

A.6 Linear Inequalities in One Variable

What you should learn

- Represent solutions of linear inequalities in one variable.
- Solve linear inequalities in one variable.
- Solve inequalities involving absolute values.
- Use inequalities to model and solve real-life problems.

Why you should learn it

Inequalities can be used to model and solve real-life problems. For instance, in Exercise 101 on page A68, you will use a linear inequality to analyze the average salary for elementary school teachers.

Introduction

Simple inequalities were discussed in Appendix A.1. There, you used the inequality symbols $<$, \leq , $>$, and \geq to compare two numbers and to denote subsets of real numbers. For instance, the simple inequality

$$x \geq 3$$

denotes all real numbers x that are greater than or equal to 3.

Now, you will expand your work with inequalities to include more involved statements such as

$$5x - 7 < 3x + 9$$

and

$$-3 \leq 6x - 1 < 3.$$

As with an equation, you **solve an inequality** in the variable x by finding all values of x for which the inequality is true. Such values are **solutions** and are said to **satisfy** the inequality. The set of all real numbers that are solutions of an inequality is the **solution set** of the inequality. For instance, the solution set of

$$x + 1 < 4$$

is all real numbers that are less than 3.

The set of all points on the real number line that represent the solution set is the **graph of the inequality**. Graphs of many types of inequalities consist of intervals on the real number line. See Appendix A.1 to review the nine basic types of intervals on the real number line. Note that each type of interval can be classified as *bounded* or *unbounded*.

Example 1 Intervals and Inequalities

Write an inequality to represent each interval, and state whether the interval is bounded or unbounded.

- $(-3, 5]$
- $(-3, \infty)$
- $[0, 2]$
- $(-\infty, \infty)$

Solution

- $(-3, 5]$ corresponds to $-3 < x \leq 5$. Bounded
- $(-3, \infty)$ corresponds to $-3 < x$. Unbounded
- $[0, 2]$ corresponds to $0 \leq x \leq 2$. Bounded
- $(-\infty, \infty)$ corresponds to $-\infty < x < \infty$. Unbounded

 **CHECKPOINT** Now try Exercise 1.

Properties of Inequalities

The procedures for solving linear inequalities in one variable are much like those for solving linear equations. To isolate the variable, you can make use of the **Properties of Inequalities**. These properties are similar to the properties of equality, but there are two important exceptions. When each side of an inequality is multiplied or divided by a negative number, the direction of the inequality symbol must be reversed. Here is an example.

$$\begin{array}{ll} -2 < 5 & \text{Original inequality} \\ (-3)(-2) > (-3)(5) & \text{Multiply each side by } -3 \text{ and reverse inequality.} \\ 6 > -15 & \text{Simplify.} \end{array}$$

Notice that if the inequality was not reversed you would obtain the false statement $6 < -15$.

Two inequalities that have the same solution set are **equivalent**. For instance, the inequalities

$$x + 2 < 5$$

and

$$x < 3$$

are equivalent. To obtain the second inequality from the first, you can subtract 2 from each side of the inequality. The following list describes the operations that can be used to create equivalent inequalities.

Properties of Inequalities

Let a , b , c , and d be real numbers.

1. Transitive Property

$$a < b \text{ and } b < c \quad \Rightarrow \quad a < c$$

2. Addition of Inequalities

$$a < b \text{ and } c < d \quad \Rightarrow \quad a + c < b + d$$

3. Addition of a Constant

$$a < b \quad \Rightarrow \quad a + c < b + c$$

4. Multiplication by a Constant

$$\text{For } c > 0, a < b \quad \Rightarrow \quad ac < bc$$

$$\text{For } c < 0, a < b \quad \Rightarrow \quad ac > bc \quad \text{Reverse the inequality.}$$

Each of the properties above is true if the symbol $<$ is replaced by \leq and the symbol $>$ is replaced by \geq . For instance, another form of the multiplication property would be as follows.

$$\text{For } c > 0, a \leq b \quad \Rightarrow \quad ac \leq bc$$

$$\text{For } c < 0, a \leq b \quad \Rightarrow \quad ac \geq bc$$

Solving a Linear Inequality in One Variable

The simplest type of inequality is a **linear inequality** in one variable. For instance, $2x + 3 > 4$ is a linear inequality in x .

