A.5 Solving Equations

What you should learn

- Identify different types of equations.
- Solve linear equations in one variable and equations that lead to linear equations.
- Solve quadratic equations by factoring, extracting square roots, completing the square, and using the Quadratic Formula.
- Solve polynomial equations of degree three or greater.
- Solve equations involving radicals.
- Solve equations with absolute values.

Why you should learn it

Linear equations are used in many real-life applications. For example, in Exercise 185 on page A58, linear equations can be used to model the relationship between the length of a thigh-bone and the height of a person, helping researchers learn about ancient cultures.

Equations and Solutions of Equations

An **equation** in x is a statement that two algebraic expressions are equal. For example

$$3x - 5 = 7, \; x^2 - x - 6 = 0, \text{ and } \sqrt{2x} = 4$$

are equations. To **solve** an equation in x means to find all values of x for which the equation is true. Such values are **solutions.** For instance, $x = 4$ is a solution of the equation

$$3x - 5 = 7$$

because $3(4) - 5 = 7$ is a true statement.

The solutions of an equation depend on the kinds of numbers being considered. For instance, in the set of rational numbers, $x^2 = 10$ has no solution because there is no rational number whose square is 10. However, in the set of real numbers, the equation has the two solutions $x = \sqrt{10}$ and $x = -\sqrt{10}$.

An equation that is true for *every* real number in the *domain* of the variable is called an **identity.** The domain is the set of all real numbers for which the equation is defined. For example

$$x^2 - 9 = (x + 3)(x - 3) \qquad \text{Identity}$$

is an identity because it is a true statement for any real value of x. The equation

$$\frac{x}{3x^2} = \frac{1}{3x} \qquad \text{Identity}$$

where $x \neq 0$, is an identity because it is true for any nonzero real value of x.

An equation that is true for just *some* (or even none) of the real numbers in the domain of the variable is called a **conditional equation.** For example, the equation

$$x^2 - 9 = 0 \qquad \text{Conditional equation}$$

is conditional because $x = 3$ and $x = -3$ are the only values in the domain that satisfy the equation. The equation $2x - 4 = 2x + 1$ is conditional because there are no real values of x for which the equation is true. Learning to solve conditional equations is the primary focus of this section.

Linear Equations in One Variable

Definition of a Linear Equation

A **linear equation in one variable** x is an equation that can be written in the standard form

$$ax + b = 0$$

where a and b are real numbers with $a \neq 0$.

A linear equation has exactly one solution. To see this, consider the following steps. (Remember that $a \neq 0$.)

$$ax + b = 0 \quad \text{Write original equation.}$$

$$ax = -b \quad \text{Subtract } b \text{ from each side.}$$

$$x = -\frac{b}{a} \quad \text{Divide each side by } a.$$

To solve a conditional equation in x, isolate x on one side of the equation by a sequence of **equivalent** (and usually simpler) **equations,** each having the same solution(s) as the original equation. The operations that yield equivalent equations come from the Substitution Principle (see Appendix A.1) and simplification techniques.

Generating Equivalent Equations

An equation can be transformed into an *equivalent equation* by one or more of the following steps.

	Given Equation	*Equivalent Equation*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) *each* side of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) *each* side of the equation by the same *nonzero* quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

STUDY TIP

After solving an equation, you should check each solution in the original equation. For instance, you can check the solution to Example 1(a) as follows.

$$3x - 6 = 0 \quad \text{Write original equation.}$$

$$3(2) - 6 \stackrel{?}{=} 0 \quad \text{Substitute 2 for } x.$$

$$0 = 0 \quad \text{Solution checks. ✓}$$

Try checking the solution to Example 1(b).

Example 1 Solving a Linear Equation

a. $3x - 6 = 0$ Original equation

$3x = 6$ Add 6 to each side.

$x = 2$ Divide each side by 3.

b. $5x + 4 = 3x - 8$ Original equation

$2x + 4 = -8$ Subtract $3x$ from each side.

$2x = -12$ Subtract 4 from each side.

$x = -6$ Divide each side by 2.

CHECKPOINT Now try Exercise 13.

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms and multiply every term by the LCD. This process will clear the original equation of fractions and produce a simpler equation to work with.

Example 2 An Equation Involving Fractional Expressions

Solve $\dfrac{x}{3} + \dfrac{3x}{4} = 2$.

