

## Section 2.6 Rational Functions

**Objective:** In this lesson you learned how to determine the domains of rational functions, find asymptotes of rational functions, and sketch the graphs of rational functions.

Course Number

Instructor

Date

### Important Vocabulary

Define each term or concept.

**Rational function**

**Vertical asymptote**

**Horizontal asymptote**

**Slant (or oblique) asymptote**

### I. Introduction (Page 184)

The domain of a rational function of  $x$  includes all real numbers except . . .

To find the domain of a rational function of  $x$ , . . .

#### *What you should learn*

How to find the domains of rational functions

**Example 1:** Find the domain of the function  $f(x) = \frac{1}{x^2 - 9}$ .

### II. Horizontal and Vertical Asymptotes (Pages 185–186)

The notation “ $f(x) \rightarrow 5$  as  $x \rightarrow \infty$ ” means . . .

Describe the end behavior of a rational function in relation to its horizontal asymptote.

#### *What you should learn*

How to find the horizontal and vertical asymptotes of graphs of rational functions

Let  $f$  be the rational function given by

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$$

where  $N(x)$  and  $D(x)$  have no common factors.

- 1) The graph of  $f$  has vertical asymptotes at \_\_\_\_\_  
\_\_\_\_\_.
- 2) The graph of  $f$  has one or no horizontal asymptote determined by \_\_\_\_\_  
\_\_\_\_\_.
  - a) If  $n < m$ , the graph of  $f$  has \_\_\_\_\_  
\_\_\_\_\_.
  - b) If  $n = m$ , the graph of  $f$  has \_\_\_\_\_  
\_\_\_\_\_.
  - c) If  $n > m$ , the graph of  $f$  has \_\_\_\_\_  
\_\_\_\_\_.

**Example 2:** Find the asymptotes of the function

$$f(x) = \frac{2x - 1}{x^2 - x - 6}$$

### III. Analyzing Graphs of Rational Functions

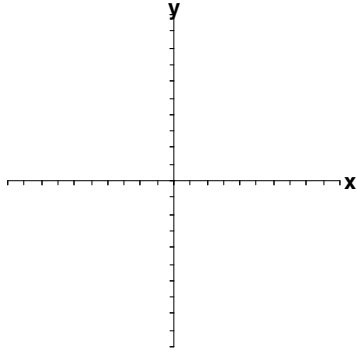
(Pages 187–189)

To sketch the graph of the rational function  $f(x) = N(x)/D(x)$ ,

where  $N(x)$  and  $D(x)$  are polynomials with no common factors, . . .

***What you should learn***  
How to analyze and sketch graphs of rational functions

**Example 3:** Sketch the graph of  $f(x) = \frac{3x}{x+4}$ .



#### IV. Slant Asymptotes (Page 190)

To find the equation of a slant asymptote, . . .

***What you should learn***  
How to sketch graphs of rational functions that have slant asymptotes

**Example 4:** Decide whether each of the following rational functions has a slant asymptote. If so, find the equation of the slant asymptote.

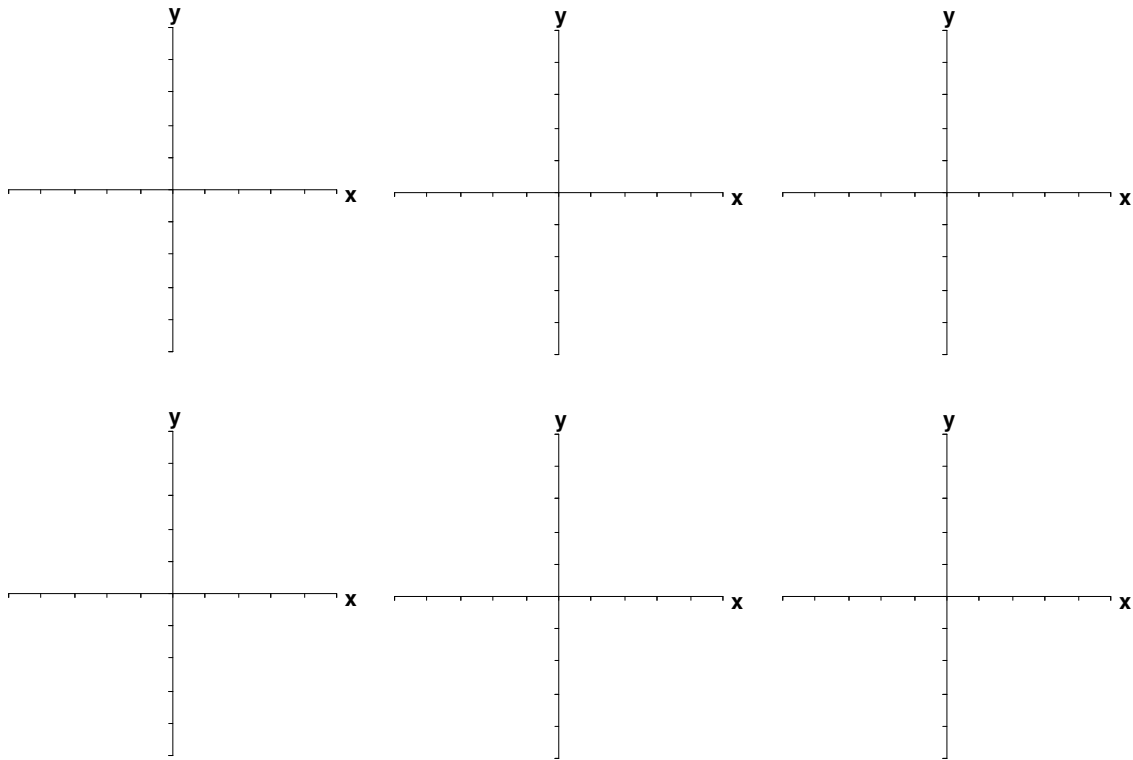
$$(a) f(x) = \frac{x^3 - 1}{x^2 + 3x + 5} \quad (b) f(x) = \frac{3x^3 + 2}{2x - 5}$$

#### V. Applications of Rational Functions (Pages 191–192)

Give an example of asymptotic behavior that occurs in real life.

***What you should learn***  
How to use rational functions to model and solve real-life problems

Additional notes



**Homework Assignment**

Page(s)

Exercises