In the following examples, pay special attention to the steps in which the inequality symbol is reversed. Remember that when you multiply or divide by a negative number, you must reverse the inequality symbol.

Example 2 Solving Linear Inequalities

Solve each inequality.

a. $5x - 7 > 3x + 9$

b. $1 - \frac{3x}{2} \geq x - 4$

STUDY TIP

Checking the solution set of an inequality is not as simple as checking the solutions of an equation. You can, however, get an indication of the validity of a solution set by substituting a few convenient values of x .

Solution

a. $5x - 7 > 3x + 9$

Write original inequality.

$$2x - 7 > 9$$

Subtract $3x$ from each side.

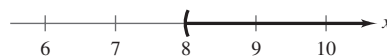
$$2x > 16$$

Add 7 to each side.

$$x > 8$$

Divide each side by 2.

The solution set is all real numbers that are greater than 8, which is denoted by $(8, \infty)$. The graph of this solution set is shown in Figure A.8. Note that a parenthesis at 8 on the real number line indicates that 8 is *not* part of the solution set.



Solution interval: $(8, \infty)$

FIGURE A.8

b. $1 - \frac{3x}{2} \geq x - 4$

Write original inequality.

$$2 - 3x \geq 2x - 8$$

Multiply each side by 2.

$$2 - 5x \geq -8$$

Subtract $2x$ from each side.

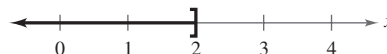
$$-5x \geq -10$$

Subtract 2 from each side.

$$x \leq 2$$

Divide each side by -5 and reverse the inequality.

The solution set is all real numbers that are less than or equal to 2, which is denoted by $(-\infty, 2]$. The graph of this solution set is shown in Figure A.9. Note that a bracket at 2 on the real number line indicates that 2 is part of the solution set.



Solution interval: $(-\infty, 2]$

FIGURE A.9



Now try Exercise 25.

Sometimes it is possible to write two inequalities as a **double inequality**. For instance, you can write the two inequalities $-4 \leq 5x - 2$ and $5x - 2 < 7$ more simply as

$$-4 \leq 5x - 2 < 7. \quad \text{Double inequality}$$

This form allows you to solve the two inequalities together, as demonstrated in Example 3.

Example 3 Solving a Double Inequality

To solve a double inequality, you can isolate x as the middle term.

$$-3 \leq 6x - 1 < 3 \quad \text{Original inequality}$$

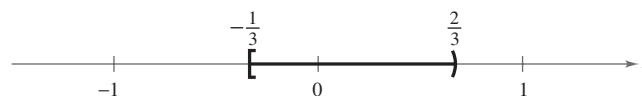
$$-3 + 1 \leq 6x - 1 + 1 < 3 + 1 \quad \text{Add 1 to each part.}$$

$$-2 \leq 6x < 4 \quad \text{Simplify.}$$

$$\frac{-2}{6} \leq \frac{6x}{6} < \frac{4}{6} \quad \text{Divide each part by 6.}$$

$$-\frac{1}{3} \leq x < \frac{2}{3} \quad \text{Simplify.}$$

The solution set is all real numbers that are greater than or equal to $-\frac{1}{3}$ and less than $\frac{2}{3}$, which is denoted by $[-\frac{1}{3}, \frac{2}{3})$. The graph of this solution set is shown in Figure A.10.



Solution interval: $[-\frac{1}{3}, \frac{2}{3})$

FIGURE A.10

CHECKPOINT Now try Exercise 37.