Solution

$\dfrac{x}{3} + \dfrac{3x}{4} = 2$	Write original equation.
$(12)\dfrac{x}{3} + (12)\dfrac{3x}{4} = (12)2$	Multiply each term by the LCD of 12.
$4x + 9x = 24$	Divide out and multiply.
$13x = 24$	Combine like terms.
$x = \dfrac{24}{13}$	Divide each side by 13.

The solution is $x = \frac{24}{13}$. Check this in the original equation.

CHECKPOINT Now try Exercise 21.

When multiplying or dividing an equation by a *variable* quantity, it is possible to introduce an extraneous solution. An **extraneous solution** is one that does not satisfy the original equation. Therefore, it is essential that you check your solutions.

Example 3 An Equation with an Extraneous Solution

Solve $\dfrac{1}{x-2} = \dfrac{3}{x+2} - \dfrac{6x}{x^2-4}$.

Solution

The LCD is $x^2 - 4$, or $(x + 2)(x - 2)$. Multiply each term by this LCD.

$$\frac{1}{x-2}(x+2)(x-2) = \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$x + 2 = 3(x - 2) - 6x, \qquad x \neq \pm 2$$

$$x + 2 = 3x - 6 - 6x$$

$$x + 2 = -3x - 6$$

$$4x = -8 \quad \Rightarrow \quad x = -2$$ Extraneous solution

In the original equation, $x = -2$ yields a denominator of zero. So, $x = -2$ is an extraneous solution, and the original equation has *no solution*.

CHECKPOINT Now try Exercise 37.

STUDY TIP

An equation with a *single fraction* on each side can be cleared of denominators by **cross multiplying,** which is equivalent to multiplying by the LCD and then dividing out. To do this, multiply the left numerator by the right denominator and the right numerator by the left denominator as follows.

$\dfrac{a}{b} = \dfrac{c}{d}$	LCD is bd.
$\dfrac{a}{b} \cdot bd = \dfrac{c}{d} \cdot bd$	Multiply by LCD.
$ad = cb$	Divide out common factors.

STUDY TIP

Recall that the least common denominator of two or more fractions consists of the product of all prime factors in the denominators, with each factor given the highest power of its occurrence in any denominator. For instance, in Example 3, by factoring each denominator you can determine that the LCD is $(x + 2)(x - 2)$.

Quadratic Equations

A **quadratic equation** in x is an equation that can be written in the general form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, with $a \neq 0$. A quadratic equation in x is also known as a **second-degree polynomial equation** in x.

You should be familiar with the following four methods of solving quadratic equations.

Solving a Quadratic Equation

Factoring: If $ab = 0$, then $a = 0$ or $b = 0$.

Example:

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \quad \Rightarrow \quad x = 3$$

$$x + 2 = 0 \quad \Rightarrow \quad x = -2$$

Square Root Principle: If $u^2 = c$, where $c > 0$, then $u = \pm\sqrt{c}$.

Example:

$$(x + 3)^2 = 16$$

$$x + 3 = \pm 4$$

$$x = -3 \pm 4$$

$$x = 1 \quad \text{or} \quad x = -7$$

Completing the Square: If $x^2 + bx = c$, then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2 \qquad \text{Add } \left(\frac{b}{2}\right)^2 \text{ to each side.}$$

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}.$$

Example:

$$x^2 + 6x = 5$$

$$x^2 + 6x + 3^2 = 5 + 3^2 \qquad \text{Add } \left(\frac{6}{2}\right)^2 \text{ to each side.}$$

$$(x + 3)^2 = 14$$

$$x + 3 = \pm\sqrt{14}$$

$$x = -3 \pm \sqrt{14}$$

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example:

$$2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

STUDY TIP

The Square Root Principle is also referred to as *extracting square roots*.

STUDY TIP

You can solve every quadratic equation by completing the square or using the Quadratic Formula.

Example 4 Solving a Quadratic Equation by Factoring

a. $2x^2 + 9x + 7 = 3$ Original equation

$2x^2 + 9x + 4 = 0$ Write in general form.

$(2x + 1)(x + 4) = 0$ Factor.

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$ Set 1st factor equal to 0.

$x + 4 = 0 \Rightarrow x = -4$ Set 2nd factor equal to 0.

The solutions are $x = -\frac{1}{2}$ and $x = -4$. Check these in the original equation.

b. $6x^2 - 3x = 0$ Original equation

$3x(2x - 1) = 0$ Factor.

$3x = 0 \Rightarrow x = 0$ Set 1st factor equal to 0.

