

# Chapter 12 Limits and an Introduction to Calculus

Course Number

Instructor

Date

## Section 12.1 Introduction to Limits

**Objective:** In this lesson you learned how to estimate limits and use properties and operations of limits.

### I. The Limit Concept and Definition of Limit (Pages 852–854)

Define **limit**.

***What you should learn***  
How to use the definition of a limit to estimate limits

Describe how to estimate the limit  $\lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{x + 2}$  numerically.

The existence or nonexistence of  $f(x)$  when  $x = c$  has no bearing on the existence of . . .

### II. Limits That Fail to Exist (Pages 855–856)

The limit of  $f(x)$  as  $x \rightarrow c$  does not exist if any of the following conditions is true:

- 1.
- 2.
- 3.

***What you should learn***  
How to determine whether limits of functions exist

Give an example of a limit that does not exist.

**III. Properties of Limits and Direct Substitution**

(Pages 857–859)

Let  $b$  and  $c$  be real numbers and let  $n$  be a positive integer. Complete each of the following properties of limits.

1.  $\lim_{x \rightarrow c} b = \underline{\hspace{2cm}}$

2.  $\lim_{x \rightarrow c} x = \underline{\hspace{2cm}}$

3.  $\lim_{x \rightarrow c} x^n = \underline{\hspace{2cm}}$

4.  $\lim_{x \rightarrow c} \sqrt[n]{x} = \underline{\hspace{2cm}}$

Let  $b$  and  $c$  be real numbers, let  $n$  be a positive integer, and let  $f$  and  $g$  be functions with the following limits.

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = K$$

Complete each of the following statements about operations with limits.

1. Scalar multiple:  $\lim_{x \rightarrow c} [b f(x)] = \underline{\hspace{2cm}}$

2. Sum or difference:  $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \underline{\hspace{2cm}}$

3. Product:  $\lim_{x \rightarrow c} [f(x) \cdot g(x)] = \underline{\hspace{2cm}}$

4. Quotient:  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \underline{\hspace{2cm}}$

5. Power:  $\lim_{x \rightarrow c} [f(x)]^n = \underline{\hspace{2cm}}$

**Example 1:** Find the limit:  $\lim_{x \rightarrow 4} 3x^2$ .

***What you should learn***

How to use properties of limits and direct substitution to evaluate limits

If  $p$  is a polynomial function and  $c$  is a real number, then

$$\lim_{x \rightarrow c} p(x) = \underline{\hspace{2cm}}.$$

If  $r$  is a rational function given by  $r(x) = p(x)/q(x)$ , and  $c$  is a real number such that  $q(c) \neq 0$ , then

$$\lim_{x \rightarrow c} r(x) = \underline{\hspace{2cm}}.$$

**Example 2:** Find the limit:  $\lim_{x \rightarrow 2} \frac{4 - x^2}{x}$ .

**Additional notes**

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**Homework Assignment**

Page(s)

Exercises