The double inequality in Example 3 could have been solved in two parts as follows.

$$-3 \leq 6x - 1 \quad \text{and} \quad 6x - 1 < 3$$

$$-2 \leq 6x \quad 6x < 4$$

$$-\frac{1}{3} \leq x \quad x < \frac{2}{3}$$

The solution set consists of all real numbers that satisfy *both* inequalities. In other words, the solution set is the set of all values of x for which

$$-\frac{1}{3} \leq x < \frac{2}{3}.$$

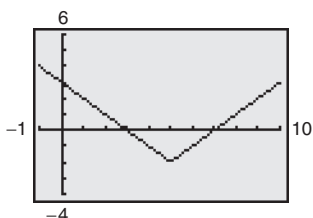
When combining two inequalities to form a double inequality, be sure that the inequalities satisfy the Transitive Property. For instance, it is *incorrect* to combine the inequalities $3 < x$ and $x \leq -1$ as $3 < x \leq -1$. This “inequality” is wrong because 3 is not less than -1 .

Technology

A graphing utility can be used to identify the solution set of the graph of an inequality. For instance, to find the solution set of $|x - 5| < 2$ (see Example 4), rewrite the inequality as $|x - 5| - 2 < 0$, enter

$$Y1 = \text{abs}(X - 5) - 2,$$

and press the *graph* key. The graph should look like the one shown below.



Notice that the graph is below the x-axis on the interval (3, 7).

Inequalities Involving Absolute Values

Solving an Absolute Value Inequality

Let x be a variable or an algebraic expression and let a be a real number such that $a \geq 0$.

- The solutions of $|x| < a$ are all values of x that lie between $-a$ and a .

$$|x| < a \quad \text{if and only if} \quad -a < x < a. \quad \text{Double inequality}$$

- The solutions of $|x| > a$ are all values of x that are less than $-a$ or greater than a .

$$|x| > a \quad \text{if and only if} \quad x < -a \quad \text{or} \quad x > a. \quad \text{Compound inequality}$$

These rules are also valid if $<$ is replaced by \leq and $>$ is replaced by \geq .

Example 4 Solving an Absolute Value Inequality

Solve each inequality.

- a. $|x - 5| < 2$ b. $|x + 3| \geq 7$

Solution

- a. $|x - 5| < 2$

Write original inequality.

$$-2 < x - 5 < 2$$

Write equivalent inequalities.

$$-2 + 5 < x - 5 + 5 < 2 + 5$$

Add 5 to each part.

$$3 < x < 7$$

Simplify.

The solution set is all real numbers that are greater than 3 and less than 7, which is denoted by (3, 7). The graph of this solution set is shown in Figure A.11.

- b. $|x + 3| \geq 7$

Write original inequality.

$$x + 3 \leq -7 \quad \text{or} \quad x + 3 \geq 7$$

Write equivalent inequalities.

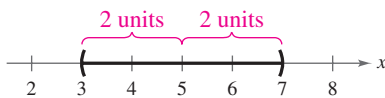
$$x + 3 - 3 \leq -7 - 3 \quad x + 3 - 3 \geq 7 - 3$$

Subtract 3 from each side.

$$x \leq -10 \quad \quad \quad x \geq 4$$

Simplify.

The solution set is all real numbers that are less than or equal to -10 or greater than or equal to 4 . The interval notation for this solution set is $(-\infty, -10] \cup [4, \infty)$. The symbol \cup is called a *union* symbol and is used to denote the combining of two sets. The graph of this solution set is shown in Figure A.12.



$|x - 5| < 2$: Solutions lie inside (3, 7)

FIGURE A.11



$|x + 3| \geq 7$: Solutions lie outside $(-10, 4)$

FIGURE A.12

STUDY TIP

Note that the graph of the inequality $|x - 5| < 2$ can be described as all real numbers *within* two units of 5, as shown in Figure A.11.

