## Part III: Texas Instruments TI-86 Graphics Calculator

### III.1 Getting started with the TI-86

***III.1.1 Basics:*** Press the ON key to begin using your TI-86 calculator. If you need to adjust the display contrast, first press 2nd, then press and hold ▼ (the *down* arrow key) to lighten or ▲ (the *up* arrow key) to darken. As you press and hold ▼ or ▲, an integer between 0 (lightest) and 9 (darkest) appears in the upper right corner of the display. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF.

Check the TI-86's settings by pressing 2nd MODE. If necessary, use the arrow keys to move the blinking cursor to a setting you want to change. Press ENTER to select a new setting. To start, select the options along the left side of the MODE menu as illustrated in Figure III.1: normal display, floating decimals, radian measure, rectangular coordinates, function graphs, decimal number system, rectangular vectors, and differentiation type. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing EXIT or 2nd QUIT or CLEAR.

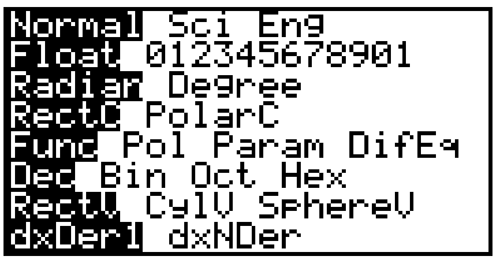


Figure III.1: MODE menu

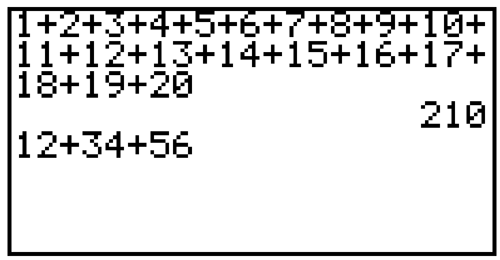


Figure III.2: Home screen

***III.1.2 Editing:*** One advantage of the TI-86 is that up to 8 lines are visible at one time, so you can *see* a long calculation. For example, type this sum (see Figure III.2):

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20

Then press ENTER to see the answer.

Often we do not notice a mistake until we see how unreasonable an answer is. The TI-86 permits you to redisplay an entire calculation, edit it easily, then execute the *corrected* calculation.

Suppose you had typed 12 + 34 + 56 as in Figure III.2 but had *not yet* pressed ENTER, when you realize that 34 should have been 74. Simply press ◄ (the *left* arrow key) as many times as necessary to move the blinking cursor left to 3, then type 7 to write over it. On the other hand, if 34 should have been 384, move the cursor back to 4, press 2nd INS (the cursor changes to a blinking underline) and then type 8 (inserts at the cursor position and the other characters are pushed to the right). If the 34 should have been 3 only, move the cursor to 4, and press DEL to delete it.

***Technology Tip:*** To move quickly to the *beginning* of an expression you are currently editing, press ▲ (the *up* arrow key); to jump to the *end* of that expression, press ▼ (the *down* arrow key).

Even if you had pressed ENTER, you may still edit the previous expression. Press 2nd and then ENTRY to *recall* the last expression that was entered. Now you can change it. In fact, the TI-86 retains many prior entries in a "last entry" storage area. Press 2nd ENTRY repeatedly until the previous line you want replaces the current line.

  

***Technology Tip:*** When you need to evaluate a formula for different values of a variable, use the editing feature to simplify the process. For example, suppose you want to find the balance in an investment account if there is now $5000 in the account and interest is compounded annually at the rate of 8.5%. The formula for the balance is $P\left(1 + \frac{r}{n}\right)^{nt}$, where $P$ = principal, $r$ = rate of interest (expressed as a decimal), $n$ = number of times interest is compounded each year, and $t$ = number of years. In our example, this becomes $5000(1 + .085)^t$. Here are the keystrokes for finding the balance after $t = 3$, 5, and 10 years (results are shown in Figure III.3).

| *Years* | *Keystrokes* | *Balance* |
|---|---|---|
| 3 | 5000 (1 + .085) ^ 3 ENTER | $6386.45 |
| 5 | 2nd ENTRY ◀ 5 ENTER | $7518.28 |
| 10 | 2nd ENTRY ◀ 10 ENTER | $11,304.92 |

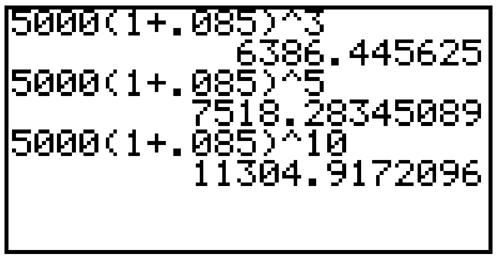


Figure III.3: Editing expressions

Then to find the balance from the same initial investment but after 5 years when the annual interest rate is 7.5%, press these keys to change the last calculation above: 2nd ENTRY ◀ DEL ◀ 5 ◀ ◀ ◀ ◀ ◀ 7 ENTER.

***III.1.3 Key Functions:*** Most keys on the TI-86 offer access to more than one function, just as the keys on a computer keyboard can produce more than one letter ("g" and "G") or even quite different characters ("5" and "%"). The primary function of a key is indicated on the key itself, and you access that function by a simple press on the key.

To access the *second* function to the *left* above a key, first press 2nd (the cursor changes to a blinking ↑) and *then* press the key. For example, to calculate $\sqrt{25}$, press 2nd √ 25 ENTER.

When you want to use a capital letter or other characters printed to the *right* above a key, first press ALPHA (the cursor changes to a blinking **A**) and then the key. For example, to use the letter K in a formula, press ALPHA K. If you need several letters in a row, press ALPHA twice in succession, which is like the **CAPS LOCK** key on a computer keyboard, and then press all the letters you want. Remember to press ALPHA when you are finished and want to restore the keys to their primary functions. To type lowercase letters, press 2nd alpha (the cursor changes to a blinking a). To lock in lowercase letters, press 2nd alpha 2nd alpha or 2nd alpha ALPHA. To unlock from lowercase, press ALPHA ALPHA (you'll see the cursor change from blinking a to blinking A and then to standard blinking rectangle).

***III.1.4 Order of Operations:*** The TI-86 performs calculations according to the standard algebraic rules. Working outwards from inner parentheses, calculations are performed from left to right. Powers and roots are evaluated first, followed by multiplications and divisions, and then additions and subtractions.

Note that the TI-86 distinguishes between *subtraction* and the *negative sign*. If you wish to enter a negative number, it is necessary to use (-) key. For example, you would evaluate $-5 - (4 \cdot -3)$ by pressing (-) 5 – (4 × (-) 3 ) ENTER to get 7.

  

Enter these expressions to practice using your TI-86.

| *Expression* | *Keystrokes* | *Display* |
|---|---|---|
| $7 - 5 \cdot 3$ | 7 – 5 × 3 ENTER | -8 |
| $(7 - 5) \cdot 3$ | (7 – 5) × 3 ENTER | 6 |
| $120 - 10^2$ | 120 – 10 $x^2$ ENTER | 20 |
| $(120 - 10)^2$ | (120 – 10) $x^2$ ENTER | 12100 |
| $\frac{24}{2^3}$ | 24 ÷ 2 ^ 3 ENTER | 3 |
| $\left(\frac{24}{2}\right)^3$ | (24 ÷ 2) ^ 3 ENTER | 1728 |
| $(7 - -5) \cdot -3$ | (7 – (-) 5 ) × (-) 3 ENTER | -36 |

***III.1.5 Algebraic Expressions and Memory:*** Your calculator can evaluate expressions such as $\frac{N(N+1)}{2}$ *after* you have entered a value for $N$. Suppose you want $N = 200$. Press 200 STO ► N ENTER to store the value 200 in memory location $N$. (The STO ► key prepares the TI-86 for alphabetical entry, so it is not necessary to press ALPHA also.) Whenever you use $N$ in an expression, the calculator will substitute the value 200 until you make a change by storing *another* number in $N$. Next enter the expression $\frac{N(N+1)}{2}$ by typing ALPHA N ( ALPHA N + 1 ) ÷ 2 ENTER. For $N = 200$, you will find that $\frac{N(N+1)}{2} = 20{,}100$.

The contents of any memory location may be revealed by typing just its letter name and then ENTER. And the TI-86 retains memorized values even when it is turned off, so long as its batteries are good.

A variable name in the TI-86 can be a single letter, or a string of up to eight characters that begins with a letter followed by other letters, numerals, and various symbols. Variable names are case sensitive, which means that length and Length and LENGTH may represent *different* quantities.

***III.1.6 Repeated Operations with ANS:*** The result of your *last* calculation is always stored in memory location ANS and replaces any previous result. This makes it easy to use the answer from one computation in another computation. For example, press 30 + 15 ENTER so that 45 is the last result displayed. Then press 2nd ANS ÷ 9 ENTER and get 5 because $45 \div 9 = 5$.

With a function like division, you press the ÷ key *after* you enter an argument. For such functions, whenever you would start a new calculation with the previous answer followed by pressing the function key, you may press just the function key. So instead of 2nd ANS ÷ 9 in the previous example, you could have pressed simply ÷ 9 to achieve the same result. This technique also works for these functions: + – × $x^2$ ^ $x^{-1}$.

Here is a situation where this is especially useful. Suppose a person makes $5.85 per hour and you are asked to calculate earnings for a day, a week, and a year. Execute the given keystrokes to find the person's incomes during these periods (results are shown in Figure III.4.):

| *Pay period* | *Keystrokes* | *Balance* |
|---|---|---|
| 8-hour day | 5.85 × 8 ENTER | $46.80 |
| 5-day week | × 5 ENTER | $234 |
| 52-week year | × 52 ENTER | $12,168 |

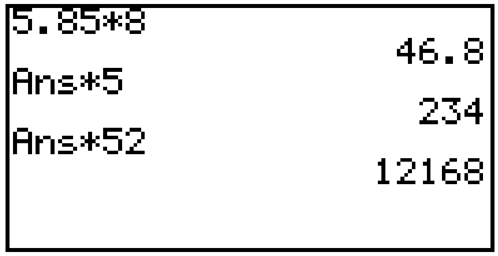


Figure III.4: ANS variable

***III.1.7 The MATH Menu:*** Operators and functions associated with a scientific calculator are available either immediately from the keys of the TI-86 or by the 2nd keys. You have direct key access to common arithmetic operations ($x^2$, 2nd $\sqrt{\ }$, 2nd $x^{-1}$, ^), trigonometric functions (SIN, COS, TAN) and their inverses ( 2nd $\text{SIN}^{-1}$, 2nd $\text{COS}^{-1}$, 2nd $\text{TAN}^{-1}$ ), exponential and logarithmic functions (LOG, 2nd $10^x$, LN, 2nd $e^x$), and a famous constant (2nd $\pi$).

A significant difference between the TI-86 graphing calculators and most scientific calculators is that TI-86 requires the argument of a function *after* the function, as you would see a formula written in your textbook. For example, on the TI-86 you calculate $\sqrt{16}$ by pressing the keys 2nd $\sqrt{\ }$ 16 in that order.

Here are keystrokes for basic mathematical operations. Try them for practice on your TI-86.

| *Expression* | *Keystrokes* | *Display* |
|---|---|---|
| $\sqrt{3^2 + 4^2}$ | 2nd $\sqrt{\ }$ ( 3 $x^2$ + 4 $x^2$) ENTER | 5 |
| $2\frac{1}{3}$ | 2 + 3 2nd $x^{-1}$ ENTER | 2.33333333333 |
| log 200 | LOG 200 ENTER | 2.30102999566 |
| $2.34 \cdot 10^5$ | 2.34 × 2nd $10^x$ 5 ENTER | 234000 |

Additional mathematical operations and functions are available from the MATH menu. Press 2nd MATH to see the various options that are listed across the bottom of the screen (Figure III.5). These options are activated by pressing corresponding menu keys, F1 through F5.


Figure III.5: Basic MATH menu

For example, pressing F1 brings up the NUM menu of numerical functions. Note that the basic MATH menu items have moved up a line; these options are now available by pressing 2nd M1 through 2nd M5. As an example, determine $|-5|$ by pressing 2nd MATH F1 *[NUM]* and then F5 *[abs]* (-) 5 ENTER (see Figure III.6).

  

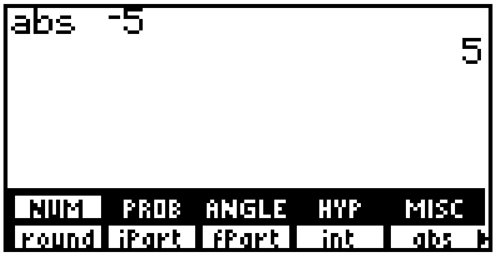


Figure III.6: MATH NUM menu

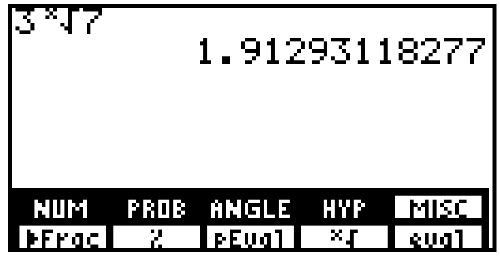


Figure III.7: MATH MISC menu

Next calculate $\sqrt[3]{7}$ by first pressing 2nd MATH F5 (when the MATH NUM menu is displayed, as in Figure III.6, just press 2nd M5) to access the MISC menu of miscellaneous mathematical functions. The arrow at the right end of this menu indicates there are more items that you can access. You may press the MORE key repeatedly to move down the row of options and back again. To calculate $\sqrt[3]{7}$, press 2nd MATH F5 MORE 3 F4 *[ $\sqrt[x]{\ }$ ]* 7 ENTER; this will result in 1.91293118277 (Figure III.7). To leave the MATH menu and take no other action, press EXIT twice.

The *factorial* of a nonnegative integer is the *product* of *all* the integers from 1 up to the given integer. The symbol for factorial is the exclamation point. So 4! (pronounced *four factorial*) is $1 \cdot 2 \cdot 3 \cdot 4 = 24$. You will learn more about applications of factorials in your textbook, but for now use the TI-86 to calculate 4! The factorial command is located in the MATH menu's PROB sub-menu. To compute 4!, press these keystrokes: 2nd MATH F2 *[PROB]* 4 F1 *[!]* ENTER.

On the TI-86 it is possible to do calculations with complex numbers. The complex number $a + bi$ is represented as an ordered pair $(a, b)$. Using ordered pair notation to divide $2 + 3i$ by $4 - 2i$, press ( 2 , 3 ) ÷ ( 4 , (-) 2 ) ENTER. The result is displayed as $(.1, .8)$ for $.1 + .8i$ (Figure III.8).

To find the complex conjugate of $4 + 5i$ press 2nd CPLX F1 *[conj]* ( 4 , 5 ) ENTER (Figure III.8).