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ Set 2nd factor equal to 0.

The solutions are $x = 0$ and $x = \frac{1}{2}$. Check these in the original equation.

CHECKPOINT Now try Exercise 57.

Note that the method of solution in Example 4 is based on the Zero-Factor Property from Appendix A.1. Be sure you see that this property works *only* for equations written in general form (in which the right side of the equation is zero). So, all terms must be collected on one side *before* factoring. For instance, in the equation $(x - 5)(x + 2) = 8$, it is *incorrect* to set each factor equal to 8. Try to solve this equation correctly.

Example 5 Extracting Square Roots

Solve each equation by extracting square roots.

a. $4x^2 = 12$ **b.** $(x - 3)^2 = 7$

Solution

a. $4x^2 = 12$ Write original equation.

$x^2 = 3$ Divide each side by 4.

$x = \pm\sqrt{3}$ Extract square roots.

When you take the square root of a variable expression, you must account for both positive and negative solutions. So, the solutions are $x = \sqrt{3}$ and $x = -\sqrt{3}$. Check these in the original equation.

b. $(x - 3)^2 = 7$ Write original equation.

$x - 3 = \pm\sqrt{7}$ Extract square roots.

$x = 3 \pm \sqrt{7}$ Add 3 to each side.

The solutions are $x = 3 \pm \sqrt{7}$. Check these in the original equation.

CHECKPOINT Now try Exercise 77.

When solving quadratic equations by completing the square, you must add $(b/2)^2$ to *each side* in order to maintain equality. If the leading coefficient is *not* 1, you must divide each side of the equation by the leading coefficient *before* completing the square, as shown in Example 7.

Example 6 **Completing the Square: Leading Coefficient Is 1**

Solve $x^2 + 2x - 6 = 0$ by completing the square.

Solution

$$x^2 + 2x - 6 = 0 \qquad \text{Write original equation.}$$

$$x^2 + 2x = 6 \qquad \text{Add 6 to each side.}$$

$$x^2 + 2x + 1^2 = 6 + 1^2 \qquad \text{Add } 1^2 \text{ to each side.}$$

(half of 2)^2

$$(x + 1)^2 = 7 \qquad \text{Simplify.}$$

$$x + 1 = \pm\sqrt{7} \qquad \text{Take square root of each side.}$$

$$x = -1 \pm \sqrt{7} \qquad \text{Subtract 1 from each side.}$$

The solutions are $x = -1 \pm \sqrt{7}$. Check these in the original equation.

CHECKPOINT Now try Exercise 85.

Example 7 **Completing the Square: Leading Coefficient Is Not 1**

$$3x^2 - 4x - 5 = 0 \qquad \text{Original equation}$$

$$3x^2 - 4x = 5 \qquad \text{Add 5 to each side.}$$

$$x^2 - \frac{4}{3}x = \frac{5}{3} \qquad \text{Divide each side by 3.}$$

$$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{5}{3} + \left(-\frac{2}{3}\right)^2 \qquad \text{Add } \left(-\tfrac{2}{3}\right)^2 \text{ to each side.}$$

$\left(\text{half of } -\frac{4}{3}\right)^2$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{19}{9} \qquad \text{Simplify.}$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9} \qquad \text{Perfect square trinomial.}$$

$$x - \frac{2}{3} = \pm\frac{\sqrt{19}}{3} \qquad \text{Extract square roots.}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3} \qquad \text{Solutions}$$

CHECKPOINT Now try Exercise 91.

STUDY TIP

When using the Quadratic Formula, remember that *before* the formula can be applied, you must first write the quadratic equation in general form.

Example 8 The Quadratic Formula: Two Distinct Solutions

Use the Quadratic Formula to solve $x^2 + 3x = 9$.

Solution

$$x^2 + 3x = 9 \qquad \text{Write original equation.}$$

$$x^2 + 3x - 9 = 0 \qquad \text{Write in general form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-9)}}{2(1)} \qquad \text{Substitute } a = 1, b = 3, \text{ and } c = -9.$$

$$x = \frac{-3 \pm \sqrt{45}}{2} \qquad \text{Simplify.}$$

$$x = \frac{-3 \pm 3\sqrt{5}}{2} \qquad \text{Simplify.}$$

The equation has two solutions:

$$x = \frac{-3 + 3\sqrt{5}}{2} \quad \text{and} \quad x = \frac{-3 - 3\sqrt{5}}{2}.$$

Check these in the original equation.

