Chapter 3 Exponential and Logarithmic Functions

Course/Section Lesson Number Date

Section 3.4 Exponential and Logarithmic Equations

Section Objectives: Students will know how to solve exponential and logarithmic equations.

I. Introduction (p. 246) Pace: 5 minutes

Review the following one-to-one and inverse properties, which will be key in solving exponential and logarithmic equations.

One-to-One Properties

1. $a^x = a^y$ if and only if $x = y$.
2. $\log_a x = \log_a y$ if and only if $x = y$.

Inverse Properties

1. $\log_a a^x = x$
2. $a^{\log_a x} \quad x$

II. Solving Exponential Equations (pp. 247−248) Pace: 15 minutes

State that two very general keys to solving exponential equations are:

1. Isolate the exponential expression.
2. Use the second one-to-one property from above.

Example 1. Solve each equation and round your answer to three decimal places.

a)

$$4e^{2x} \quad 16 \qquad e^{2x} \quad 4 \qquad \ln e^{2x} \quad \ln 4 \qquad 2x \quad \ln 4$$

$$x \quad \frac{\ln 4}{2} \quad 0.693$$

b)

$$5e^{x \quad 2} \quad 8 \quad 14 \qquad 5e^{x \quad 2} \quad 22 \qquad e^{x \quad 2} \quad \frac{22}{5} \qquad \ln e^{x \quad 2} \quad \ln\frac{22}{5}$$

$$x \quad 2 \quad \ln\frac{22}{5} \qquad x \quad \ln\frac{22}{5} \quad 2 \quad 0.518$$

c)

$$2 \; 3^x \quad 1 \quad 10 \qquad 3^x \quad 1 \quad 5 \qquad 3^x \quad 4 \qquad \ln 3^x \quad \ln 4 \qquad x\ln 3 \quad \ln 4$$

$$x \quad \frac{\ln 4}{\ln 3} \quad 1.262$$

d)

$$e^{2x} \quad e^x \quad 20 \quad 0 \qquad e^x \quad 4 \;\; e^x \quad 5 \quad 0$$

$$e^x \quad 4 \quad 0 \qquad e^x \quad 4 \text{ or } e^x \quad 5 \quad 0 \qquad e^x \quad 5 \qquad x \quad \ln 5 \quad 1.609$$

III. Solving Logarithmic Equations (pp. 249−250) Pace: 15 minutes

State that there are two basic ways of solving logarithmic equations.

1. Isolate the logarithmic expression and then write the equation in equivalent exponential form.
2. Get a single logarithmic expression with the same base on each side of the equation; then use the one-to-one property.

Example 2. Solve the following logarithmic equations and round your answers to three decimal places.

a) $2\log x \quad 5 \qquad \log x \quad \frac{5}{2} \qquad x \quad 10^{5/2} \quad 316.228$

b)

$$\ln\sqrt{x \quad 2} \quad \ln x \qquad \sqrt{x \quad 2} \quad x \qquad x \quad 2 \quad x^2 \qquad x^2 \quad x \quad 2 \quad 0$$

$$x \quad 2 \;\; x \quad 1 \quad 0$$

$$x \quad 2 \quad 0 \qquad x \quad 2$$

$$x \quad 1 \quad 0 \qquad x \qquad 1$$

−1 cannot be a solution because of the domain of the logarithmic function.

c)

$$\log x \quad \log x \quad 3 \quad 1 \qquad \log\frac{x}{x \quad 3} \quad 1 \qquad \frac{x}{x \quad 3} \quad 10^1$$

$$x \quad 10x \quad 30 \qquad 9x \qquad 30 \qquad x \quad \frac{10}{3}$$

IV. Applications (pp. 251−252) Pace: 10 minutes

Example 3. How long would it take for an investment to double if the interest were compounded continuously at 8%?

$$2P \quad Pe^{0.08t}$$

$$2 \quad e^{0.08t}$$

$$\ln 2 \quad \ln e^{0.08t}$$

$$\ln 2 \quad 0.08t$$

$$t \quad \frac{\ln 2}{0.08} \quad 8.66 \text{ years}$$

Example 4. You have \$50,000 to invest. You need to have \$350,000 to retire in thirty years. At what continuously compounded interest rate would you need to invest to reach your goal?

$$350,000 \quad 50,000e^{r\ 30}$$

$$7 \quad e^{30r}$$

$$\ln 7 \quad \ln e^{30r}$$

$$\ln 7 \quad 30r$$

$$r \quad \frac{\ln 7}{30} \quad 6.5\%$$