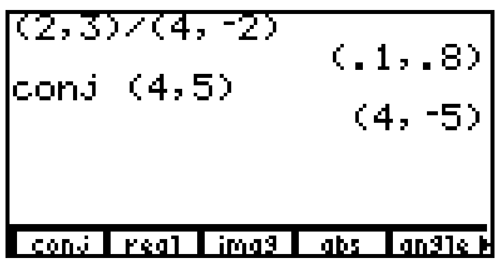


Figure III.8: Complex number calculations

## III.2 Functions and Graphs

***III.2.1 Evaluating Functions:*** Suppose you received a monthly salary of $1975 plus a commission of 10% of sales. Let $x$ = your sales in dollars; then your wages $W$ in dollars are given by the equation $W = 1975 + .10x$. If your January sales were $2230 and your February sales were $1865, what was your income during those months?

Here's one method to use your TI-86 to perform this task. Press the GRAPH key and then F1 to select y(x)= to get access to the function editing screen (Figure III.9). You may enter as many as 99 different functions for the TI-86 to use at one time (if sufficient memory is available). Press F4 *[DELf]* as many times as necessary to delete any functions that may be there already. Then with the cursor on the top line to the right of y1= enter the expression $1975 + .10x$ by pressing these keys: 1975 + .10 F1 *[x]*. As you see, the

  

TI-86 uses lowercase letters for its graphing variables, just like your mathematics textbook. Note that pressing F1 in this situation is the same as pressing either x-VAR or 2nd alpha X. The x-VAR key lets you enter the variable x easily without having to use the ALPHA key. Now press 2nd QUIT to return to the main calculations screen.

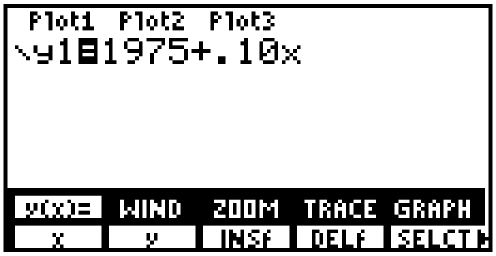


Figure III.9: y(x)= screen

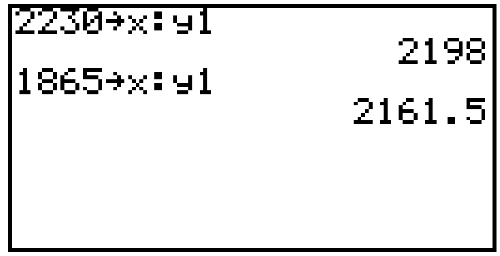


Figure III.10: Evaluating a function

Assign the value 2230 to the variable *x* by using these keystrokes (see Figure III.10): 2230 STO ► x-VAR. Then press 2nd : to allow another expression to be entered on the same command line. Next press the following keystrokes to evaluate y1 and find January's wages: 2nd alpha Y ALPHA ALPHA 1 ENTER. It is not necessary to repeat all these steps to find the February wages. Simply press 2nd ENTRY to recall the entire previous line, change 2230 to 1865, and press ENTER. Each time the TI-86 evaluates the function y1, it uses the *current* value of *x*.

Like your textbook, the TI-86 uses standard function notation. So, to evaluate $y_1(2230)$ when $y_1(x) = 1975 + .10x$, press 2nd alpha Y 1 (2230) ENTER (see Figure III.11). Then to evaluate $y_1(1865)$, press 2nd ENTRY to recall the last line, change 2230 to 1865, and press ENTER.

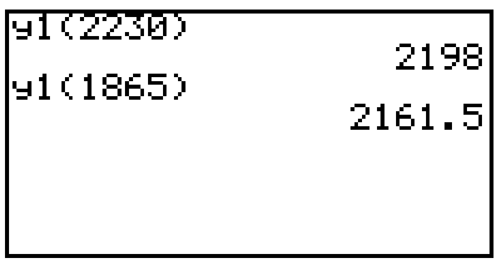


Figure III.11: Function notation

You may also have the TI-86 make a table of values for the function. Press TABLE to get the TABLE menu across the bottom of the screen and then press F2 *[TBLST]* to set up the table (Figure III.12). Move the blinking cursor onto Ask beside Indpnt:, then press ENTER. This configuration permits you to input values for *x* one at a time. Now press F1 *[TABLE]*, enter 2230 in the x column, and press ENTER (see Figure III.13). Continue to enter additional values for x and the calculator automatically completes the table with corresponding values of y1. Press 2nd QUIT to leave the TABLE screen.

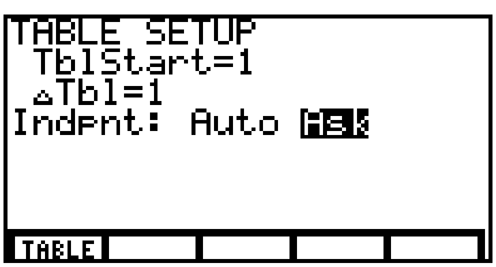


Figure III.12: TBLST screen

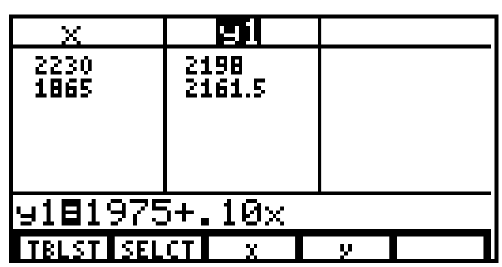


Figure III.13: Table of values

  

For a table containing values for $x = 1, 2, 3, 4, 5$, and so on, set TblStart = 1 to start at $x = 1$, ΔTbl = 1 to increment in steps of 1, and Indpnt to Auto.

***Technology Tip:*** The TI-86 does not require multiplication to be expressed between variables, so $xxx$ means $x^3$. It is often easier to press two or three $x$'s together than to search for the square key or the powers key. Of course, expressed multiplication is also not required between a constant and variable. So, to enter $2x^3 + 3x^2 - 4x + 5$ in the TI-86, you might save keystrokes and press just these keys: 2 x-VAR x-VAR x-VAR + 3 x-VAR x-VAR − 4 x-VAR + 5.

***III.2.2 Functions in a Graph Window:*** Once you have entered a function in the y(x)= screen of the TI-86, just press 2nd M5 *[GRAPH]* to see its graph. The ability to draw a graph contributes substantially to our ability to solve problems.

For example, here is how to graph $y = -x^3 + 4x$. First press GRAPH y(x)= and delete anything that may be there by moving with the arrow keys to any of the existing functions and pressing F4 *[DELf]*. Then, with the cursor on the top line to the right of y1, press (-) x-VAR ^ 3 + 4 x-VAR to enter the function (as in Figure III.14). Now press 2nd M5 and the TI-86 changes to a window with the graph of $y = -x^3 + 4x$ (Figure III.15). To remove the menu from the bottom of the display screen, press CLEAR.

While the TI-86 is calculating coordinates for a plot, it displays a busy indicator at the top right of the graph window.

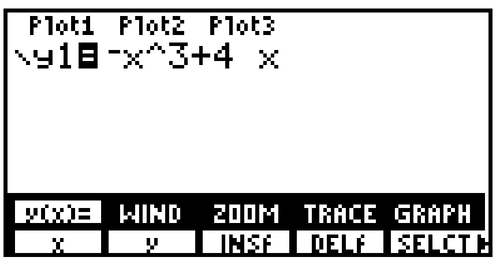


Figure III.14: y(x)= screen

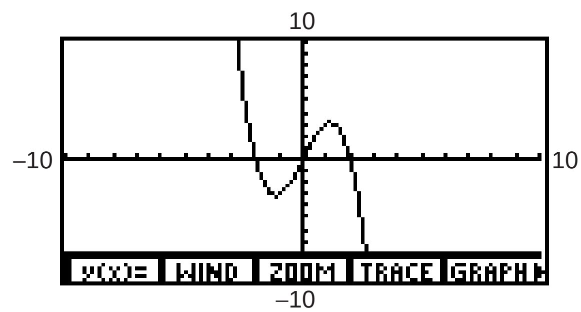


Figure III.15: Graph of $y = -x^3 + 4x$

Your graph window may look like the one in Figure III.15 or it may be different. Because the graph of $y = -x^3 + 4x$ extends infinitely far left and right and also infinitely far up and down, the TI-86 can display only a piece of the actual graph. This displayed rectangular part is called a *viewing window*. You can easily change the viewing window to enhance your investigation of a graph.

The viewing window in Figure III.15 shows the part of the graph that extends horizontally from −10 to 10 and vertically from −10 to 10. Press F2 *[WIND]* to see information about your viewing window. Figure III.16 shows the WINDOW screen that corresponds to the viewing window in Figure III.15. This is the *standard* viewing window for the TI-86.

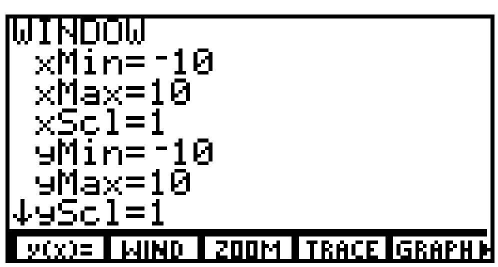


Figure III.16: Standard WINDOW

The variables xMin and xMax are the minimum and maximum *x*-values of the viewing window. yMin and yMax are its minimum and maximum *y*-values.

xScl and yScl set the spacing between tick marks on the axes.

Scrolling to the bottom of the screen, xRes sets pixel resolution (1 through 8) for function graphs.

***Technology Tip:*** Small xRes values improve graph resolution, but may cause the TI-86 to draw graphs more slowly.

Use the arrow keys ▲ and ▼ to move up and down from one line to another in this list; pressing the ENTER key will move down the list. Press CLEAR to delete the current value and then enter a new value. You may also edit the entry as you would edit an expression. Remember that a minimum *must* be less than the corresponding maximum or the TI-86 will issue an error message. Also, remember to use the (-) key, not – (which is subtraction), when you want to enter a negative value. Figures III.15–16, III.17–18, and III.19–20 show different WINDOW screens and the corresponding viewing window for each one.

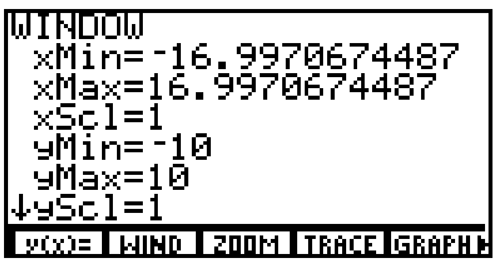


Figure III.17: Square window

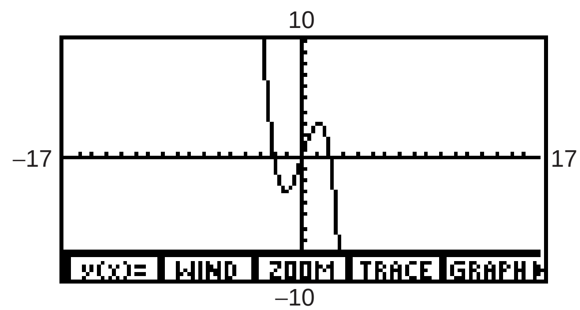


Figure III.18: Graph of $y = -x^3 + 4x$

To initialize the viewing window quickly to the standard viewing window (Figure III.16), press F3 *[ZOOM]* F4 *[ZSTD]*. To set the viewing window quickly to a square window (Figure III.17), press F3 MORE F2 *[ZSQR]*. More information about square windows is presented later in Section III.2.4.

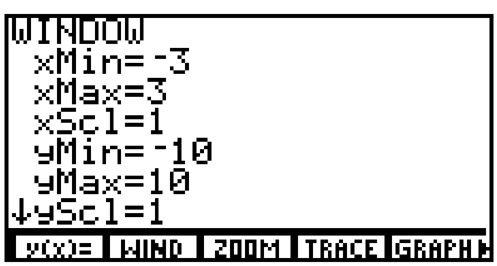


Figure III.19: Custom window

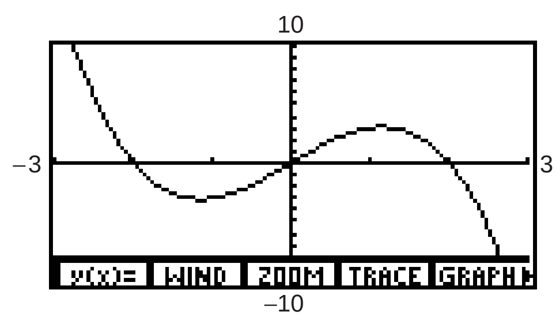


Figure III.20: Graph of $y = -x^3 + 4x$

Sometimes you may wish to display grid points corresponding to tick marks on the axes. This and other graph format options may be changed by pressing GRAPH MORE F3 *[FORMT]* to display the FORMAT menu (Figure III.21). Use arrow keys to move the blinking cursor to GridOn; press ENTER and then F5 to redraw the graph. Figure III.22 shows the same graph as in Figure III.20 but with the grid turned on. In general, you'll want the grid turned *off*, so do that now by pressing GRAPH MORE F3, use the arrow keys to move the blinking cursor to GridOff, and press ENTER and EXIT.

  

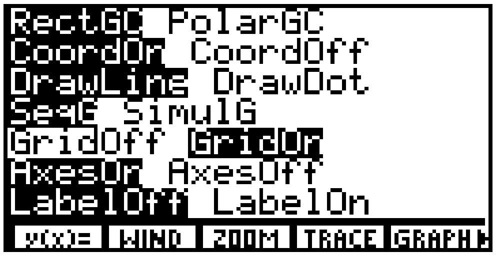


Figure III.21: FORMAT menu

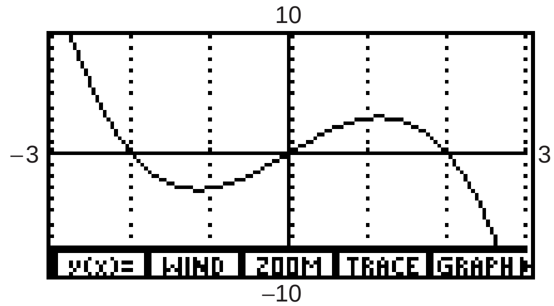


Figure III.22: Grid turned on for $y = -x^3 + 4x$

***Technology Tip:*** On the TI-86, the style of your graph can be changed by changing the icon to the left of y1 on the y(x)= screen. To change the icon enter the y(x)= screen and move your cursor onto the line of the function whose style you want to change. Then press MORE and F3 *[STYLE]* repeatedly to scroll through the different styles available.

***III.2.3 Graphing Step and Piecewise-Defined Functions:*** The greatest integer function, written $[\![x]\!]$, gives the greatest *integer* less than or equal to a number $x$. On the TI-86, the greatest integer function is called int and is located under the NUM sub-menu of the MATH menu (see Figure III.5). So, calculate $[\![6.78]\!] = 6$ by pressing 2nd MATH F1 F4 *[int]* 6.78 ENTER.