CHECKPOINT Now try Exercise 101.

Example 9 The Quadratic Formula: One Solution

Use the Quadratic Formula to solve $8x^2 - 24x + 18 = 0$.

Solution

$$8x^2 - 24x + 18 = 0 \qquad \text{Write original equation.}$$

$$4x^2 - 12x + 9 = 0 \qquad \text{Divide out common factor of 2.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)} \qquad \text{Substitute } a = 4, b = -12, \text{ and } c = 9.$$

$$x = \frac{12 \pm \sqrt{0}}{8} = \frac{3}{2} \qquad \text{Simplify.}$$

This quadratic equation has only one solution: $x = \frac{3}{2}$. Check this in the original equation.

CHECKPOINT Now try Exercise 105.

Note that Example 9 could have been solved without first dividing out a common factor of 2. Substituting $a = 8$, $b = -24$, and $c = 18$ into the Quadratic Formula produces the same result.

Polynomial Equations of Higher Degree

The methods used to solve quadratic equations can sometimes be extended to solve polynomial equations of higher degree.

STUDY TIP

A common mistake that is made in solving an equation such as that in Example 10 is to divide each side of the equation by the variable factor x^2. This loses the solution $x = 0$. When solving an equation, always write the equation in general form, then factor the equation and set each factor equal to zero. Do not divide each side of an equation by a variable factor in an attempt to simplify the equation.

Example 10 Solving a Polynomial Equation by Factoring

Solve $3x^4 = 48x^2$.

Solution

First write the polynomial equation in general form with zero on one side, factor the other side, and then set each factor equal to zero and solve.

$3x^4 = 48x^2$ Write original equation.

$3x^4 - 48x^2 = 0$ Write in general form.

$3x^2(x^2 - 16) = 0$ Factor out common factor.

$3x^2(x + 4)(x - 4) = 0$ Write in factored form.

$3x^2 = 0 \Rightarrow x = 0$ Set 1st factor equal to 0.

$x + 4 = 0 \Rightarrow x = -4$ Set 2nd factor equal to 0.

$x - 4 = 0 \Rightarrow x = 4$ Set 3rd factor equal to 0.

You can check these solutions by substituting in the original equation, as follows.

Check

$3(0)^4 = 48(0)^2$ 0 checks. ✓

$3(-4)^4 = 48(-4)^2$ -4 checks. ✓

$3(4)^4 = 48(4)^2$ 4 checks. ✓

So, you can conclude that the solutions are $x = 0$, $x = -4$, and $x = 4$.

✓CHECKPOINT Now try Exercise 135.

Example 11 Solving a Polynomial Equation by Factoring

Solve $x^3 - 3x^2 - 3x + 9 = 0$.

Solution

$x^3 - 3x^2 - 3x + 9 = 0$ Write original equation.

$x^2(x - 3) - 3(x - 3) = 0$ Factor by grouping.

$(x - 3)(x^2 - 3) = 0$ Distributive Property

$x - 3 = 0 \Rightarrow x = 3$ Set 1st factor equal to 0.

$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$ Set 2nd factor equal to 0.

The solutions are $x = 3$, $x = \sqrt{3}$, and $x = -\sqrt{3}$. Check these in the original equation.

✓CHECKPOINT Now try Exercise 143.

Equations Involving Radicals

Operations such as squaring each side of an equation, raising each side of an equation to a rational power, and multiplying each side of an equation by a variable quantity all can introduce extraneous solutions. So, when you use any of these operations, checking your solutions is crucial.

Example 12 Solving Equations Involving Radicals

a. $\sqrt{2x+7} - x = 2$ Original equation

$\sqrt{2x+7} = x + 2$ Isolate radical.

$2x + 7 = x^2 + 4x + 4$ Square each side.

$0 = x^2 + 2x - 3$ Write in general form.

$0 = (x+3)(x-1)$ Factor.

$x + 3 = 0 \Rightarrow x = -3$ Set 1st factor equal to 0.

$x - 1 = 0 \Rightarrow x = 1$ Set 2nd factor equal to 0.

By checking these values, you can determine that the only solution is $x = 1$.

b. $\sqrt{2x-5} - \sqrt{x-3} = 1$ Original equation

$\sqrt{2x-5} = \sqrt{x-3} + 1$ Isolate $\sqrt{2x-5}$.

$2x - 5 = x - 3 + 2\sqrt{x-3} + 1$ Square each side.

