quad 8y \quad 1 \end{aligned}$$

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}^{1}\begin{bmatrix} 8 \\ 1 \end{bmatrix} \begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \end{bmatrix} \begin{bmatrix} 59 \\ 22 \end{bmatrix}$$

Discuss the *Technology* on page 607 of the text.