

## Section 8.5 Applications of Matrices and Determinants

**Objective:** In this lesson you learned how to use Cramer's Rule to solve systems of linear equations and how to use determinants and matrices to model and solve problems.

Course Number

Instructor

Date

### I. Cramer's Rule (Pages 619–621)

**Cramer's Rule** states that if a system of  $n$  linear equations in  $n$  variables has a coefficient matrix  $A$  with a nonzero determinant  $|A|$ , the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \dots, x_n = \frac{|A_n|}{|A|}$$

where the  $i$ th column of  $A_i$  is \_\_\_\_\_

\_\_\_\_\_.

Cramer's Rule does not apply if the determinant of the coefficient matrix is \_\_\_\_\_, in which case the system has either no solution or \_\_\_\_\_.

**Example 1:** Use Cramer's Rule to solve the system of linear equations.

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

#### *What you should learn*

How to use Cramer's Rule to solve systems of linear equations

### II. Area of a Triangle (Page 622)

The area of a triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol  $\pm$  indicates that the appropriate sign should be chosen to yield a positive area.

**Example 2:** Find the area of a triangle whose vertices are  $(-3, 1)$ ,  $(2, 4)$ , and  $(5, -3)$ .

#### *What you should learn*

How to use determinants to find the areas of triangles

**III. Lines in a Plane** (Pages 623–624)

Three points  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  are collinear (lie on the same line) if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

**Example 3:** Determine whether the points  $(-2, 4)$ ,  $(0, 3)$ , and  $(8, -1)$  are collinear.

An equation of the line passing through the distinct points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

**Example 4:** Find an equation of the line passing through the points  $(-2, 9)$  and  $(3, -1)$ .

***What you should learn***  
How to use a determinant to test for collinear points and find an equation of a line passing through two points

**IV. Cryptography** (Pages 625–627)

A cryptogram is . . .

***What you should learn***  
How to use matrices to encode and decode messages

To use matrix multiplication to encode and decode messages, . . .

**Homework Assignment**

Page(s)

Exercises