CHECKPOINT Now try Exercise 49.

Applications

A problem-solving plan can be used to model and solve real-life problems that involve inequalities, as illustrated in Example 5.

Example 5 Comparative Shopping



You are choosing between two different cell phone plans. Plan A costs \$49.99 per month for 500 minutes plus \$0.40 for each additional minute. Plan B costs \$45.99 per month for 500 minutes plus \$0.45 for each additional minute. How many *additional* minutes must you use in one month for plan B to cost more than plan A?

Solution

Verbal

Model:

$$\text{Monthly cost for plan B} > \text{Monthly cost for plan A}$$

Labels: Minutes used (over 500) in one month = m (minutes)
 Monthly cost for plan A = $0.40m + 49.99$ (dollars)
 Monthly cost for plan B = $0.45m + 45.99$ (dollars)

$$\text{Inequality: } 0.45m + 45.99 > 0.40m + 49.99$$

$$0.05m > 4$$

$$m > 80 \text{ minutes}$$

Plan B costs more if you use more than 80 additional minutes in one month.



CHECKPOINT Now try Exercise 91.

Example 6 Accuracy of a Measurement



You go to a candy store to buy chocolates that cost \$9.89 per pound. The scale that is used in the store has a state seal of approval that indicates the scale is accurate to within half an ounce (or $\frac{1}{32}$ of a pound). According to the scale, your purchase weighs one-half pound and costs \$4.95. How much might you have been undercharged or overcharged as a result of inaccuracy in the scale?

Solution

Let x represent the *true* weight of the candy. Because the scale is accurate to within half an ounce (or $\frac{1}{32}$ of a pound), the difference between the exact weight (x) and the scale weight ($\frac{1}{2}$) is less than or equal to $\frac{1}{32}$ of a pound. That is, $|x - \frac{1}{2}| \leq \frac{1}{32}$. You can solve this inequality as follows.

$$-\frac{1}{32} \leq x - \frac{1}{2} \leq \frac{1}{32}$$

$$\frac{15}{32} \leq x \leq \frac{17}{32}$$

$$0.46875 \leq x \leq 0.53125$$

In other words, your “one-half pound” of candy could have weighed as little as 0.46875 pound (which would have cost \$4.64) or as much as 0.53125 pound (which would have cost \$5.25). So, you could have been overcharged by as much as \$0.31 or undercharged by as much as \$0.30.



CHECKPOINT Now try Exercise 105.

A.6 Exercises

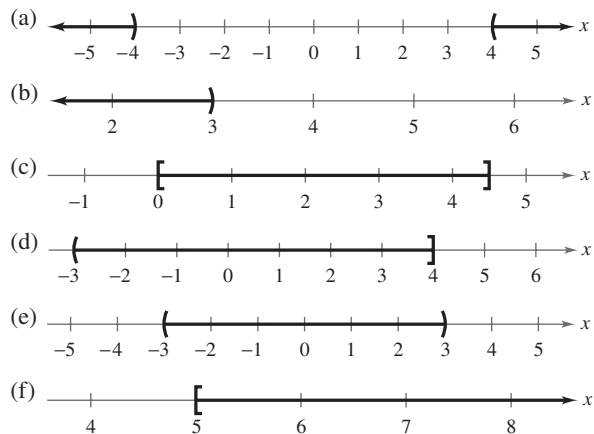
VOCABULARY CHECK: Fill in the blanks.

- The set of all real numbers that are solutions to an inequality is the _____ of the inequality.
- The set of all points on the real number line that represent the solution set of an inequality is the _____ of the inequality.
- To solve a linear inequality in one variable, you can use the properties of inequalities, which are identical to those used to solve equations, with the exception of multiplying or dividing each side by a _____ number.
- Two inequalities that have the same solution set are _____.
- It is sometimes possible to write two inequalities as one inequality, called a _____ inequality.
- The symbol \cup is called a _____ symbol and is used to denote the combining of two sets.