$2x - 5 = x - 2 + 2\sqrt{x-3}$ Combine like terms.

$x - 3 = 2\sqrt{x-3}$ Isolate $2\sqrt{x-3}$.

$x^2 - 6x + 9 = 4(x-3)$ Square each side.

$x^2 - 10x + 21 = 0$ Write in general form.

$(x-3)(x-7) = 0$ Factor.

$x - 3 = 0 \Rightarrow x = 3$ Set 1st factor equal to 0.

$x - 7 = 0 \Rightarrow x = 7$ Set 2nd factor equal to 0.

The solutions are $x = 3$ and $x = 7$. Check these in the original equation.

CHECKPOINT Now try Exercise 155.

STUDY TIP

When an equation contains two radicals, it may not be possible to isolate both. In such cases, you may have to raise each side of the equation to a power at *two* different stages in the solution, as shown in Example 12(b).

Example 13 Solving an Equation Involving a Rational Exponent

$(x-4)^{2/3} = 25$ Original equation

$\sqrt[3]{(x-4)^2} = 25$ Rewrite in radical form.

$(x-4)^2 = 15{,}625$ Cube each side.

$x - 4 = \pm 125$ Take square root of each side.

$x = 129, \quad x = -121$ Add 4 to each side.

CHECKPOINT Now try Exercise 163.

Equations with Absolute Values

To solve an equation involving an absolute value, remember that the expression inside the absolute value signs can be positive or negative. This results in *two* separate equations, each of which must be solved. For instance, the equation

$$|x - 2| = 3$$

results in the two equations $x - 2 = 3$ and $-(x - 2) = 3$, which implies that the equation has two solutions: $x = 5$ and $x = -1$.

Example 14 Solving an Equation Involving Absolute Value

Solve $|x^2 - 3x| = -4x + 6$.

Solution

Because the variable expression inside the absolute value signs can be positive or negative, you must solve the following two equations.

First Equation

$x^2 - 3x = -4x + 6$ Use positive expression.

$x^2 + x - 6 = 0$ Write in general form.

$(x + 3)(x - 2) = 0$ Factor.

$x + 3 = 0 \Rightarrow x = -3$ Set 1st factor equal to 0.

$x - 2 = 0 \Rightarrow x = 2$ Set 2nd factor equal to 0.

Second Equation

$-(x^2 - 3x) = -4x + 6$ Use negative expression.

$x^2 - 7x + 6 = 0$ Write in general form.

$(x - 1)(x - 6) = 0$ Factor.

$x - 1 = 0 \Rightarrow x = 1$ Set 1st factor equal to 0.

$x - 6 = 0 \Rightarrow x = 6$ Set 2nd factor equal to 0.

Check

$|(-3)^2 - 3(-3)| \stackrel{?}{=} -4(-3) + 6$ Substitute -3 for x.

$18 = 18$ -3 checks. ✓

$|(2)^2 - 3(2)| \stackrel{?}{=} -4(2) + 6$ Substitute 2 for x.

$2 \neq -2$ 2 does not check.

$|(1)^2 - 3(1)| \stackrel{?}{=} -4(1) + 6$ Substitute 1 for x.

$2 = 2$ 1 checks. ✓

$|(6)^2 - 3(6)| \stackrel{?}{=} -4(6) + 6$ Substitute 6 for x.

$18 \neq -18$ 6 does not check.

The solutions are $x = -3$ and $x = 1$.

✓CHECKPOINT Now try Exercise 181.

A.5 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. An ________ is a statement that equates two algebraic expressions.
2. To find all values that satisfy an equation is to ________ the equation.
3. There are two types of equations, ________ and ________ equations.
4. A linear equation in one variable is an equation that can be written in the standard from ________.
5. When solving an equation, it is possible to introduce an ________ solution, which is a value that does not satisfy the original equation.
6. An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is a ________ ________, or a second-degree polynomial equation in x.
7. The four methods that can be used to solve a quadratic equation are ________, ________, ________, and the ________.

In Exercises 1–10, determine whether the equation is an identity or a conditional equation.

1. $2(x - 1) = 2x - 2$
2. $3(x + 2) = 5x + 4$
3. $-6(x - 3) + 5 = -2x + 10$
4. $3(x + 2) - 5 = 3x + 1$
5. $4(x + 1) - 2x = 2(x + 2)$
6. $-7(x - 3) + 4x = 3(7 - x)$
7. $x^2 - 8x + 5 = (x - 4)^2 - 11$
8. $x^2 + 2(3x - 2) = x^2 + 6x - 4$
9. $3 + \dfrac{1}{x + 1} = \dfrac{4x}{x + 1}$
10. $\dfrac{5}{x} + \dfrac{3}{x} = 24$

In Exercises 11–26, solve the equation and check your solution.