To graph $y = [\![x]\!]$, go into the y(x)= menu, move beside y1, and press CLEAR 2nd MATH F1 F4 EXIT F1 2nd M5 *[GRAPH]*. Figure III.23 shows this graph in a viewing window from $-5$ to 5 in both directions.

The true graph of the greatest integer function is a step graph, like the one in Figure III.24. For the graph of $y = [\![x]\!]$, a segment should *not* be drawn between every pair of successive points. You can change from DrawLine to DrawDot format on the TI-86 by opening the GRAPH FORMT menu. Another option is to change the style of the graph from line to dot by using the method described in the above Technology Tip. (The style will be indicated by three dots along a diagonal.)


Figure III.23: DrawLine graph of $y = [\![x]\!]$

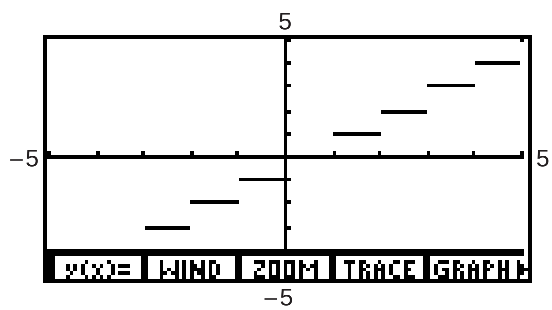


Figure III.24: DrawDot graph of $y = [\![x]\!]$

Make sure to change your TI-86 back to DrawLine, because most of the functions that you will be graphing should be viewed this way.

The TI-86 can graph piecewise-defined functions by using the options in the TEST menu (Figure III.25) that is displayed across the bottom of the screen by pressing 2nd TEST. Each TEST function returns the value 1 if the statement is true, and the value 0 if the statement is false.

  

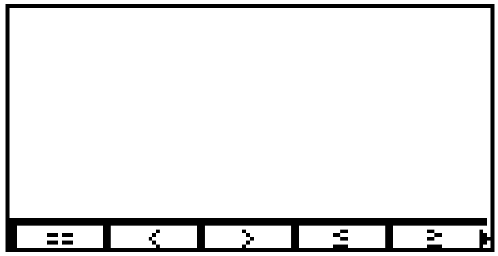


Figure III.25: 2nd TEST menu

For example, to graph the function $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$ (using DrawDot), enter the following keystrokes for y1 in the y(x)= screen: ( x-VAR $x^2$ + 1) ( x-VAR 2nd TEST F2 *[<]* 0 ) + ( x-VAR – 1 ) (x-VAR F5 *[≥]* 0 ) (Figure III.26). Press EXIT to remove the TEST menu. Then change to DrawDot and press F5 to display the graph. Figure III.27 shows this graph in a viewing window from $-5$ to 5 in both directions.

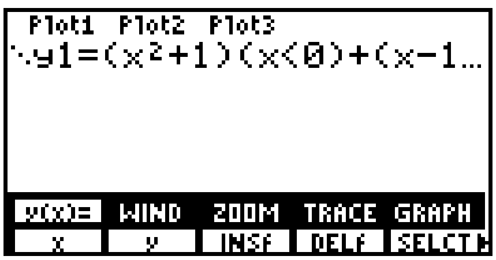


Figure III.26: Piecewise-defined function

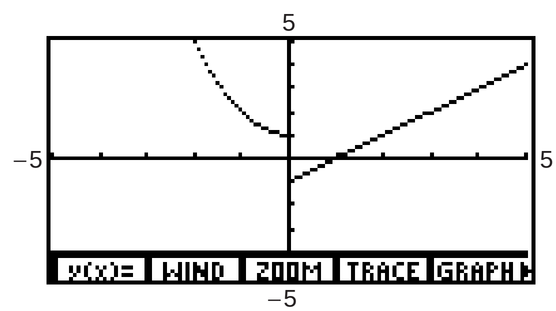


Figure III.27: Piecewise-defined graph

***III.2.4 Graphing a Circle:*** Here is a useful technique for graphs that are not functions, but that can be "split" into a top part and a bottom part, or into multiple parts. Suppose you wish to graph the circle whose equation is $x^2 + y^2 = 36$. First solve for $y$ and get an equation for the top semicircle, $y = \sqrt{36 - x^2}$, and for the bottom semicircle, $y = -\sqrt{36 - x^2}$. Then graph the two semicircles simultaneously.

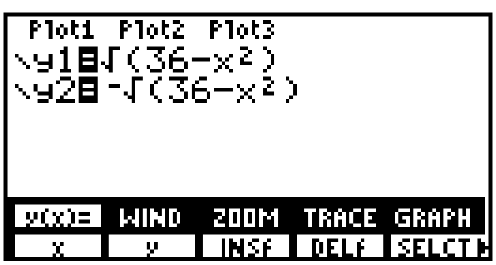


Figure III.28: Two semicircles

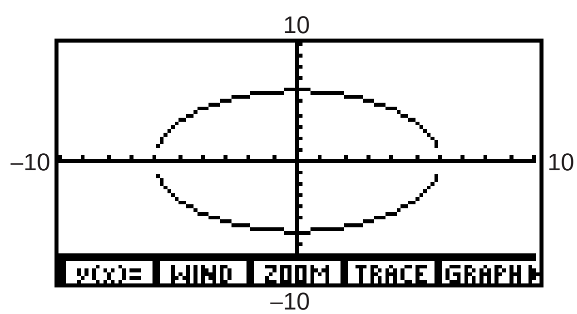


Figure III.29: Circle's graph – standard WINDOW

Use the following keystrokes to draw the circle's graph. Enter $\sqrt{36 - x^2}$ as y1 and $-\sqrt{36 - x^2}$ as y2 (see Figure III.28) by pressing GRAPH F1 CLEAR 2nd $\sqrt{\ }$ ( 36 $-$ x-VAR $x^2$ ) ENTER CLEAR (-) 2nd $\sqrt{\ }$ ( 36 $-$ x-VAR $x^2$ ). Then press 2nd M5 to draw them both (Figure III.29).

If your range were set to the standard viewing window, your graph would look like Figure III.29. Now this does *not* look like a circle, because the units along the axes are not the same. This is where the square viewing window is important. Press F3 MORE F2 and see a graph that appears more circular.

  

***Technology Tip:*** Another way to get a square graph is to change the window variables so that the value of yMax - yMin is approximately $\frac{10}{17}$ times xMax - xMin. For example, see the WINDOW in Figure III.30 and the corresponding graph in Figure III.31. This method works because the dimensions of the TI-86's display are such that the ratio of vertical to horizontal is approximately $\frac{10}{17}$.

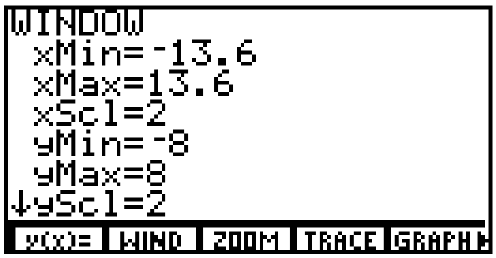


Figure III.30: $\dfrac{\text{vertical}}{\text{horizontal}} = \dfrac{16}{27.2} = \dfrac{10}{17}$

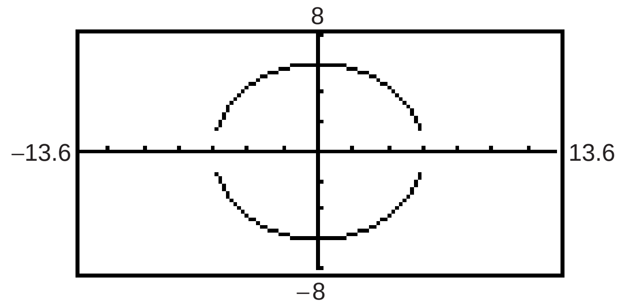


Figure III.31: A "square" circle

The two semicircles in Figure III.31 do not connect because of an idiosyncrasy in the way the TI-86 plots a graph.

Back when you entered $\sqrt{36 - x^2}$ as y1 and $-\sqrt{36 - x^2}$ as y2, you could have entered −y1 as y2 and saved some keystrokes. Try this by going to the y(x)= screen and pressing to move the cursor down to $y_2$. Then press CLEAR (-) 2nd alpha Y 1. The graph should be just as it was before.

***III.2.5 TRACE:*** Graph $y = -x^3 + 4x$ from Section III.2.2 in the standard viewing window. (Remember to clear any other functions in the y(x)= screen.) Press any of the arrow keys and see the cursor move from the center of the viewing window. The coordinates of the cursor's location are displayed at the bottom of the screen, as in Figure III.32, in floating decimal format. (Recall that if you have a menu at the bottom of the screen, you can remove it by pressing CLEAR.) This cursor is called a *free-moving cursor* because it can move from dot to dot *anywhere* in the graph window.

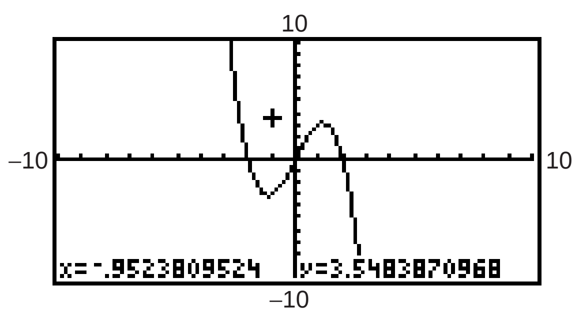


Figure III.32: Free-moving cursor

Remove the free-moving cursor and its coordinates from the window by pressing CLEAR, ENTER, or GRAPH (this also restores the GRAPH menu). Press an arrow key again and the free-moving cursor will reappear at the same point you left it.

With the GRAPH menu active at the bottom of the screen, press F4 *[TRACE]* to enable the left and right arrow keys to move the cursor from point to point along the graph of the function. The cursor is no longer free-moving, but is now constrained to the function. The coordinates that are displayed belong to points on the function's graph, so the *y*-coordinate is the calculated value of the function at the corresponding *x*-coordinate (Figure III.33).

  

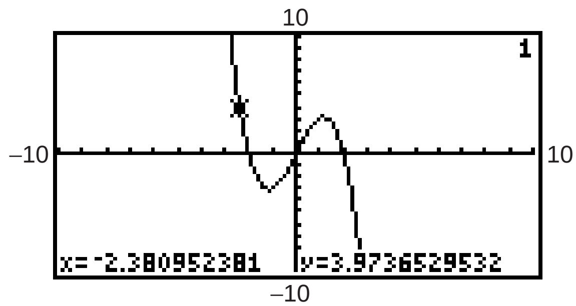


Figure III.33: TRACE

Now plot a second function, $y = -.25x$, along with $y = -x^3 + 4x$. Press GRAPH F1 and enter $-.25x$, for y2, then press 2nd M5 to see both functions.

Note that in Figure III.34 the equal signs next to y1 and y2 are *both* highlighted. This means *both* functions will be graphed as shown in Figure III.35. In the y(x)= screen, move the cursor to y1 and press F5 *[SELCT]* to turn off the selection of the function. The equal sign beside y1 should no longer be highlighted (see Figure III.36). Now press 2nd M5 and see that only y2 is plotted (Figure III.37).

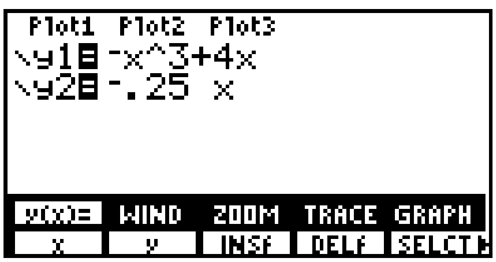


Figure III.34: Two functions

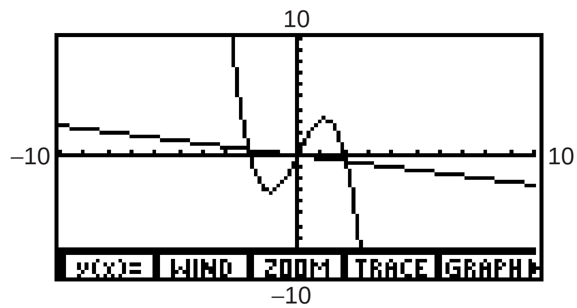


Figure III.35: $y = -x^3 + 4x$ and $y = -.25x$

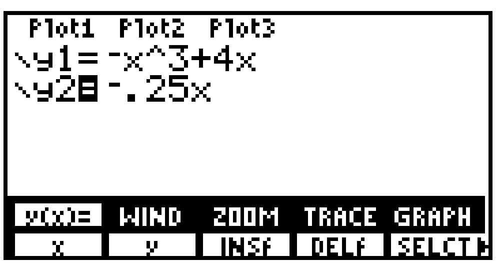


Figure III.36: y(x)= screen with only y2 active

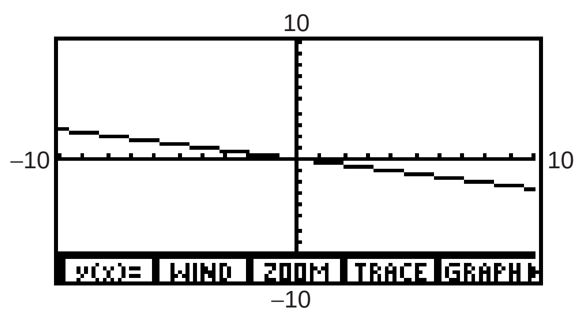


Figure III.37: Graph of $y = -.25x$

Many different functions can be stored in the y(x)= list and any combination of them may be graphed simultaneously. You can make a function active or inactive for graphing by pressing SELCT to highlight (activate) or remove the highlight (deactivate). Now go back to the y(x)= screen and do what is needed in order to graph y1 but not y2.

Now activate both functions so that both graphs are plotted. Press GRAPH F4 and the cursor appears first on the graph of $y = -x^3 + 4x$ because it is higher up in the y(x)= list. You know that the cursor is on this function, y1, because the number 1 is displayed in the upper right corner of the screen (see Figure III.33). Press the up ▲ or down ▼ arrow key to move the cursor vertically to the graph of $y = -.25x$. Now the number 2 is displayed in the upper right corner of the screen. Next press the left and right arrow keys to trace along the graph of $y = -.25x$. When more than one function is plotted, you can move the trace cursor vertically from one graph to another with the ▲ and ▼ keys.

***Technology Tip:*** Trace along the graph of $y = -.25x$ and press and hold either ◀ or ▶. Eventually you will reach the left or right edge of the window. Keep pressing the arrow key and the TI-86 will allow you to continue the trace by panning the viewing window. Check the WINDOW screen to see that xMin and xMax are automatically updated.

If you trace along the graph of $y = -x^3 + 4x$, the cursor will eventually move *above* or *below* the viewing window. The cursor's coordinates on the graph will still be displayed, though the cursor itself can no longer be seen. When you are tracing along a graph, press ENTER and the window will quickly pan over so that the cursor's position on the function is centered in a new viewing window. This feature is especially helpful when you trace near or beyond the edge of the current viewing window.