In Exercises 1–6, (a) write an inequality that represents the interval and (b) state whether the interval is bounded or unbounded.

- | | |
|--------------------|-------------------|
| 1. $[-1, 5]$ | 2. $(2, 10]$ |
| 3. $(11, \infty)$ | 4. $[-5, \infty)$ |
| 5. $(-\infty, -2)$ | 6. $(-\infty, 7]$ |

In Exercises 7–12, match the inequality with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



- | | |
|--------------------|---------------------------------|
| 7. $x < 3$ | 8. $x \geq 5$ |
| 9. $-3 < x \leq 4$ | 10. $0 \leq x \leq \frac{9}{2}$ |
| 11. $ x < 3$ | 12. $ x > 4$ |

In Exercises 13–18, determine whether each value of x is a solution of the inequality.

- | Inequality | Values | |
|-------------------|-----------------------|------------------------|
| 13. $5x - 12 > 0$ | (a) $x = 3$ | (b) $x = -3$ |
| | (c) $x = \frac{5}{2}$ | (d) $x = \frac{3}{2}$ |
| 14. $2x + 1 < -3$ | (a) $x = 0$ | (b) $x = -\frac{1}{4}$ |
| | (c) $x = -4$ | (d) $x = -\frac{3}{2}$ |

- | Inequality | Values | |
|---------------------------------|--------------|-----------------------|
| 15. $0 < \frac{x-2}{4} < 2$ | (a) $x = 4$ | (b) $x = 10$ |
| | (c) $x = 0$ | (d) $x = \frac{7}{2}$ |
| 16. $-1 < \frac{3-x}{2} \leq 1$ | (a) $x = 0$ | (b) $x = -5$ |
| | (c) $x = 1$ | (d) $x = 5$ |
| 17. $ x - 10 \geq 3$ | (a) $x = 13$ | (b) $x = -1$ |
| | (c) $x = 14$ | (d) $x = 9$ |
| 18. $ 2x - 3 < 15$ | (a) $x = -6$ | (b) $x = 0$ |
| | (c) $x = 12$ | (d) $x = 7$ |

In Exercises 19–44, solve the inequality and sketch the solution on the real number line. (Some inequalities have no solutions.)

- | | |
|---|-----------------|
| 19. $4x < 12$ | 20. $10x < -40$ |
| 21. $-2x > -3$ | 22. $-6x > 15$ |
| 23. $x - 5 \geq 7$ | |
| 24. $x + 7 \leq 12$ | |
| 25. $2x + 7 < 3 + 4x$ | |
| 26. $3x + 1 \geq 2 + x$ | |
| 27. $2x - 1 \geq 1 - 5x$ | |
| 28. $6x - 4 \leq 2 + 8x$ | |
| 29. $4 - 2x < 3(3 - x)$ | |
| 30. $4(x + 1) < 2x + 3$ | |
| 31. $\frac{3}{4}x - 6 \leq x - 7$ | |
| 32. $3 + \frac{2}{7}x > x - 2$ | |
| 33. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$ | |
| 34. $9x - 1 < \frac{3}{4}(16x - 2)$ | |
| 35. $3.6x + 11 \geq -3.4$ | |
| 36. $15.6 - 1.3x < -5.2$ | |
| 37. $1 < 2x + 3 < 9$ | |
| 38. $-8 \leq -(3x + 5) < 13$ | |

39. $-4 < \frac{2x-3}{3} < 4$

40. $0 \leq \frac{x+3}{2} < 5$

41. $\frac{3}{4} > x + 1 > \frac{1}{4}$

42. $-1 < 2 - \frac{x}{3} < 1$

43. $3.2 \leq 0.4x - 1 \leq 4.4$

44. $4.5 > \frac{1.5x+6}{2} > 10.5$

In Exercises 45–60, solve the inequality and sketch the solution on the real number line. (Some inequalities have no solution.)

