

Chapter 2 Project ► Finding Points of Intersection Graphically

Example ► Approximating Points of Intersection

Approximate the points of intersection of the circle and parabola given by

$$x^2 + y^2 - 3x + 5y - 11 = 0 \quad \text{and} \quad y = x^2 - 4x + 5$$

using the *intersect* feature of a graphing utility.

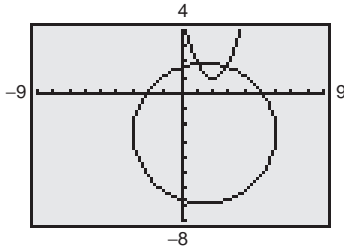
Solution

Begin by writing the circle as the union of two functions.

$$y = \frac{1}{2}(-5 + \sqrt{69 + 12x - 4x^2}) \quad \text{Top half of circle}$$

$$y = \frac{1}{2}(-5 - \sqrt{69 + 12x - 4x^2}) \quad \text{Bottom half of circle}$$

$$y = x^2 - 4x + 5 \quad \text{Parabola}$$



Next, graph all three functions in the same viewing window, as shown in the figure at the left. Using the *intersect* feature of the graphing utility, you can estimate that the points of intersection are approximately (1.1, 1.9) and (2.8, 1.7).



Chapter Project Investigations

- Using a setting of $1.05 \leq x \leq 1.06$ and $1.89 \leq y \leq 1.90$, graph the top half of the circle and the parabola in the example in the same viewing window. Then use the *intersect* feature to approximate (accurate to three decimal places) the y -coordinate of the point of intersection that is shown on the screen.
- Another method for finding the points of intersection is to substitute $x^2 - 4x + 5$ for y in the equation of the circle to get a fourth-degree polynomial equation. Graph this polynomial function.
 - Find a setting that allows you to approximate the solution $x \approx 1.055$ of the polynomial equation to two more decimal places.
 - Find a setting that allows you to approximate the solution $x \approx 2.841$ to two more decimal places.
- Use a graphing utility to find the points of intersection of the circle and the parabola given by $x^2 + y^2 - 5x + 4y - 13 = 0$ and $y = x^2 - 3x + 2$.
- The *market equilibrium* of a commodity is the quantity (and corresponding price) at which the supply of the commodity and the demand for the commodity are equal. The supply and demand curves for a business dealing with wheat are $p = 1.45 + 0.00014x^2$ and $p = (2.388 - 0.007x)^2$, respectively, where p is the price (in dollars per bushel) and x is the quantity (in bushels per day). Use a graphing utility to graph the supply and demand equations and find the market equilibrium. (*Hint*: The market equilibrium is the point of intersection of the graphs for $x > 0$.)