The TI-86's display has 127 horizontal columns of pixels and 63 vertical rows. So when you trace a curve across a graph window, you are actually moving from xMin to xMax in 126 equal jumps, each called $\Delta x$. You would calculate the size of each jump to be $\Delta x = \frac{\text{xMax} - \text{xMin}}{126}$. Sometimes you may want the jumps to be friendly numbers like 0.1 or 0.25 so that, when you trace along the curve, the $x$-coordinates will be incremented by such a convenient amount. Just set your viewing window for a particular increment $\Delta x$ by making $\text{xMax} = \text{xMin} + 126 \cdot \Delta x$. For example, if you want $\text{xMin} = -15$ and $\Delta x = .25$, set $\text{xMax} = -15 + 125 \cdot .25 = 16.5$. Likewise, set $\text{yMax} = \text{yMin} + 62 \cdot \Delta y$ if you want the vertical increment to be some special $\Delta y$.

To center your window around a particular point, $(h, k)$, and also have a certain $\Delta x$, set $\text{xMin} = h - 63 \cdot \Delta x$ and $\text{xMax} = h + 63 \cdot \Delta x$. Likewise, make $\text{yMin} = k - 31 \cdot \Delta y$ and $\text{yMax} = k + 31 \cdot \Delta x$. For example, to center a window around the origin $(0, 0)$, with both horizontal and vertical increments of 0.25, set the range so that $\text{xMin} = 0 - 63 \cdot 0.25 = -15.75$, $\text{xMax} = 0 + 63 \cdot 0.25 = 15.75$, $\text{yMin} = 0 - 31 \cdot 0.25 = -7.75$, and $\text{yMax} = 0 + 31 \cdot 0.25 = 7.75$.

See the benefit by first graphing $y = x^2 + 2x + 1$ in a standard viewing window. Trace near its $y$-intercept, which is $(0, 1)$, and move towards its $x$-intercept, which is $(-1, 0)$. Then change to a viewing window that extends from $-6.3$ to 6.3 horizontally and from $-3.1$ to 3.1 vertically (center at the origin, $\Delta x$ and $\Delta y$ both .1), and trace again near its $y$-intercept. The TI-86 makes it easy to get this particular viewing window: press GRAPH F3 MORE F4 *[ZDECM]*.

***III.2.6 ZOOM:*** Plot again the two graphs for $y = -x^3 + 4x$ and for $y = -.25x$. There appears to be an intersection near $x = 2$. The TI-86 provides several ways to enlarge the view around this point. You can change the viewing window directly by pressing GRAPH F2 and editing the values of xMin, xMax, yMin, and yMax. Figure III.39 shows a new viewing window for the range displayed in Figure III.38. The cursor has been moved near the point of intersection; move your cursor closer to get the best approximation possible for the coordinates of the intersection.

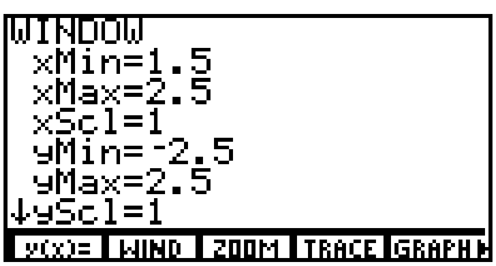


Figure III.38: New WINDOW

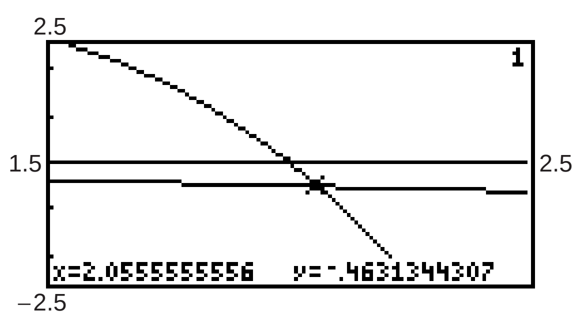


Figure III.39: Closer view

  

A more efficient method for enlarging the view is to draw a new viewing window with the cursor. Start again with a graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$ in a standard viewing window (press GRAPH F3 F4 for the standard viewing window).

Now imagine a small rectangular box around the intersection point, near $x = 2$. Press GRAPH F3 F1 *[BOX]* to draw a box to define this new viewing window. Use the arrow keys to move the cursor, whose coordinates are displayed at the bottom of the window, to one corner of the new viewing window you imagine.

Press ENTER to fix the corner where you moved the cursor; it changes shape and becomes a blinking square (Figure III.40). Use the arrow keys again to move the cursor to the diagonally opposite corner of the new rectangle (Figure III.41), then press ENTER. Then press CLEAR to remove the menu from the bottom of the screen. The rectangular area you have enclosed will now enlarge to fill the graph window (Figure III.42).

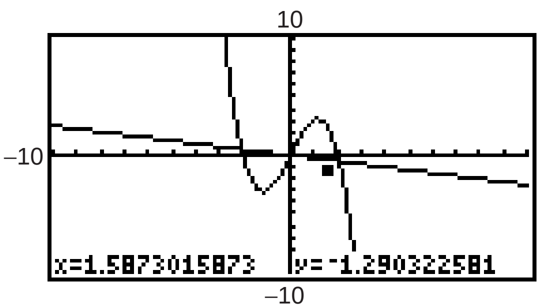


Figure III.40: One corner selected

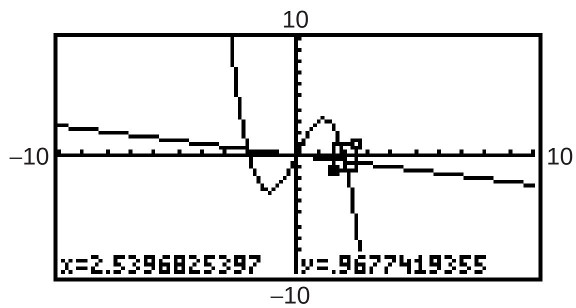


Figure III.41: Box drawn

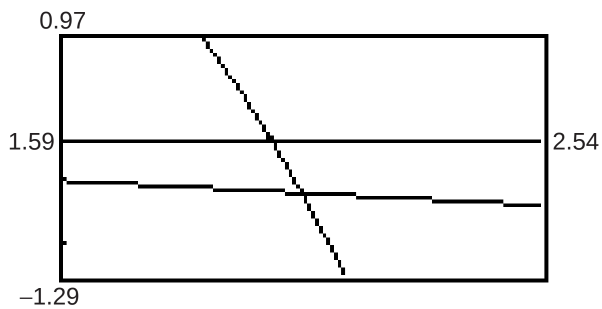


Figure III.42: New viewing window

You may cancel the zoom any time *before* you press this last ENTER. Press EXIT or GRAPH to interrupt the zoom and return to the current graph window. Even if you did execute the zoom, you may still return to the previous viewing window by pressing F5 *[ZPREV]* in the ZOOM menu.

You can also quickly magnify a graph around the cursor's location. Return once more to the standard window for the graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$. Press GRAPH F3 F2 *[ZIN]* and then press arrow keys to move the cursor as close as you can to the point of intersection near $x = 2$ (see Figure III.43). Then press ENTER and the calculator draws a magnified graph, centered at the cursor's position (Figure III.44). The range variables are changed to reflect this new viewing window. Look in the WINDOW menu to verify this.

  

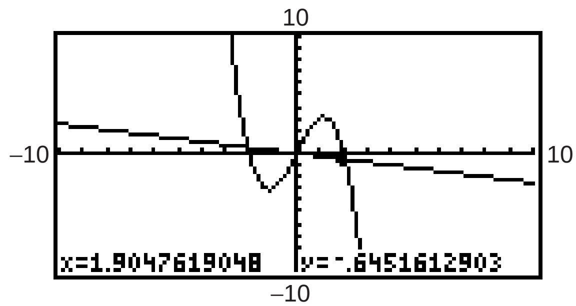


Figure III.43: Before a zoom in

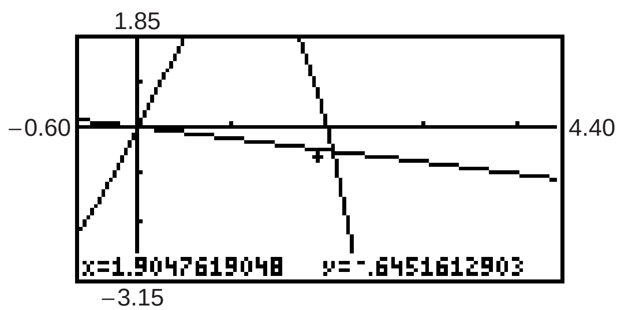


Figure III.44: After a zoom in

Selecting in the ZOOM menu, the TI-86 can zoom in (press F2 *[ZIN]* ) or zoom out (press F3 *[ZOUT]* ). Zoom out to see a larger view of the graph, centered at the cursor position. You can change the horizontal and vertical scale of the magnification by pressing GRAPH F3 MORE MORE F2 *[ZFACT]* (see Figure III.45) and editing xFact and yFact, the horizontal and vertical magnification factors.

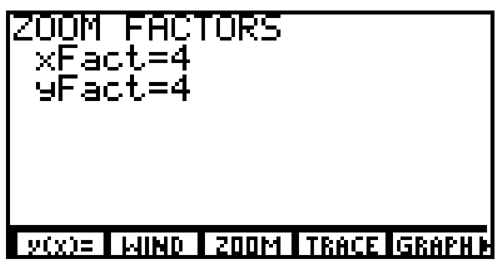


Figure III.45: Set zoom factors

The default zoom factor is 4 in both directions. It is not necessary for xFact and yFact to be equal. Sometimes, you may prefer to zoom in one direction only, so the other factor should be set to 1. As usual, press GRAPH or EXIT to leave the ZOOM FACTORS menu.

***Technology Tip:*** The TI-86 remembers the window it displayed before a zoom. So, if you should zoom in too much and lose the curve, press GRAPH F3 F5 *[ZPREV]* to go back to the window before. If you want to execute a series of zooms but then return to a particular window, press GRAPH F3 MORE MORE MORE F1 *[ZSTO]* to store the current window's dimensions. Later, press GRAPH F3 MORE MORE F1 *[ZRCL]* to recall the stored window.

***II.2.7 Value:*** Graph $y = -x^3 + 4x$ in the standard viewing window (Figure III.15). The TI-86 can calculate the value of this function for any given $x$ (between the xMin and xMax values).

  

Press GRAPH MORE MORE F1 *[EVAL]*. The graph of the function is displayed and you are prompted to enter a value for $x$. Press 1 ENTER. The $x$-value you entered and its corresponding $y$-value are shown at the bottom of the screen and the cursor is located at the point $(1, 3)$ on the graph (see Figure III.46).

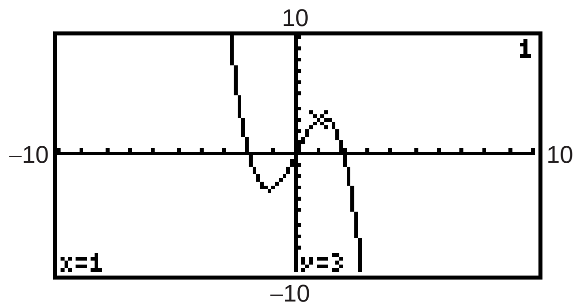


Figure III.46: Finding a value

Note that if you have more than one graph on the screen, the upper right corner of the TI-86 screen will display the numeral corresponding to the equation of the function in the y(x)= list whose value is being calculated. Press the up ▲ or down ▼ arrow key to move the cursor vertically between functions at the entered $x$-value.

***II.2.8 Relative Minimums and Maximums:*** Graph $y = -x^3 + 4x$ once again in the standard viewing window. This function appears to have a relative minimum near $x = -1$ and a relative maximum near $x = 1$. You may zoom and trace to approximate these extreme values.

First trace along the curve near the relative minimum. Notice by how much the $x$-values and $y$-values change as you move from point to point. Trace along the curve until the $y$-coordinate is as *small* as you can get it, so that you are as close as possible to the relative minimum, and zoom in (press GRAPH F3 F2 or use a zoom box). Now trace again along the curve and, as you move from point to point, see that the coordinates change by smaller amounts than before. Keep zooming and tracing until you find the coordinates of the relative minimum point as accurately as you need them, approximately $(-1.15, -3.08)$.

Follow a similar procedure to find the relative maximum. Trace along the curve until the $y$-coordinate is as *great* as you can get it, so that you are as close as possible to the relative maximum, and zoom in. The relative maximum point on the graph of $y = -x^3 + 4x$ is approximately $(1.15, 3.08)$.

The TI-86 can automatically find the relative maximum and relative minimum points. Press GRAPH MORE F1 *[MATH]* to display the MATH menu across the bottom of the screen (Figure III.47). Choose F4 *[FMIN]* to calculate the minimum value of the function and F5 *[FMAX]* for the maximum. You will be prompted to trace the cursor along the graph first to a point *left* of the minimum/maximum (press ENTER to set this *left bound*). Then move to a point *right* of the minimum/maximum and set a *right bound* and press ENTER. Note the two arrows near the top of the display marking the left and right bounds (as in Figure III.48).

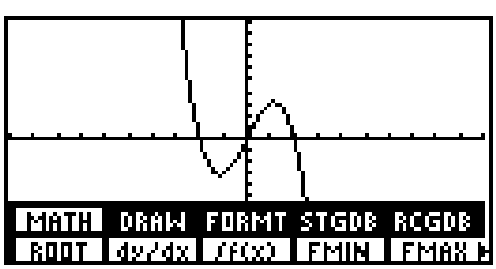


Figure III.47: MATH menu

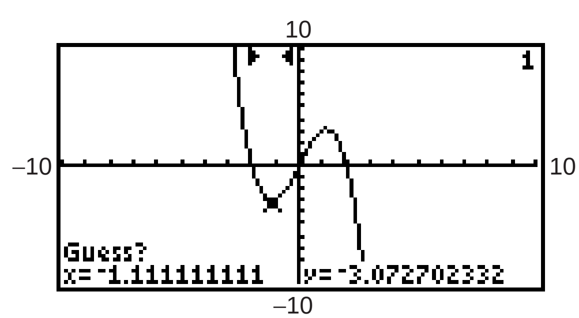


Figure III.48: Finding a minimum

  

Next move the cursor along the graph between the two bounds and as close to the minimum/maximum as you can. This serves as a *guess* for the TI-86 to start its search. Good choices for the left bound, right bound, and guess can help the calculator work more efficiently and quickly. Press ENTER and the coordinates of the relative minimum/maximum point will be displayed (see Figure III.49).

