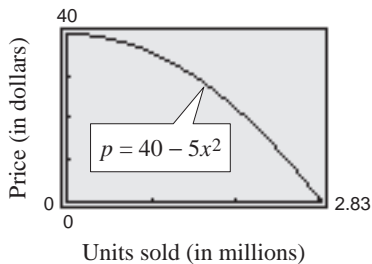


Chapter 1 Project ► A Graphical Approach to Maximization



In business, a **demand function** gives the price per unit p in terms of the number of units sold x . The demand function whose graph is shown at the left is

$$p = 40 - 5x^2, \quad 0 \leq x \leq \sqrt{8} \quad \text{Demand function}$$

where x is measured in millions of units. Note that as the price decreases, the number of units sold increases. The **revenue** R (in millions of dollars) is determined by multiplying the number of units sold by the price per unit. So,

$$R = xp = x(40 - 5x^2), \quad 0 \leq x \leq \sqrt{8}. \quad \text{Revenue function}$$

Example ► Finding the Maximum Revenue

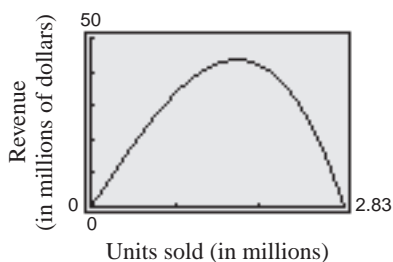


Use a graphing utility to graph the revenue function $R = 40x - 5x^3$ for $0 \leq x \leq \sqrt{8}$. How many units should be sold to obtain a maximum revenue? What price per unit should be charged to obtain a maximum revenue?

Solution

To begin, you need to determine a viewing window that will display the part of the graph that is important to this problem. The domain is given, so you can set the x -boundaries of the graph between 0 and $\sqrt{8}$. To determine the y -boundaries, however, you need to experiment a little. After calculating several values of R , you could decide to use y -boundaries between 0 and 50, as shown in the graph at the left. Next, you can use the *trace* feature to find that the maximum revenue of about \$43.5 million occurs when x is approximately 1.63 million units. To find the price per unit that corresponds to this maximum revenue, you can substitute $x = 1.63$ into the demand function to obtain

$$p = 40 - 5(1.63)^2 \approx \$26.72.$$



Chapter Project Investigations

1. For the demand function $p = 40 - 5x^2$, match each of the points $(0, 40)$ and $(\sqrt{8}, 0)$ with statement (a) or (b). Explain your reasoning.

- (a) No one will buy the product at this price.
- (b) You cannot give more than this number away.



2. Use a graphing utility to zoom in on the maximum point of the revenue function in the example. (Use a setting of $1.62 \leq x \leq 1.65$ and $43.5 \leq y \leq 43.6$.) Use the *trace* feature to improve the accuracy of the approximation obtained in the example. Do you think this improved accuracy is appropriate in the context of this particular problem? Does it change the price?



3. For the revenue function discussed in the example, the cost of producing each unit is \$15, so the total cost of producing x million units is $C = 15x$. Use a graphing utility to graph the profit function $P = R - C$ to determine how many units should be sold to obtain a maximum profit. What price per unit should be charged to obtain a maximum profit?