45. $|x| < 6$

46. $|x| > 4$

47. $\left|\frac{x}{2}\right| > 1$

48. $\left|\frac{x}{5}\right| > 3$

49. $|x-5| < -1$

50. $|x-7| < -5$

51. $|x-20| \leq 6$

52. $|x-8| \geq 0$

53. $|3-4x| \geq 9$

54. $|1-2x| < 5$

55. $\left|\frac{x-3}{2}\right| \geq 4$

56. $\left|1 - \frac{2x}{3}\right| < 1$

57. $|9-2x|-2 < -1$

58. $|x+14|+3 > 17$

59. $2|x+10| \geq 9$

60. $3|4-5x| \leq 9$



Graphical Analysis In Exercises 61–68, use a graphing utility to graph the inequality and identify the solution set.

61. $6x > 12$

62. $3x - 1 \leq 5$

63. $5 - 2x \geq 1$

64. $3(x+1) < x+7$

65. $|x-8| \leq 14$

66. $|2x+9| > 13$

67. $2|x+7| \geq 13$

68. $\frac{1}{2}|x+1| \leq 3$



Graphical Analysis In Exercises 69–74, use a graphing utility to graph the equation. Use the graph to approximate the values of x that satisfy each inequality.

Equation	Inequalities	
69. $y = 2x - 3$	(a) $y \geq 1$	(b) $y \leq 0$
70. $y = \frac{2}{3}x + 1$	(a) $y \leq 5$	(b) $y \geq 0$
71. $y = -\frac{1}{2}x + 2$	(a) $0 \leq y \leq 3$	(b) $y \geq 0$
72. $y = -3x + 8$	(a) $-1 \leq y \leq 3$	(b) $y \leq 0$
73. $y = x - 3 $	(a) $y \leq 2$	(b) $y \geq 4$
74. $y = \left \frac{1}{2}x + 1\right $	(a) $y \leq 4$	(b) $y \geq 1$

In Exercises 75–80, find the interval(s) on the real number line for which the radicand is nonnegative.

75. $\sqrt{x-5}$

76. $\sqrt{x-10}$

77. $\sqrt{x+3}$

78. $\sqrt{3-x}$

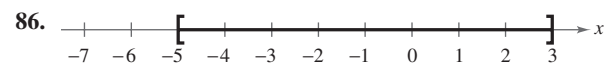
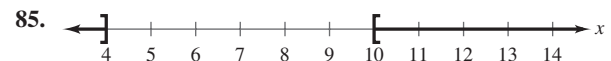
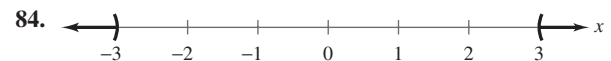
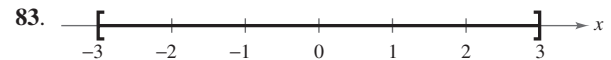
79. $\sqrt[4]{7-2x}$

80. $\sqrt[4]{6x+15}$

81. **Think About It** The graph of $|x-5| < 3$ can be described as all real numbers within three units of 5. Give a similar description of $|x-10| < 8$.

82. **Think About It** The graph of $|x-2| > 5$ can be described as all real numbers more than five units from 2. Give a similar description of $|x-8| > 4$.

In Exercises 83–90, use absolute value notation to define the interval (or pair of intervals) on the real number line.



87. All real numbers within 10 units of 12

88. All real numbers at least five units from 8

89. All real numbers more than four units from -3

90. All real numbers no more than seven units from -6

91. **Checking Account** You can choose between two types of checking accounts at your local bank. Type A charges a monthly service fee of \$6 plus \$0.25 for each check written. Type B charges a monthly service fee of \$4.50 plus \$0.50 for each check written. How many checks must you write in a month in order for the monthly charges for type A to be less than that for type B?