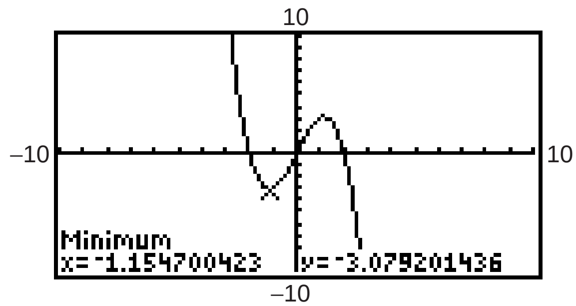


Figure III.49: Relative minimum on $y = -x^3 + 4x$

Note that if you have more than one graph on the screen, the upper right corner of the TI-86 screen will display the numeral corresponding to the equation of the function in the y(x)= list whose minimum/maximum is being calculated.

***II.2.9 Inverse Functions:*** The TI-86 draws the inverse function of a one-to-one function. Graph $y = x^3 + 1$ as y1 in the standard viewing window. Next, press GRAPH MORE F2 *[DRAW]* to display the DRAW menu across the bottom of the screen (see Figure III.50). Then press MORE MORE MORE F3 *[DrInv]* 2nd ALPHA Y 1 ENTER (see Figure III.51). These keystrokes instruct the TI-86 to draw the inverse function of y1. The original function and its inverse function will be displayed (see Figure III.52). Note that the calculator must be in function mode in order to use DrInv.

To clear the graph of the inverse function, press MORE F2 MORE MORE F1 *[CLDRW]*.

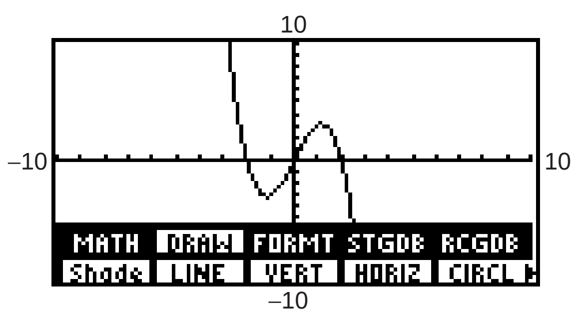


Figure III.50: DRAW menu

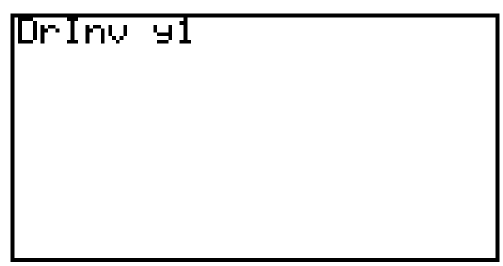


Figure III.51: DrInv

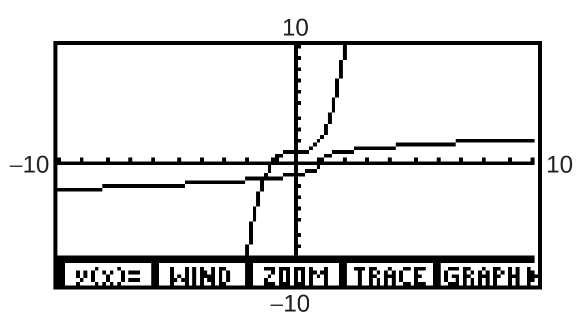


Figure III.52: Graph of $y = x^3 + 1$ and its inverse function

  

***II.2.10 Tangent Lines:*** Once again, graph $y = x^3 + 1$ in the standard viewing window (see Figure III.50). The TI-86 can draw the tangent line to a graph of a function at a specified point.

Press GRAPH MORE F2 MORE MORE MORE F2 *[TanLn]* 2nd ALPHA Y 1 , 1 ) ENTER (see Figure III.53). These keystrokes instruct the TI-86 to draw the tangent line to the graph of y1 at $x = 1$. The graph of the original function and the tangent line to the graph at $x = 1$ will be displayed (see Figure III.54).

To clear the tangent line, press MORE F2 MORE MORE F1.

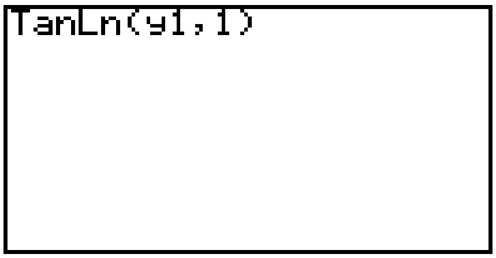


Figure III.53: Tangent line

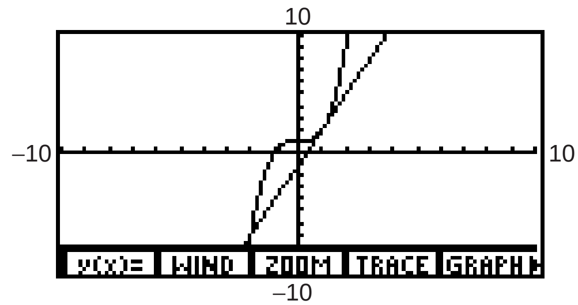


Figure III.54: Graph of $y = x^3 + 1$ and tangent line at $x = 1$

## III.3 Solving Equations and Inequalities

***II.3.1 Intercepts and Intersections:*** Tracing and zooming are also used to locate an $x$-intercept of a graph, where a curve crosses the $x$-axis. For example, the graph of $y = x^3 - 8x$ crosses the $x$-axis three times (see Figure III.55). After tracing over to the $x$-intercept point that is farthest to the left, zoom in (Figure III.56). Continue this process until you have located all three intercepts with as much accuracy as you need. The three $x$-intercepts of $y = x^3 - 8x$ are approximately $-2.828$, 0, and 2.828.

***Technology Tip:*** As you zoom in, you may also wish to change the spacing between tick marks on the $x$-axis so that the viewing window shows scale marks near the intercept point. Then the accuracy of your approximation will be such that the error is less than the distance between two tick marks. Change the $x$-scale on the TI-86 from the WINDOW menu. Move the cursor down to xScl and enter an appropriate value.

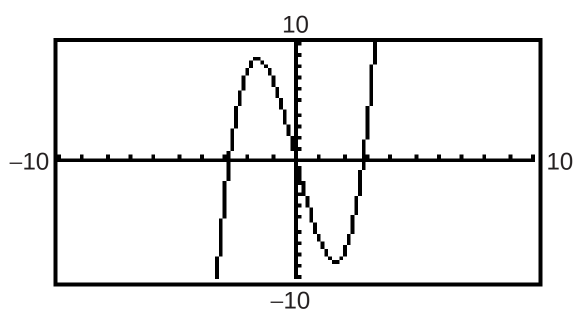


Figure III.55: Graph of $y = x^3 - 8x$

  

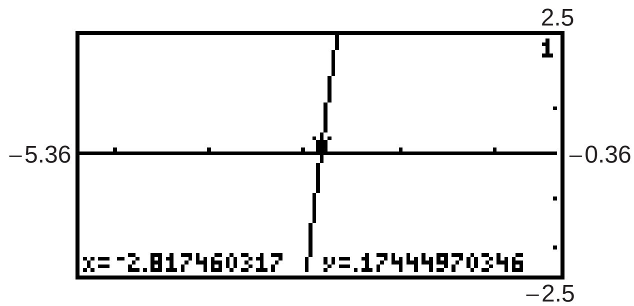


Figure III.56: Near an *x*-intercept of $y = x^3 - 8x$

The *x*-intercept of a function's graph is a *root* of the function. So press GRAPH MORE F1 to display the MATH menu across the bottom of the screen (Figure III.47) and choose F1 *[ROOT]* to find a root of this function. Set a left bound, right bound, and guess as described in Section III.2.8. The TI-86 shows the coordinates of the point and indicates that it is a root (Figure III.57).

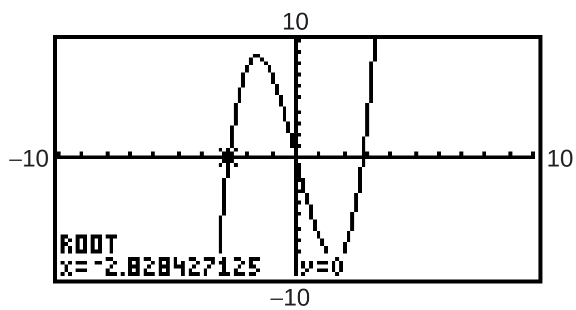


Figure III.57: A root of $y = x^3 - 8x$

TRACE and ZOOM are especially important for locating the intersection points of two graphs, say the graphs of $y = -x^3 + 4x$ and $y = -.25x$. Trace along one of the graphs until you arrive close to an intersection point. Then press ▲ or ▼ to jump to the other graph. Notice that the *x*-coordinate does not change, but the *y*-coordinate is likely to be different (see Figures III.58 and III.59).

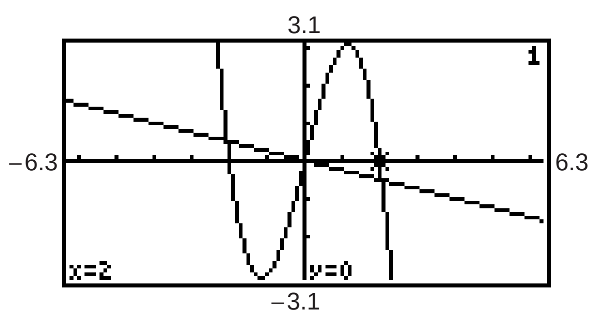


Figure III.58: Trace on $y = -x^3 + 4x$

  

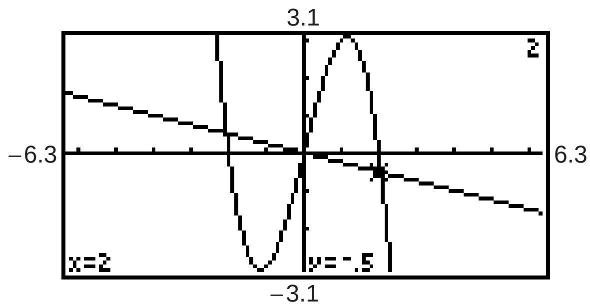


Figure III.59: Trace on $y = -.25x$

When the two $y$-coordinates are as close as they can get, you have come as close as you now can to the point of intersection. So zoom in around the intersection point, then trace again until the two $y$-coordinates are as close as possible. Continue this process until you have located the point of intersection with as much accuracy as necessary. The points of intersection are approximately $(-2.062, 0.515)$, $(0, 0)$, and $(2.062, 0.515)$.

You can also find the point of intersection of two graphs by pressing GRAPH MORE F1 *[MATH]* MORE F3 *[ISECT]*. Trace with the cursor first along one graph near the intersection and press ENTER; then trace with the cursor along the other graph and press ENTER. Marks + are placed on the graphs at these points. Finally, move the cursor near the point of intersection and press ENTER again. Coordinates of the intersection will be displayed at the bottom of the window. More will be said about the intersect feature in Section III.3.3.

***III.3.2 Solving Equations by Graphing:*** Suppose you need to solve the equation $24x^3 - 36x + 17 = 0$. First graph $y = 24x^3 - 36x + 17$ in a window large enough to exhibit *all* its $x$-intercepts, corresponding to all the equation's real zeros (roots). Then use zoom and trace, or the TI-86's root finder, to locate each one. In fact, this equation has just one real solution, $x \approx -1.414$.

Remember that when an equation has more than one $x$-intercept, it may be necessary to change the viewing window a few times to locate all of them.

***Technology Tip:*** To solve an equation like $24x^3 + 17 = 36x$, you may first rewrite it in general form, $24x^3 - 36x + 17 = 0$, and proceed as above. However, you may also graph the *two* functions $y = 24x^3 + 17$ and $y = 36x$, then zoom and trace to locate their point of intersection.

***III.3.3 Solving Systems by Graphing:*** The solutions to a system of equations correspond to the points of intersection of their graphs (Figure III.60). For example, to solve the system $y = 2x + 5$ and $y = -2x + 1$, first graph them together. Then use zoom and trace or the ISECT option in the MATH menu to locate their point of intersection, which is $(-1, 3)$ (see Figure III.61).

The solutions of the system of two equations $y = 2x + 5$ and $y = -2x + 1$ correspond to the solutions of the single equation $2x + 5 = -2x + 1$, which simplifies to $4x + 4 = 0$. So you may also graph $y = 4x + 4$ and find its $x$-intercept to solve the system.

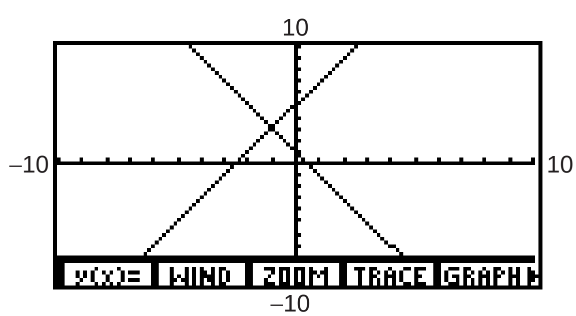


Figure III.60: Solving a system of equations

  

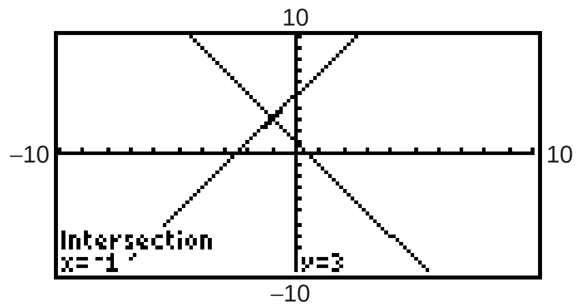


Figure III.61: The point of intersection is $(-1, 3)$.

***II.3.4 Solving Inequalities by Graphing:*** Consider the inequality $1 - \frac{3x}{2} \geq x - 4$. To solve it with your TI-86, graph the two functions $y = 1 - \frac{3x}{2}$ and $y = x - 4$ (Figure III.62). First locate their point of intersection, at $x = 2$. The inequality is true when the graph of $y = 1 - \frac{3x}{2}$ lies *above* the graph of $y = x - 4$, and that occurs for $x < 2$. So the solution is $x \leq 2$, or $(-\infty, 2]$.

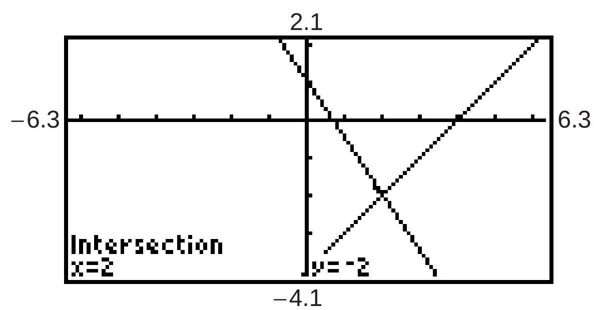


Figure III.62: Solving $1 - \frac{3x}{2} \geq x - 4$

The TI-86 is capable of shading the region above or below a graph or between two graphs. For example, to graph $y \geq x^2 - 1$, first graph the function $y = x^2 - 1$ as y1. Then press GRAPH MORE F2 F1 *[Shade]* 2nd alpha Y 1 , 10 ) ENTER (see Figure III.63). These keystrokes instruct the TI-86 to shade the region *above* $y = x^2 - 1$ and *below* $y = 10$ (chosen because this is the greatest $y$-value in the graph window) using the default shading option of vertical lines. The result is shown in Figure III.64.

