

Chapter 3 Polynomial and Rational Functions

Section 3.1

Polynomial function with degree n – Let n be a nonnegative integer and let $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ is called a polynomial function of x with degree n

Constant function – A polynomial function with degree 0. That is, $f(x) = a, a \neq 0$.

Linear function – A polynomial function with degree 1. That is, $f(x) = mx + b, m \neq 0$.

Quadratic function – Let a, b , and c be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ is called a quadratic function.

Parabola – Name for a special type of “U” shaped curve that is the graph of a quadratic function

Axis of symmetry – A line about which a parabola is symmetric

Vertex – The point where the axis intersects the parabola

Standard form of a quadratic function – The standard form of a quadratic function is $f(x) = a(x - h)^2 + k, a \neq 0$.

Section 3.2

Continuous – The graph of a polynomial function has no breaks, holes, or gaps

Leading Coefficient Test – As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ eventually rises or falls in the following manner:

1. When n is odd:
 - a. If the leading coefficient is positive, the graph falls to the left and rises to the right
 - b. If the leading coefficient is negative, the graph rises to the left and falls to the right
2. When n is even:
 - a. If the leading coefficient is positive, the graph rises to the left and right
 - b. If the leading coefficient is negative, the graph falls to the left and right

Extrema – The minimums and maximums of a function

Relative minimum – The least value of a function on an interval

Relative maximum – The greatest value of a function on an interval

Repeated zero – If $(x - a)^k$ is a factor of a polynomial, then $x = a$ is a repeated zero

Multiplicity – The number of times a zero is repeated

Intermediate Value Theorem – Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$

Section 3.3

Long division of polynomials – A procedure for dividing two polynomials similar to long division in arithmetic

Division Algorithm – If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = d(x)q(x) + r(x)$ where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$.

Synthetic division – A shortcut for long division of polynomials when dividing by divisors of the form $x - k$

Remainder Theorem – If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$

Factor Theorem – A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$

Section 3.4

Rational Zero Test – If the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ has integer coefficients, every rational zero of f has the form

$$\text{Rational zero} = \frac{p}{q}$$

where p and q have no common factors other than 1, and p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n

Upper bound – A real number b is an upper bound for the real zeros of f if no real zeros of f are greater than b

Lower bound – A real number b is a lower bound for the real zeros of f if no real zeros of f are less than b

Section 3.4

Fundamental Theorem of Algebra – If $f(x)$ is a polynomial of degree n , where $n > 0$, f has at least one zero in the complex number system

Linear Factorization Theorem – If $f(x)$ is a polynomial of degree n , where $n > 0$, f has precisely n linear factors $f(x) = a_n(x - c_1)(x - c_2) \dots (x - c_n)$ where c_1, c_2, \dots, c_n are complex numbers

Conjugates – A pair of complex numbers of the form $a + bi$ and $a - bi$

Section 3.5

Rational function – A function that can be written in the form: $f(x) = \frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial

Vertical asymptote – The line $x = a$ is a vertical asymptote of the graph of f if $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ as $x \rightarrow a$, either from the right or from the left.

Horizontal asymptote – The line $y = b$ is a horizontal asymptote of the graph of f if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or $x \rightarrow -\infty$

Section 3.6

Slant (or oblique) asymptote – If the degree of the numerator of a rational function is exactly one more than the degree of the denominator, then the line determined by the quotient of the denominator into the numerator is a slant asymptote of the graph of the rational function