92. Copying Costs Your department sends its copying to the photocopy center of your company. The center bills your department \$0.10 per page. You have investigated the possibility of buying a departmental copier for \$3000. With your own copier, the cost per page would be \$0.03. The expected life of the copier is 4 years. How many copies must you make in the four-year period to justify buying the copier?

93. Investment In order for an investment of \$1000 to grow to more than \$1062.50 in 2 years, what must the annual interest rate be? [$A = P(1 + rt)$]

94. Investment In order for an investment of \$750 to grow to more than \$825 in 2 years, what must the annual interest rate be? [$A = P(1 + rt)$]

95. Cost, Revenue, and Profit The revenue for selling x units of a product is $R = 115.95x$. The cost of producing x units is

$$C = 95x + 750.$$

To obtain a profit, the revenue must be greater than the cost. For what values of x will this product return a profit?

96. Cost, Revenue, and Profit The revenue for selling x units of a product is $R = 24.55x$. The cost of producing x units is

$$C = 15.4x + 150,000.$$

To obtain a profit, the revenue must be greater than the cost. For what values of x will this product return a profit?

97. Daily Sales A doughnut shop sells a dozen doughnuts for \$2.95. Beyond the fixed costs (rent, utilities, and insurance) of \$150 per day, it costs \$1.45 for enough materials (flour, sugar, and so on) and labor to produce a dozen doughnuts. The daily profit from doughnut sales varies between \$50 and \$200. Between what levels (in dozens) do the daily sales vary?

98. Weight Loss Program A person enrolls in a diet and exercise program that guarantees a loss of at least $1\frac{1}{2}$ pounds per week. The person's weight at the beginning of the program is 164 pounds. Find the maximum number of weeks before the person attains a goal weight of 128 pounds.



99. Data Analysis: IQ Scores and GPA The admissions office of a college wants to determine whether there is a relationship between IQ scores x and grade-point averages y after the first year of school. An equation that models the data the admissions office obtained is

$$y = 0.067x - 5.638.$$

- (a) Use a graphing utility to graph the model.
- (b) Use the graph to estimate the values of x that predict a grade-point average of at least 3.0.



100. Data Analysis: Weightlifting You want to determine whether there is a relationship between an athlete's weight x (in pounds) and the athlete's maximum bench-press weight y (in pounds). The table shows a sample of data from 12 athletes.

- (a) Use a graphing utility to plot the data.

Athlete's weight, x	Bench-press weight, y
165	170
184	185
150	200
210	255
196	205
240	295
202	190
170	175
185	195
190	185
230	250
160	155

- (b) A model for the data is $y = 1.3x - 36$. Use a graphing utility to graph the model in the same viewing window used in part (a).
- (c) Use the graph to estimate the values of x that predict a maximum bench-press weight of at least 200 pounds.
- (d) Verify your estimate from part (c) algebraically.
- (e) Use the graph to write a statement about the accuracy of the model. If you think the graph indicates that an athlete's weight is not a particularly good indicator of the athlete's maximum bench-press weight, list other factors that might influence an individual's maximum bench-press weight.

101. Teachers' Salaries The average salary S (in thousands of dollars) for elementary school teachers in the United States from 1990 to 2002 is approximated by the model

$$S = 1.05t + 31.0, \quad 0 \leq t \leq 12$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: National Education Association)

- (a) According to this model, when was the average salary at least \$32,000, but not more than \$42,000?
- (b) According to this model, when will the average salary exceed \$48,000?