11. $x + 11 = 15$
12. $7 - x = 19$
13. $7 - 2x = 25$
14. $7x + 2 = 23$
15. $8x - 5 = 3x + 20$
16. $7x + 3 = 3x - 17$
17. $2(x + 5) - 7 = 3(x - 2)$
18. $3(x + 3) = 5(1 - x) - 1$
19. $x - 3(2x + 3) = 8 - 5x$
20. $9x - 10 = 5x + 2(2x - 5)$
21. $\dfrac{5x}{4} + \dfrac{1}{2} = x - \dfrac{1}{2}$
22. $\dfrac{x}{5} - \dfrac{x}{2} = 3 + \dfrac{3x}{10}$
23 $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$
24. $\dfrac{3x}{2} + \dfrac{1}{4}(x - 2) = 10$
25. $0.25x + 0.75(10 - x) = 3$
26. $0.60x + 0.40(100 - x) = 50$

In Exercises 27–48, solve the equation and check your solution. (If not possible, explain why.)

27. $x + 8 = 2(x - 2) - x$
28. $8(x + 2) - 3(2x + 1) = 2(x + 5)$
29. $\dfrac{100 - 4x}{3} = \dfrac{5x + 6}{4} + 6$
30. $\dfrac{17 + y}{y} + \dfrac{32 + y}{y} = 100$
31. $\dfrac{5x - 4}{5x + 4} = \dfrac{2}{3}$
32. $\dfrac{10x + 3}{5x + 6} = \dfrac{1}{2}$
33. $10 - \dfrac{13}{x} = 4 + \dfrac{5}{x}$
34. $\dfrac{15}{x} - 4 = \dfrac{6}{x} + 3$
35. $3 = 2 + \dfrac{2}{z + 2}$
36. $\dfrac{1}{x} + \dfrac{2}{x - 5} = 0$
37. $\dfrac{x}{x + 4} + \dfrac{4}{x + 4} + 2 = 0$
38. $\dfrac{7}{2x + 1} - \dfrac{8x}{2x - 1} = -4$
39. $\dfrac{2}{(x - 4)(x - 2)} = \dfrac{1}{x - 4} + \dfrac{2}{x - 2}$
40. $\dfrac{4}{x - 1} + \dfrac{6}{3x + 1} = \dfrac{15}{3x + 1}$
41. $\dfrac{1}{x - 3} + \dfrac{1}{x + 3} = \dfrac{10}{x^2 - 9}$
42. $\dfrac{1}{x - 2} + \dfrac{3}{x + 3} = \dfrac{4}{x^2 + x - 6}$
43. $\dfrac{3}{x^2 - 3x} + \dfrac{4}{x} = \dfrac{1}{x - 3}$
44. $\dfrac{6}{x} - \dfrac{2}{x + 3} = \dfrac{3(x + 5)}{x^2 + 3x}$

45. $(x + 2)^2 + 5 = (x + 3)^2$

46. $(x + 1)^2 + 2(x - 2) = (x + 1)(x - 2)$

47. $(x + 2)^2 - x^2 = 4(x + 1)$

48. $(2x + 1)^2 = 4(x^2 + x + 1)$

In Exercises 49–54, write the quadratic equation in general form.

49. $2x^2 = 3 - 8x$ **50.** $x^2 = 16x$

51. $(x - 3)^2 = 3$ **52.** $13 - 3(x + 7)^2 = 0$

53. $\frac{1}{5}(3x^2 - 10) = 18x$ **54.** $x(x + 2) = 5x^2 + 1$

In Exercises 55–68, solve the quadratic equation by factoring.

55. $6x^2 + 3x = 0$ **56.** $9x^2 - 1 = 0$

57. $x^2 - 2x - 8 = 0$ **58.** $x^2 - 10x + 9 = 0$

59. $x^2 + 10x + 25 = 0$ **60.** $4x^2 + 12x + 9 = 0$

61. $3 + 5x - 2x^2 = 0$ **62.** $2x^2 = 19x + 33$

63. $x^2 + 4x = 12$ **64.** $-x^2 + 8x = 12$

65. $\frac{3}{4}x^2 + 8x + 20 = 0$ **66.** $\frac{1}{8}x^2 - x - 16 = 0$

67. $x^2 + 2ax + a^2 = 0$, a is a real number

68. $(x + a)^2 - b^2 = 0$, a and b are real numbers

In Exercises 69–82, solve the equation by extracting square roots.

