Chapter 3 Exponential and Logarithmic Functions

Course/Section
Lesson Number
Date

Section 3.3 Properties of Logarithms

Section Objectives: Students will know how to rewrite logarithmic functions with a different base, use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

I. Change of Base (p. 239) Pace: 5 minutes

1. Note that our calculators have only two buttons for logarithmic functions, base 10 and base *e*. State the **Change-of-Base Formula**.
   Let *a*, *b*, and *x* be positive real numbers such that $a \quad 1$ and $b \quad 1$. Then

$$\log_a x \quad \frac{\log_b x}{\log_b a} \quad \frac{\log x}{\log a} \quad \frac{\ln x}{\ln a}.$$

Example 1. Evaluate the following.

a) $\log_5 18 \quad \frac{\ln 18}{\ln 5} \quad 1.7959$

b) $\log_2 42 \quad \frac{\ln 42}{\ln 2} \quad 5.3923$

II. Properties of Logarithms (p. 240) Pace: 10 minutes

State that logarithms are exponents, since $y = \log_a x$ if and only if $x = a^y$. Thus the following **properties of logarithms** are similar to the exponent properties.

Let a be a positive real number such that $a \quad 1$, let n be a real number, and let u and v be positive real numbers. Then

1. $\log_a(uv) = \log_a u + \log_a v$
2. $\log_a(u/v) = \log_a u - \log_a v$
3. $\log_a u^n = n\log_a u$

Discuss the *Historical Note* on page 240 of the text.

III. Rewriting Logarithmic Expression (p. 241) Pace: 10 minutes

Example 2. Expand the logarithmic expression.

a)

$$\log \; 2x^3y^4 \quad \log 2 \quad \log x^3 \quad \log y^4$$
$$\log 2 \quad 3\log x \quad 4\log y$$

b)

$$\ln\frac{\sqrt{x \quad 5}}{y^2} \quad \ln\sqrt{x \quad 5} \quad \ln y^2$$
$$\frac{1}{2}\ln \; x \quad 5 \quad 2\ln y$$

Example 3. Condense the logarithmic expression.

$$2\log x - 3\log y + \frac{1}{2}\log z = \log x^2 - \log y^3 + \log\sqrt{z}$$

a)

$$= \log\frac{x^2}{y^3} + \log\sqrt{z}$$

$$= \log\frac{x^2\sqrt{z}}{y^3}$$

b)

$$\frac{1}{3}\ \ 2\ln x \quad 4\ln y \quad \ln(z \quad 2) \qquad \frac{1}{3}\ \ \ln x^2 \quad \ln y^4 \quad \ln\ z \quad 2$$

$$\frac{1}{3}\ \ \ln\frac{x^2}{y^4} \quad \ln\ z \quad 2$$

$$\frac{1}{3}\ \ \ln\frac{x^2}{y^4\ \ z \quad 2}$$

$$\ln\sqrt[3]{\frac{x^2}{y^4\ \ z \quad 2}}$$

IV. Application (p. 242) Pace: 5 minutes

Example 4. On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R \quad \frac{\ln I \quad \ln I_0}{\ln 10}$$

where I_0 is the minimum intensity used for comparison. Write this as a single common logarithmic expression.

$$R \quad \frac{\ln I \quad \ln I_0}{\ln 10}$$

$$\frac{\frac{\log I}{\log e} \quad \frac{\log I_0}{\log e}}{\frac{\log 10}{\log e}}$$

$$\frac{\log I \quad \log I_0}{1}$$

$$\log\frac{I}{I_0}$$