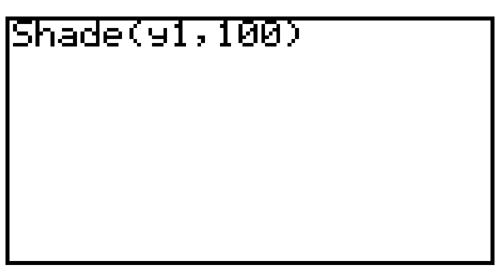


Figure III.63: DRAW Shade

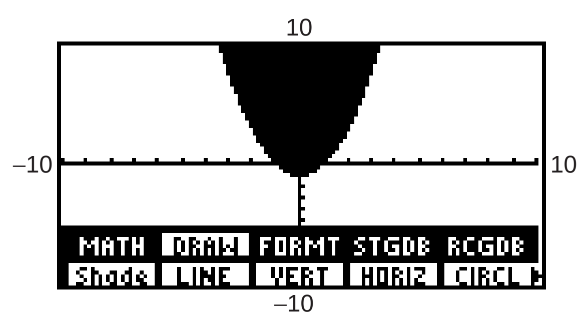


Figure III.64: Graph of $y \geq x^2 - 1$

To clear the shading when you are already in the DRAW menu, press MORE MORE F1.

  

Now use shading to solve the previous inequality, $1 - \frac{3x}{2} \geq x - 4$. The function whose graph forms the lower boundary is named *first* in the SHADE command (see Figure III.65). To enter this in your TI-86, press these keys: GRAPH MORE F2 F1 x-VAR − 4 , 1 − 3 x-VAR ÷ 2 ) ENTER (Figure III.66). The shading extends left from $x = -2$, so the solution to $1 - \frac{3x}{2} \geq x - 4$ is $x \leq 2$, or $(-\infty, 2]$ (see Figure III.66).

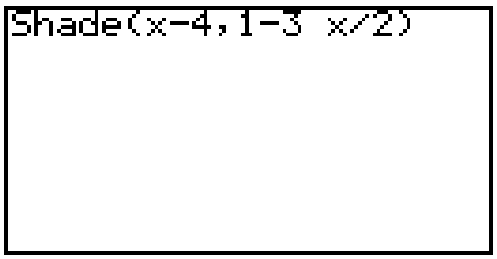


Figure III.65: DRAW Shade command

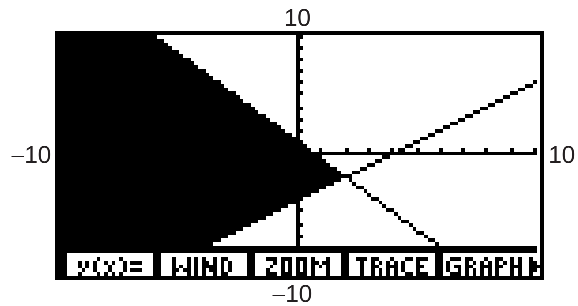


Figure III.66: Graph of $1 - \frac{3x}{2} \geq x - 4$

More information about the DRAW menu is in the TI-86 manual.

## III.4 Trigonometry

***II.4.1 Degrees and Radians:*** The trigonometric functions can be applied to angles measured either in radians or degrees, but you should take care that the TI-86 is configured for whichever measure you need. Press 2nd MODE to see the current settings. Press ▼ twice and move down to the third line of the mode menu where angle measure is selected. Then press ◄ or ► to move between the displayed options. When the blinking cursor is on the measure you want, press ENTER to select it. Then press EXIT, CLEAR , or 2nd QUIT to leave the mode menu.

It's a good idea to check the angle measure setting before executing a calculation that depends on a particular measure. You may change a mode setting at any time and not interfere with pending calculations. Try the following keystrokes to see this in action.

| *Expression* | *Keystrokes* | *Display* |
|---|---|---|
| sin 45° | 2nd MODE ▼ ▼ ► ENTER<br>EXIT SIN 45 ENTER | .707106781187 |
| sin $\pi$° | SIN 2nd $\pi$ ENTER | .054803665149 |
| sin $\pi$ | 2nd MODE ▼ ▼ ENTER<br>EXIT SIN 2ND $\pi$ ENTER | 0 |
| sin 45 | SIN 45 ENTER | .850903524534 |
| sin $\frac{\pi}{6}$ | SIN ( 2nd $\pi$ ÷ 6 ) ENTER | .5 |

The first line of keystrokes sets the TI-86 in degree mode and calculates the sine of 45 *degrees*. While the calculator is still in degree mode, the second line of keystrokes calculates the sine of $\pi$ *degrees*, approximately 3.1415°. The third line changes to radian mode just before calculating the sine of $\pi$ *radians*. The fourth line calculates the sine of 45 *radians*. Finally, the fifth line calculates the sine of $\frac{\pi}{6}$ *radians* (the calculator remains in radian mode).

  

The TI-86 makes it possible to mix degrees and radians in a calculation. Execute these keystrokes to calculate $\tan 45° + \sin \frac{\pi}{6}$ as shown in Figure III.67. TAN 45 2nd MATH F3 *[ANGLE]* F1 *[°]* + SIN ( 2nd $\pi$ ÷ 6 ) F2 *[ r]* ENTER. Do you get 1.5 whether your calculator is set *either* in degree mode *or* in radian mode?


Figure III.67: Angle measure

***II.4.2 Graphs of Trigonometric Functions:*** When you graph a trigonometric function, you need to pay careful attention to the viewing window and to your angle measure configuration. For example, graph $y = \frac{\sin 30x}{30}$ in the standard viewing window in radian mode. Trace along the curve to see where it is. Zoom in to a better window, or use the period and amplitude to establish better WINDOW values.

***Technology Tip:*** Because $\pi \approx 3.1$, when in radian mode, set Xmin = 0 and Xmax = 6.3 to cover the interval from 0 to $2\pi$.

Next graph $y = \tan x$ in the standard window first, then press F3 MORE F3 *[ZTRIG]* to change to a special window for trigonometric functions in which the xScl is $\frac{\pi}{2} \approx 1.5708$ or 90° and the vertical range is from $-4$ to 4. The TI-86 plots consecutive points and then connects them with a segment, so the graph is not exactly what you should expect. You may wish to change from DrawLine to DrawDot (see Section III.2.3) when you plot the tangent function.

## III.5 Scatter Plots

***III.5.1 Entering Data:*** This table shows total prize money (in millions of dollars) awarded at the Indianapolis 500 race from 1995 to 2003. (*Source:* Indy Racing League)

| *Year* | *1995* | *1996* | *1997* | *1998* | *1999* | *2000* | *2001* | *2002* | *2003* |
|---|---|---|---|---|---|---|---|---|---|
| Prize (in millions) | \$8.06 | \$8.11 | \$8.61 | \$8.72 | \$9.05 | \$9.48 | \$9.61 | \$10.03 | \$10.15 |

We'll now use the TI-86 to construct a scatter plot that represents these points and to find a linear model that approximates the given data.

The TI-86 holds data in up to 20 *lists*. To enter the list editor either press 2nd STAT F2 *[EDIT]* or press 2nd LIST F4 *[EDIT]*. Before entering this new data, clear all data lists. This can be done in the list editor by highlighting each list title (xStat, yStat, etc.) and pressing CLEAR ENTER.

Now enter the data. Instead of entering the full year, let $x = 5$ represent 1995, $x = 6$ represent 1996, and so on. Here are the keystrokes for the first three years: 5 ENTER 6 ENTER 7 ENTER and so on, then press ▶ to move to the first element of the next list (yStat) and press 8.06 ENTER 8.11 ENTER 8.61 and so on (see Figure III.68). Press 2nd QUIT when you have finished.

  

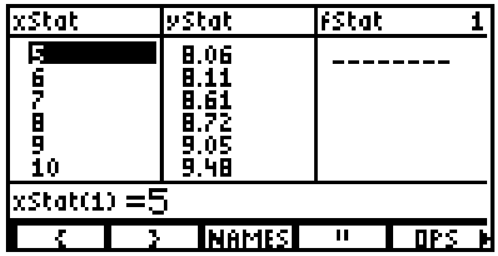


Figure III.68: Entering data points

You may edit statistical data in the same way you edit expressions in the home screen. Move the cursor to any value you wish to change, then type the correction. To insert or delete data, move the cursor over the data point you wish to add or delete. Press 2nd INS and a new data point is created; press DEL and the data point is deleted.

***III.5.2 Plotting Data:*** Once all the data points have been entered, press 2nd STAT F3 *[PLOT]* to display the STAT PLOTS screen. Press F1 *[PLOT1]* ENTER to turn PLOT1 on, select the other options shown in Figure III.69, and press GRAPH F5 *[GRAPH]*. (Make sure that you have cleared or turned off any functions in the y(x)= screen, or those functions will be graphed simultaneously.) Figure III.70 shows this plot in a window from 0 to 15 in both directions. You may now press TRACE to move from data point to data point.

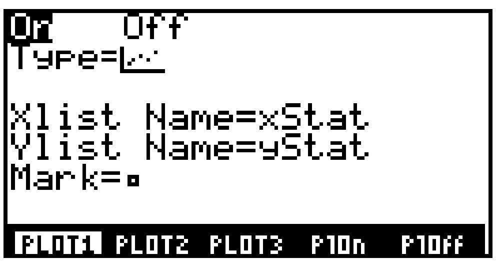


Figure III.69: Plot1 menu

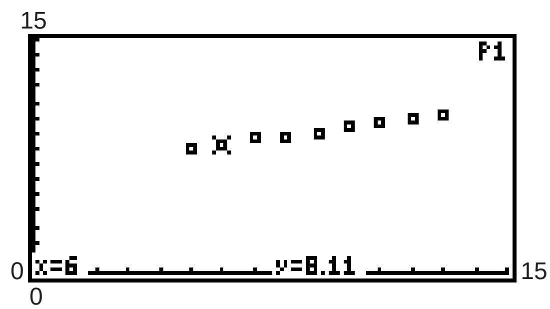


Figure III.70: Scatter plot

To draw the scatter plot in a window adjusted automatically to include all the data you entered, press GRAPH F3 MORE F5 *[ZDATA]* CLEAR.

When you no longer want to see the scatter plot, either enter the PLOT1 menu and select OFF or press GRAPH F1, move the cursor to the highlighted Plot1 at the top of the screen, and press ENTER. PLOT1 should no longer be highlighted. The TI-86 still retains all the data you entered.

***III.5.3 Regression Line:*** The TI-86 calculates slope and *y*-intercept for the line that best fits all the data. After the data points have been entered, enter 1 for each entry under the fStat column. Then quit the list editor and press 2nd STAT F1 *[CALC]* F3 *[LinR]* ENTER to calculate a linear regression model with the slope named b and the *y*-intercept named a (Figure III.71).

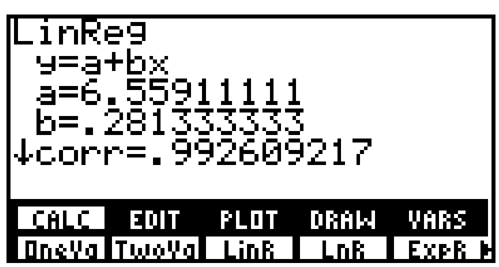


Figure III.71: Linear regression model

  

***Technology Tip:*** The number corr (between −1 and 1) is called the *correlation coefficient* and measures how well the linear regression equation fits the data. The closer |r| is to 1, the better the fit; the closer |r| is to 0, the worse the fit.

Turn PLOT1 on again, if it is not currently displayed. Graph the regression line $y = a + bx$ by entering the y(x)= editor, inactivating any existing functions, moving to a free line or clear one, and then pressing 2nd STAT F5 *[VARS]* MORE MORE F2 *[ReqEq]* ENTER GRAPH F5 CLEAR. See how well this line fits with your data points (Figure III.72).

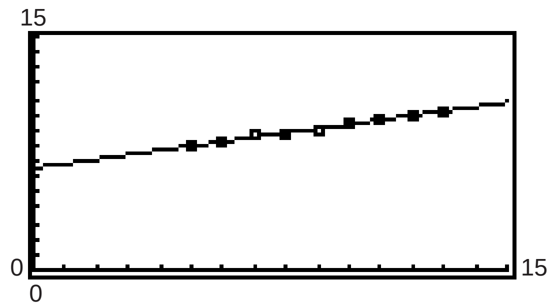


Figure III.72: Linear regression line

***III.5.4 Other Regression Models:*** After data points have been entered, you can choose from 10 different regression models. They are all located in the CALC sub-menu of the STAT menu.

## III.6 Matrices

***III.6.1 Making a Matrix:*** The TI-86 can display and use many different matrices, each with up to 255 rows and up to 255 columns! Here's how to store this 3 × 4 matrix $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & 4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$ in your calculator.

Press 2nd MATRX F2 *[EDIT]* to see the matrix edit menu (Figure III.73). You must first name the matrix; let's name this matrix A (the TI-86 is already set for alphabetic entry) and press ENTER to continue.

Change the dimensions of matrix A to 3 × 4 by pressing 3 ENTER 4 ENTER. Simply press ENTER or an arrow key to accept an existing dimension. The matrix shown in the window changes in size to reflect a changed dimension.

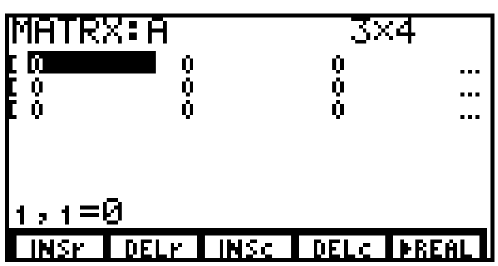


Figure III.73: Editing a matrix

  

Use the arrow keys or press ENTER repeatedly to move the cursor to a matrix element you want to change. If you press ENTER, you will move right across a row and then back to the first column of the next row. At the right edge of the screen in Figure III.73, there are dashes to indicate more columns than are shown. Go to them by pressing ▶ as many times as necessary. The ordered pair at the bottom left of the screen shows the cursor's current location within the matrix. The element in the second row and first column in Figure III.74 is highlighted, so the ordered pair at the bottom of the window is 2 , 1, and the screen shows that element's current value. Continue to enter all the elements of matrix A; press ENTER after inputting each value.

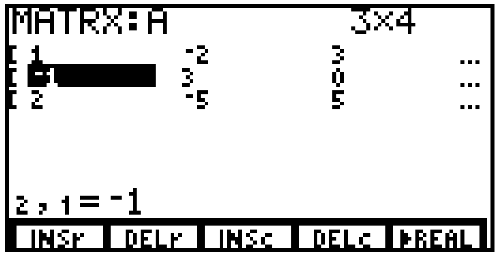


Figure III.74: Editing a matrix

When you are finished, leave the matrix editing screen by pressing 2nd QUIT to return to the home screen.