69. $x^2 = 49$ **70.** $x^2 = 169$

71. $x^2 = 11$ **72.** $x^2 = 32$

73. $3x^2 = 81$ **74.** $9x^2 = 36$

75. $(x - 12)^2 = 16$ **76.** $(x + 13)^2 = 25$

77. $(x + 2)^2 = 14$ **78.** $(x - 5)^2 = 30$

79. $(2x - 1)^2 = 18$ **80.** $(4x + 7)^2 = 44$

81. $(x - 7)^2 = (x + 3)^2$ **82.** $(x + 5)^2 = (x + 4)^2$

In Exercises 83–92, solve the quadratic equation by completing the square.

83. $x^2 + 4x - 32 = 0$ **84.** $x^2 - 2x - 3 = 0$

85. $x^2 + 12x + 25 = 0$ **86.** $x^2 + 8x + 14 = 0$

87. $9x^2 - 18x = -3$ **88.** $9x^2 - 12x = 14$

89. $8 + 4x - x^2 = 0$ **90.** $-x^2 + x - 1 = 0$

91. $2x^2 + 5x - 8 = 0$ **92.** $4x^2 - 4x - 99 = 0$

In Exercises 93–116, use the Quadratic Formula to solve the equation.

93. $2x^2 + x - 1 = 0$ **94.** $2x^2 - x - 1 = 0$

95. $16x^2 + 8x - 3 = 0$ **96.** $25x^2 - 20x + 3 = 0$

97. $2 + 2x - x^2 = 0$ **98.** $x^2 - 10x + 22 = 0$

99. $x^2 + 14x + 44 = 0$ **100.** $6x = 4 - x^2$

101. $x^2 + 8x - 4 = 0$ **102.** $4x^2 - 4x - 4 = 0$

103. $12x - 9x^2 = -3$ **104.** $16x^2 + 22 = 40x$

105. $9x^2 + 24x + 16 = 0$ **106.** $36x^2 + 24x - 7 = 0$

107. $4x^2 + 4x = 7$ **108.** $16x^2 - 40x + 5 = 0$

109. $28x - 49x^2 = 4$ **110.** $3x + x^2 - 1 = 0$

111. $8t = 5 + 2t^2$ **112.** $25h^2 + 80h + 61 = 0$

113. $(y - 5)^2 = 2y$ **114.** $(z + 6)^2 = -2z$

115. $\frac{1}{2}x^2 + \frac{3}{8}x = 2$ **116.** $\left(\frac{5}{7}x - 14\right)^2 = 8x$

In Exercises 117–124, use the Quadratic Formula to solve the equation. (Round your answer to three decimal places.)

117. $5.1x^2 - 1.7x - 3.2 = 0$

118. $2x^2 - 2.50x - 0.42 = 0$

119. $-0.067x^2 - 0.852x + 1.277 = 0$

120. $-0.005x^2 + 0.101x - 0.193 = 0$

121. $422x^2 - 506x - 347 = 0$

122. $1100x^2 + 326x - 715 = 0$

123. $12.67x^2 + 31.55x + 8.09 = 0$

124. $-3.22x^2 - 0.08x + 28.651 = 0$

In Exercises 125–134, solve the equation using any convenient method.

125. $x^2 - 2x - 1 = 0$ **126.** $11x^2 + 33x = 0$

127. $(x + 3)^2 = 81$ **128.** $x^2 - 14x + 49 = 0$

129. $x^2 - x - \frac{11}{4} = 0$ **130.** $x^2 + 3x - \frac{3}{4} = 0$

131. $(x + 1)^2 = x^2$

132. $a^2x^2 - b^2 = 0$, a and b are real numbers

133. $3x + 4 = 2x^2 - 7$

134. $4x^2 + 2x + 4 = 2x + 8$

In Exercises 135–152, find all solutions of the equation. Check your solutions in the original equation.