- 102. Egg Production** The number of eggs E (in billions) produced in the United States from 1990 to 2002 can be modeled by

$$E = 1.64t + 67.2, \quad 0 \leq t \leq 12$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: U.S. Department of Agriculture)

- (a) According to this model, when was the annual egg production 70 billion, but no more than 80 billion?
- (b) According to this model, when will the annual egg production exceed 95 billion?
- 103. Geometry** The side of a square is measured as 10.4 inches with a possible error of $\frac{1}{16}$ inch. Using these measurements, determine the interval containing the possible areas of the square.
- 104. Geometry** The side of a square is measured as 24.2 centimeters with a possible error of 0.25 centimeter. Using these measurements, determine the interval containing the possible areas of the square.
- 105. Accuracy of Measurement** You stop at a self-service gas station to buy 15 gallons of 87-octane gasoline at \$1.89 a gallon. The gas pump is accurate to within $\frac{1}{10}$ of a gallon. How much might you be undercharged or overcharged?

- 106. Accuracy of Measurement** You buy six T-bone steaks that cost \$14.99 per pound. The weight that is listed on the package is 5.72 pounds. The scale that weighed the package is accurate to within $\frac{1}{2}$ ounce. How much might you be undercharged or overcharged?

- 107. Time Study** A time study was conducted to determine the length of time required to perform a particular task in a manufacturing process. The times required by approximately two-thirds of the workers in the study satisfied the inequality

$$\left| \frac{t - 15.6}{1.9} \right| < 1$$

where t is time in minutes. Determine the interval on the real number line in which these times lie.

- 108. Height** The heights h of two-thirds of the members of a population satisfy the inequality

$$\left| \frac{h - 68.5}{2.7} \right| \leq 1$$

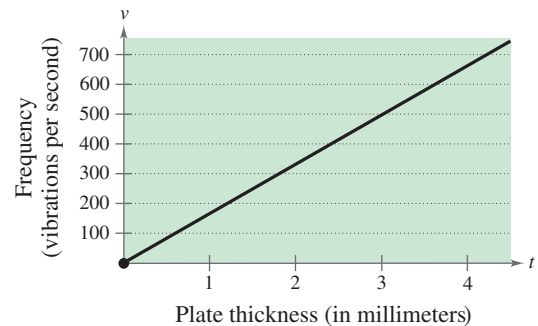
where h is measured in inches. Determine the interval on the real number line in which these heights lie.

- 109. Meteorology** An electronic device is to be operated in an environment with relative humidity h in the interval defined by $|h - 50| \leq 30$. What are the minimum and maximum relative humidities for the operation of this device?

- 110. Music** Michael Kasha of Florida State University used physics and mathematics to design a new classical guitar. He used the model for the frequency of the vibrations on a circular plate

$$v = \frac{2.6t}{d^2} \sqrt{\frac{E}{\rho}}$$

where v is the frequency (in vibrations per second), t is the plate thickness (in millimeters), d is the diameter of the plate, E is the elasticity of the plate material, and ρ is the density of the plate material. For fixed values of d , E , and ρ , the graph of the equation is a line (see figure).



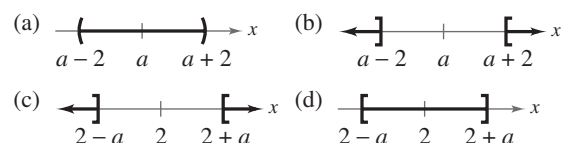
- (a) Estimate the frequency when the plate thickness is 2 millimeters.
- (b) Estimate the plate thickness when the frequency is 600 vibrations per second.
- (c) Approximate the interval for the plate thickness when the frequency is between 200 and 400 vibrations per second.
- (d) Approximate the interval for the frequency when the plate thickness is less than 3 millimeters.

Synthesis

True or False? In Exercises 111 and 112, determine whether the statement is true or false. Justify your answer.

- 111.** If a , b , and c are real numbers, and $a \leq b$, then $ac \leq bc$.
- 112.** If $-10 \leq x \leq 8$, then $-10 \geq -x$ and $-x \geq -8$.

- 113.** Identify the graph of the inequality $|x - a| \geq 2$.



- 114.** Find sets of values of a , b , and c such that $0 \leq x \leq 10$ is a solution of the inequality $|ax - b| \leq c$.