***II.6.2 Matrix Math:*** From the home screen, you can perform many calculations with matrices. To see matrix A, press ALPHA A ENTER (Figure III.75).

Perform the scalar multiplication 2A by pressing 2 ALPHA A ENTER. The resulting matrix is displayed on the screen. To create matrix B as 2A, press 2 ALPHA A STO ▶ B ENTER (see Figure III.76), or if you do this immediately after calculating 2A, press only STO ▶ B ENTER. Return to the matrix edit screen to verify that the dimensions and entries of matrix B have been changed automatically to reflect these new values.

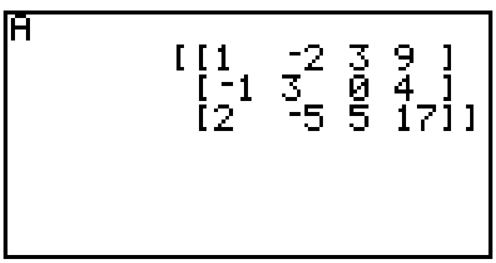


Figure III.75: Matrix A

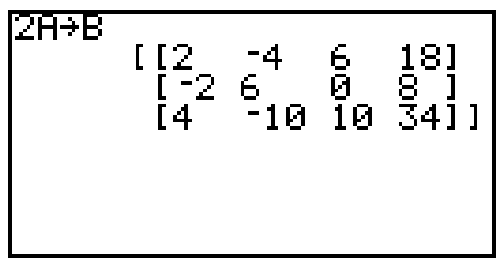


Figure III.76: Matrix B

To add two matrices, say A and B, create matrix B (with the same dimensions as A) and then press ALPHA A + ALPHA B ENTER. Subtraction is performed in a similar manner.

Now create a matrix C with dimensions of $2 \times 3$ and enter the matrix $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -5 & -1 \end{bmatrix}$ for C. For matrix multiplication of C by A, press ALPHA C × ALPHA A ENTER. If you tried to multiply A by C, your TI-86 would signal an error because the dimensions of the two matrices do not permit multiplication in this way.

***III.6.3 Row Operations:*** Here are the keystrokes necessary to perform elementary row operations on a matrix. Your textbook provides more careful explanation of the elementary row operations and their uses.

To interchange the second and third rows of the matrix A that was defined in Figure III.75, press 2nd MATRX F4 *[OPS]* MORE F2 *[rSwap]* ALPHA A , 2 , 3 ) ENTER (see Figure III.77). The format of this command is rSwap*(matrix, row1, row2)*.

To add row 2 and row 3 and *store* the results in row 3, press 2nd MATRX F4 MORE F3 *[rAdd]* ALPHA A , 2 , 3 ) ENTER. The format of this command is rAdd*(matrix, row1, row2)*.

  

To multiply row 2 by $-4$ and *store* the results in row 2, thereby replacing row 2 with new values, press 2nd MATRX F4 MORE F4 *[mulR]* (-) 4 , ALPHA A , 2 ) ENTER. The format of this command multR(*scalar, matrix, row*).

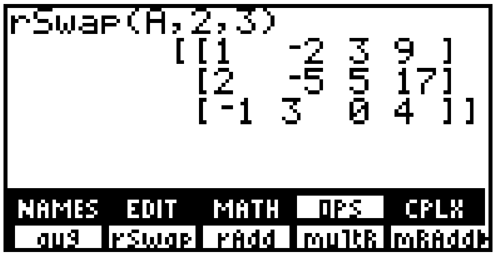


Figure III.77: Interchange rows 2 and 3

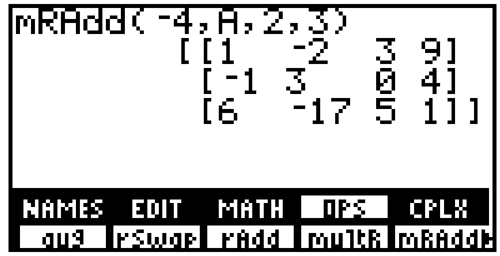


Figure III.78: Add $-4$ times row 2 to row 3

To multiply row 2 by $-4$ and *add* the results to row 3, thereby replacing row 3 with new values, press 2nd MATRX F4 MORE F5 *[mRAdd]* (-) 4 , ALPHA A , 2 , 3 ) ENTER (see Figure III.78). The format of this command is mRAdd(*scalar, matrix, row1, row2*).

***Technology Tip:*** Note that your TI-86 does *not* store a matrix obtained as the result of any row operations. So, when you need to perform several row operations in succession, it is a good idea to store the result of each one in a temporary place.

For example, use row operations to solve this system of linear equations: $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4. \\ 2x - 5y + 5z = 17 \end{cases}$

First enter this *augmented matrix* as A in your TI-86: $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$. Next store this matrix as C (press ALPHA A STO ▶ C ENTER) so you may keep the original in case you need to recall it.

Here are the row operations and their associated keystrokes. At each step, the result is stored in C and replaces the previous matrix C. The matrix in row-echelon form is shown in Figure III.79.

| *Row Operation* | *Keystrokes* |
|---|---|
| Add row 1 to row 2. | 2nd MATRX F4 MORE F3 ALPHA C , 1 , 2 ) STO ▶ C ENTER |
| Add $-2$ times row 1 to row 3. | F5 (-) 2 , ALPHA C , 1 , 3 ) STO ▶ C ENTER |
| Add row 2 to row 3. | F3 ALPHA C , 2 , 3 ) STO ▶ C ENTER |
| Multiply row 3 by $\frac{1}{2}$. | F4 1 ÷ 2 , ALPHA C , 3 ) STO ▶ C ENTER |

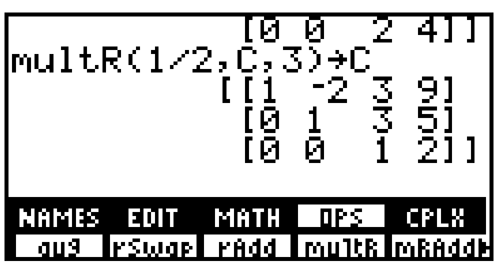


Figure III.79: Row-echelon form of matrix after row operations

So, $z = 2$, $y = -1$, and $x = 1$.

***Technology Tip:*** The TI-86 can produce a row-echelon form and the reduced row-echelon form of a matrix. The row-echelon form of matrix A is obtained by pressing 2nd MATRX F4 F4 *[ref]* ALPHA A ENTER (Figure III.80) and the reduced row-echelon form is obtained by pressing 2nd MATRX F4 F5 *[rref]* ALPHA A ENTER (Figure III.81). Note that the row-echelon form of a matrix is not unique, so your calculator may not get exactly the same matrix as you do by using row operations. However, the matrix that the TI-86 produces will result in the same solution to the system.

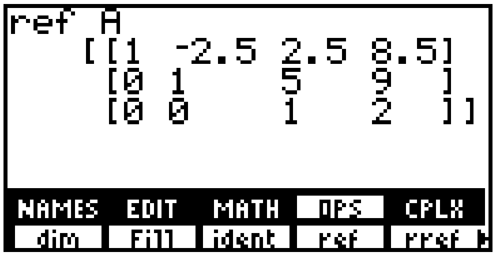


Figure III.80: Row-echelon form

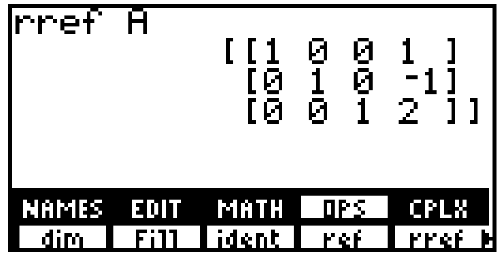


Figure III.81: Reduced row-echelon form

***III.6.4 Determinants and Inverses:*** Enter this $3 \times 3$ square matrix as A: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. To calculate its determinant $\begin{vmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{vmatrix}$, go to the home screen and press 2nd MATRX F3 F1 *[det]* ALPHA A ENTER. You should find that the determinant is 2 as shown in Figure III.82.

Because the determinant of the matrix is not zero, it has an inverse, $A^{-1}$. Press ALPHA A 2nd $x^{-1}$ ENTER to calculate the inverse of matrix A. The result is shown in Figure III.83.

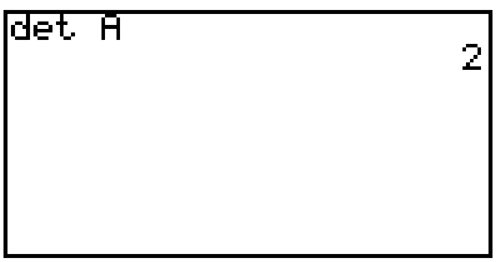


Figure III.82: Determinant of A

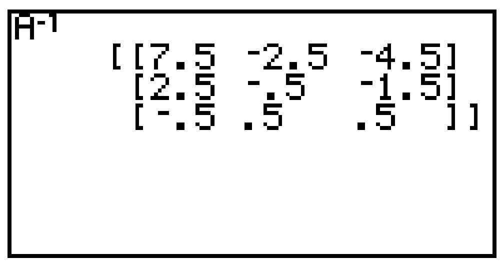


Figure III.83: Inverse of A

Now let's solve a system of linear equations by matrix inversion. Once more, consider $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$.

The coefficient matrix for this system is the matrix $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$, which was entered as matrix A in the previous example. Now enter the $3 \times 1$ matrix $\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$ as B. Then press ALPHA A 2nd $x^{-1}$ $\times$ ALPHA B ENTER to calculate the solution matrix (Figure III.84). The solution is still $x = 1$, $y = -1$, and $z = 2$.

  

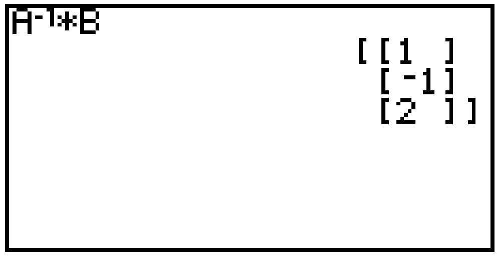


Figure III.84: Solution matrix

## III.7 Sequences

***II.7.1 Iteration with the ANS Key:*** The ANS feature permits you to perform *iteration*, the process of evaluating a function repeatedly. As an example, calculate $\frac{n-1}{3}$ for $n = 27$. Then calculate $\frac{n-1}{3}$ for $n =$ the answer to the previous calculation. Continue to use each answer as $n$ in the *next* calculation. Here are keystrokes to accomplish this iteration on TI-86 calculator, (see the results in Figure III.85). Notice that when you use ANS in place of $n$ in a formula, it is sufficient to press ENTER to continue an iteration.

| *Iteration* | *Keystrokes* | *Display* |
|---|---|---|
| 1 | 27 ENTER | 27 |
| 2 | ( 2nd ANS − 1 ) ÷ 3 ENTER | 8.66666666667 |
| 3 | ENTER | 2.55555555556 |
| 4 | ENTER | .518518518519 |
| 5 | ENTER | -.16049382716 |

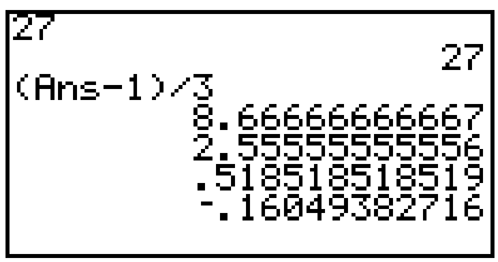


Figure III.85: Iteration

Press ENTER several more times and see what happens with this iteration. You may wish to try it again with a different starting value.

***III.7.2 Terms of Sequences:*** Another way to display the terms of a sequence is to enter the sequence and the number of terms you want listed. For example, to find the first five terms of the sequence $u_n = -n + 4$, press 2nd MATH F5 *[MISC]* F3 *[seq]* (-) ALPHA N + 4 , ALPHA N, 1, 5, 1 ) ENTER (see figure III.86). The format of this command is seq(*expression*, *variable*, *begin*, *end*, *increment* ).

  

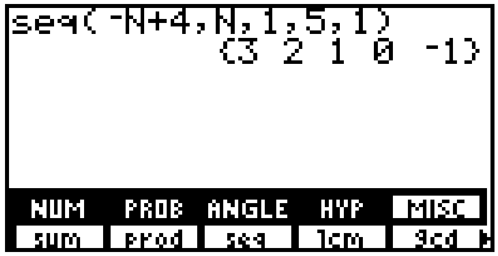


Figure III.86: Terms of sequence $u_n = -n + 4$

***III.7.3 Arithmetic and Geometric Sequences:*** Use iteration with the ANS variable to determine the $n$th term of a sequence. For example, find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Enter the first term 7, then start the progression with the recursion formula, 2nd ANS + 4 ENTER. This yields the 2nd term, so press ENTER sixteen more times to find the 18th term, 75. For a *geometric* sequence whose common ratio is 4, start the progression with 2nd ANS × 4 ENTER.

Of course, you could use the *explicit* formula for the $n$th term of an arithmetic sequence, $t_n = a + (n - 1)d$. First enter values for the variables $a$, $d$, and $n$, then evaluate the formula by pressing 2nd alpha A + ( 2nd alpha N − 1 ) 2nd alpha D ENTER. For a geometric sequence whose $n$th term is given by $t_n = a \cdot r^{n-1}$, enter values for the variables $a$, $r$, and $n$, then evaluate the formula by pressing 2nd alpha A 2nd alpha R ^ ( 2nd alpha N − 1 ) ENTER.

***III.7.4 Finding Sums and Partial Sums of Sequences:*** You can find the sum of a sequence by combining the sum command with the seq command, both found in the MATH MISC menu or the LIST OPS menu. The format of the sum( command is sum(*list*, *start*, *end)*, where the optional arguments *start* and *end* determine which elements of *list* are summed. The format of the seq( command is seq(*expression*, *variable*, *begin*, *end*, *increment*), where the optional argument *increment* indicates the difference between successive points at which *expression* is evaluated. The default *increment* on a TI-86 is 1.

For example, suppose you want to find the sum $\sum_{n=1}^{12} 4(0.3)^n$. Press 2nd MATH F5 F1 *[sum]* F3 4 ( . 3 ) ^ ALPHA N , ALPHA N , 1 , 12 ) ENTER (Figure III.87). Note that the sum command does not need a starting or ending point, because every term in the sequence is being summed. Also, any letter can be used for the variable in the sum, i.e., the N could just have easily been an A or a K.

Now calculate the sum starting at $n = 0$ by using 2nd ENTRY to edit the range. Your calculator should display a sum of approximately 5.71428480324.

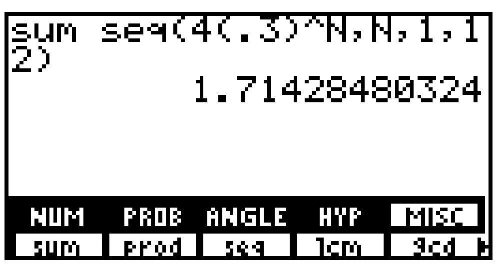


Figure III.87: $\sum_{n=1}^{12} 4(0.3)^n$

The seq( feature can also be combined with the cSum( feature to find partial sums of a series. The format of the cSum( command is cSum(*list*).

  

For example, suppose you want to find the first four partial sums of the series $\sum_{n=1}^{4} 3^{n+1}$. Press 2nd LIST F5 *[OPS]* MORE MORE F3 *[cSum]* 2nd MATH F5 F3 3 ^ ( ALPHA N + 1 ) , ALPHA N , 1 , 4 ) ) ENTER (Figure III.88).

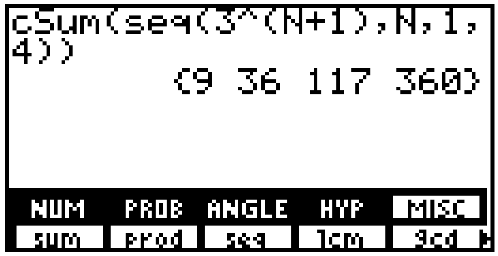


Figure III.88: Partial sums of $\sum_{n=1}^{4} 3^{n+1}$

## III.8 Parametric and Polar Graphs

***III.8.1 Graphing Parametric Equations:*** The TI-86 plots up to 99 pairs of parametric equations as easily as it plots functions. In the MODE menu (Figure III.1), go to the fifth line from the top, and change the setting to Param. Be sure, if the independent parameter is an angle measure, that the angle measure in the MODE menu is set to whichever you need, Radian or Degree.

For example, here are the keystrokes needed to graph the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$. First check that angles are currently being measured in radians and change to parametric mode. Then press GRAPH F1 *[E(t)=]* to enter the parametric equation menu. Then enter the two parametric equations by pressing ( COS F1 *[t]* ^ 3 ENTER ( SIN F1 *[t]* ^ 3 ENTER (Figure III.89). Note that when you press F1, you get a t because the calculator is in parametric mode.


Figure III.89: Parametric E(t)= menu

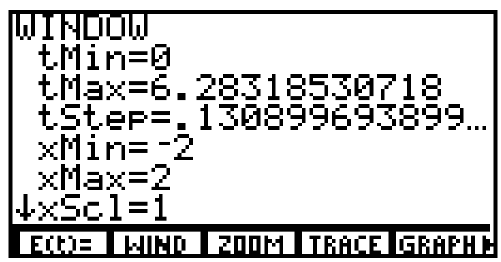


Figure III.90: Parametric WINDOW menu

Press 2nd M2 *[WIND]* to set the graphing window and to initialize the values of t. In the standard window, the values of t go from 0 to $2\pi$ in steps of $\frac{\pi}{24} \approx 0.1309$, with the view from $-10$ to 10 in both directions. In order to provide a better viewing window, press ENTER three times to move the cursor down, then set the window to extend from $-2$ to 2 in both directions (Figure III.90). Press F5 CLEAR to see the parametric graph (Figure III.91).

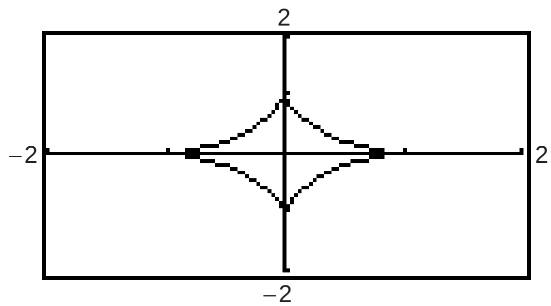


Figure III.91: Parametric graph of $x = \cos^3 t$ and $y = \sin^3 t$

You may ZOOM and TRACE along parametric graphs just as you did with function graphs. However, unlike with function graphs, the cursor will not move to values outside of the t range, so the left arrow ◀ will not work when $t = 0$, and the right arrow ▶ will not work when $t = 2\pi$. As you trace along this graph, notice that the cursor moves in the *counterclockwise* direction as t increases.

***II.8.2 Rectangular-Polar Coordinate Conversion:*** The CPLX menu (Figure III.86) provides function for converting between rectangular and polar coordinate systems. These functions use the current angle measure setting, so it is a good idea to check the default angle measure before any conversion. Of course, you may override the current angle measure setting, as explained in Section III.4.1. For the following examples, the TI-86 is set to radian measure.

Given the rectangular coordinates $(x, y) = (4, -3)$, convert *from* these rectangular coordinates *to* polar coordinates $(r, \theta)$ by pressing 2nd CPLX MORE ( 4 , (-) 3 ) F2 *[ ▶Pol]* ENTER. The values of $r$ and $\theta$ are displayed (Figure III.92).

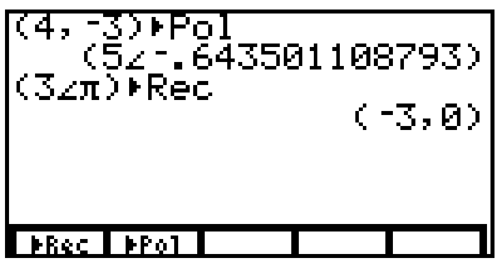


Figure III.92: Coordinate conversion

Suppose $(r, \theta) = (3, \pi)$. To convert *from* these polar coordinates *to* rectangular coordinates $(x, y)$, press 2nd CPLX MORE ( 3 2nd ∠ 2nd $\pi$ ) F1 *[ ▶Rec]* ENTER. The $x$- and $y$-coordinates are displayed (Figure III.92).

***III.8.3 Graphing Polar Equations:*** The TI-86 graphs polar functions in the form $r = f(\theta)$. In the fifth line of the MODE menu, select Pol for polar graphs. You may now graph up to 99 polar functions at a time. Be sure that the angle measure has been set to whichever you need, Radian or Degree. Here we will use radian measure.

For example, to graph $r = 4 \sin \theta$, press Graph F1 *[r(θ)=]* for the polar graph editing screen. Then enter the expression $4 \sin \theta$ for r1 by pressing 4 SIN F1 *[(θ)]*. Note that when you press F1, you get a $\theta$ because the calculator is in polar mode (see Figure III.93). Choose a good viewing window and an appropriate interval and increment for $\theta$. In Figure III.94, the viewing window is roughly "square" and extends from $-6.5$ to 6.5 horizontally and from $-4$ to 4 vertically.

  


Figure III.93: Polar r(θ)= menu

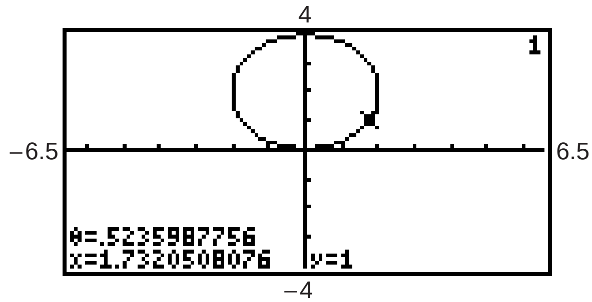


Figure III.94: Polar graph of $r = 4 \sin \theta$

Figure III.94 shows *rectangular* coordinates of the cursor's location on the graph. You may sometimes wish to trace along the curve and see *polar* coordinates of the cursor's location. The first line of the GRAPH FORMT menu (Figure III.21) has options for displaying the cursor's position in rectangular (RectGC) or polar (PolarGC) form.

## III.9 Probability and Statistics

***III.9.1 Random Numbers:*** The command rand generates a number between 0 and 1. You will find this command in the PROB (probability) sub-menu of the MATH menu. Press 2nd MATH F2 *[PROB]* F4 *[rand]* ENTER to generate a random number. Press ENTER to generate another number; keep pressing ENTER to generate more of them.

If you need a random number between, say, 0 and 10, then press 10 2nd MATH F2 F4 ENTER. To get a random number between 5 and 15, press 5 + 10 2nd MATH F2 F4 ENTER.

***III.9.2 Permutations and Combinations:*** To calculate the number of permutations of 12 objects taken 7 at a time, ${}_{12}P_7$, press 2nd MATH F2 12 F2 *[nPr]* 7 ENTER. So, ${}_{12}P_7 = 3{,}991{,}680$, as shown in Figure III.95.

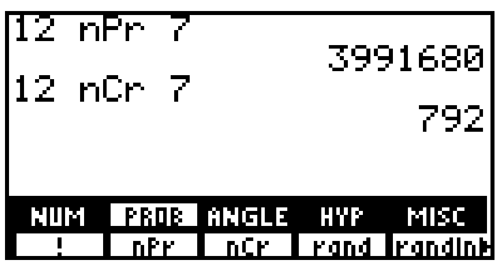


Figure III.95: ${}_{12}P_7$ and ${}_{12}C_7$

For the number of combinations of 12 objects taken 7 at a time, ${}_{12}C_7$, press 2nd MATH F2 12 F3 *[nCr]* 7 ENTER. So, ${}_{12}C_7 = 792$, as shown in Figure III.95. Note that you do not have to press 2nd MATH F2 again if the MATH PROB menu is active.

***III.9.3 Probability of Winning:*** A state lottery is configured so that each player chooses six different numbers from 1 to 40. If these six numbers match the six numbers drawn by the State Lottery Commission, the player wins the top prize. There are ${}_{40}C_6$ ways for the six numbers to be drawn. If you purchase a single lottery ticket, your probability of winning is 1 in ${}_{40}C_6$. Press 1 ÷ 40 2nd MATH F2 F3 6 ENTER to calculate your chances, but don't be disappointed.

***III.9.4 Sum of Data:*** The following data are a student's scores on 8 quizzes and 2 tests throughout an algebra course.

25, 20, 18, 89, 17, 24, 23, 22, 25, 93

To find the total points earned by the student, first enter the data using the TI-86's list editor, as shown in Figure III.96. Then press 2nd MATH F5 F1 2nd LIST F3 *[NAMES]* F2 *[xStat]* ENTER. From Figure III.97, the student earned 356 points throughout the algebra course.

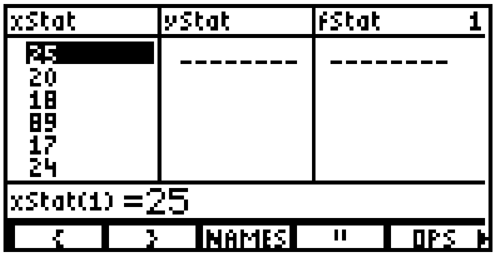


Figure III.96: List editor

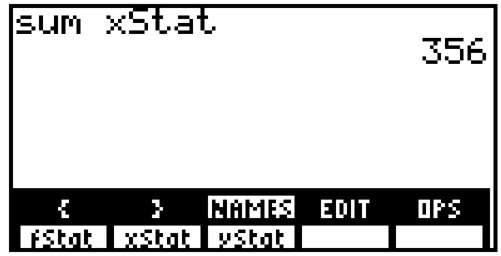


Figure III.97: Sum

***III.9.5 Statistics:*** The following data are the high temperatures (in degrees Fahrenheit) recorded in Lincoln, Nebraska from October 1, 2003 to October 12, 2003 (*Source:* University of Nebraska-Lincoln).

65, 68, 74, 79, 83, 81, 80, 80, 79, 72, 67, 71

To find the mean and median of these temperatures, use the OneVa command (one-variable statistics). First enter the data using the TI-86's list editor, as shown in Figure III.98.

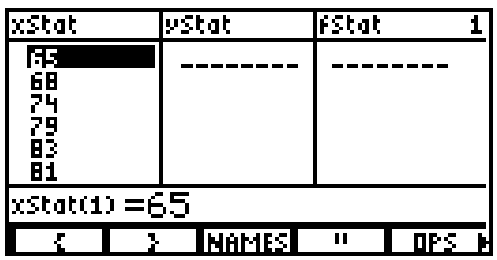


Figure III.98: List editor

Next, press 2nd STAT F1 F1 *[OneVa]* 2nd LIST F3 F2 ENTER EXIT EXIT. You should obtain a list of several different statistical values. The first line represents the mean of the data which is approximately 75°F (see Figure III.99). The second line is the sum of the data, the third line is the sum of the squares of the data, the fourth line is the sample standard deviation of the data, the fifth line is the population standard deviation of the data, the sixth line is the number of data values, the seventh line is the minimum value of the data, the eighth line is the first quartile of the data, and the ninth line is the median of the data which is 76.5°F (see Figure III.100). The tenth line is the third quartile of the data and the eleventh line is the maximum value of the data.

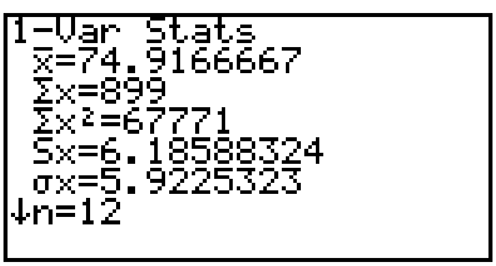


Figure III.99: OneVa

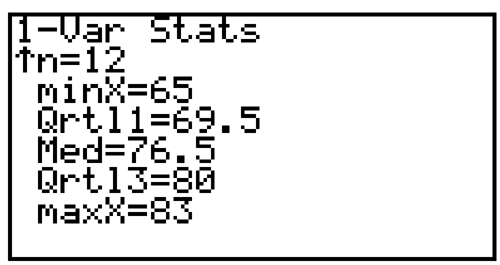


Figure III.100: OneVa

You can scroll through the list of statistical values by pressing ▲ or ▼.

  

## III.10 Programming

***III.10.1 Entering a Program:*** The TI-86 is a programmable calculator that can store sequences of commands for later replay. Press PRGM to access the programming menu. The TI-86 has space for many programs, each called by a title you give it. The title should be descriptive and can be eight characters, letters, or numerals long (but the first character must be a letter or $\theta$).

In the program, each line begins with a colon : supplied automatically by the calculator. Any command you could enter directly in the TI-86's home screen can be entered as a line in a program. There are also special programming commands.

You may interrupt programming input at any stage by pressing 2nd QUIT. To return later for more editing, press PRGM F2 *[EDIT]* and enter its name. The names of the available programs are also displayed alphabetically at the bottom of the screen; press MORE to advance through the listing. Then press the function key corresponding to the name to select the program.

You may remove a program from memory by pressing 2nd MEM F2 *[DELET]* MORE F5 *[PRGM]*. Then move the cursor to the program's name and press ENTER to delete the entire program.

***III.10.2 Executing a Program:*** To execute a program you entered, press PRGM F1 *[NAMES].* The names of the available programs are displayed alphabetically at the bottom of the screen; press MORE to advance through the listing. Press the function key corresponding to the name to select the program, and press ENTER to execute it.

If you need to interrupt a program during execution, press ON.

The instruction manual for your TI-86 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.