135. $4x^4 - 18x^2 = 0$ **136.** $20x^3 - 125x = 0$

137. $x^4 - 81 = 0$ **138.** $x^6 - 64 = 0$

139. $x^3 + 216 = 0$ **140.** $27x^3 - 512 = 0$

141. $5x^3 + 30x^2 + 45x = 0$

142. $9x^4 - 24x^3 + 16x^2 = 0$

143. $x^3 - 3x^2 - x + 3 = 0$

144. $x^3 + 2x^2 + 3x + 6 = 0$

145. $x^4 - x^3 + x - 1 = 0$

146. $x^4 + 2x^3 - 8x - 16 = 0$

147. $x^4 - 4x^2 + 3 = 0$ **148.** $x^4 + 5x^2 - 36 = 0$

149. $4x^4 - 65x^2 + 16 = 0$ **150.** $36t^4 + 29t^2 - 7 = 0$

151. $x^6 + 7x^3 - 8 = 0$ **152.** $x^6 + 3x^3 + 2 = 0$

In Exercises 153–184, find all solutions of the equation. Check your solutions in the original equation.

153. $\sqrt{2x} - 10 = 0$

154. $4\sqrt{x} - 3 = 0$

155. $\sqrt{x - 10} - 4 = 0$

156. $\sqrt{5 - x} - 3 = 0$

157. $\sqrt[3]{2x + 5} + 3 = 0$

158. $\sqrt[3]{3x + 1} - 5 = 0$

159. $-\sqrt{26 - 11x} + 4 = x$

160. $x + \sqrt{31 - 9x} = 5$

161. $\sqrt{x + 1} = \sqrt{3x + 1}$

162. $\sqrt{x + 5} = \sqrt{x - 5}$

163. $(x - 5)^{3/2} = 8$

164. $(x + 3)^{3/2} = 8$

165. $(x + 3)^{2/3} = 8$

166. $(x + 2)^{2/3} = 9$

167. $(x^2 - 5)^{3/2} = 27$

168. $(x^2 - x - 22)^{3/2} = 27$

169. $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$

170. $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$

171. $x = \dfrac{3}{x} + \dfrac{1}{2}$

172. $\dfrac{4}{x} - \dfrac{5}{3} = \dfrac{x}{6}$

173. $\dfrac{1}{x} - \dfrac{1}{x + 1} = 3$

174. $\dfrac{4}{x + 1} - \dfrac{3}{x + 2} = 1$

175. $\dfrac{20 - x}{x} = x$

176. $4x + 1 = \dfrac{3}{x}$

177. $\dfrac{x}{x^2 - 4} + \dfrac{1}{x + 2} = 3$

178. $\dfrac{x + 1}{3} - \dfrac{x + 1}{x + 2} = 0$

179. $|2x - 1| = 5$

180. $|3x + 2| = 7$

181. $|x| = x^2 + x - 3$

182. $|x^2 + 6x| = 3x + 18$

183. $|x + 1| = x^2 - 5$

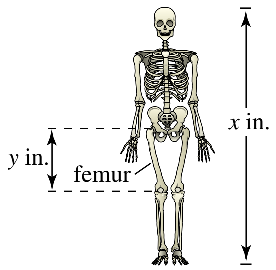
184. $|x - 10| = x^2 - 10x$

185. ***Anthropology*** The relationship between the length of an adult's femur (thigh bone) and the height of the adult can be approximated by the linear equations

$y = 0.432x - 10.44$ Female

$y = 0.449x - 12.15$ Male

where y is the length of the femur in inches and x is the height of the adult in inches (see figure).



(a) An anthropologist discovers a femur belonging to an adult human female. The bone is 16 inches long. Estimate the height of the female.

(b) From the foot bones of an adult human male, an anthropologist estimates that the person's height was 69 inches. A few feet away from the site where the foot bones were discovered, the anthropologist discovers a male adult femur that is 19 inches long. Is it likely that both the foot bones and the thigh bone came from the same person?

(c) Complete the table to determine if there is a height of an adult for which an anthropologist would not be able to determine whether the femur belonged to a male or a female.

Height, x	Female femur length, y	Male femur length, y
60		
70		
80		
90		
100		
110		

186. ***Operating Cost*** A delivery company has a fleet of vans. The annual operating cost C per van is

$C = 0.32m + 2500$

where m is the number of miles traveled by a van in a year. What number of miles will yield an annual operating cost of \$10,000?

187. ***Flood Control*** A river has risen 8 feet above its flood stage. The water begins to recede at a rate of 3 inches per hour. Write a mathematical model that shows the number of feet above flood stage after t hours. If the water continually recedes at this rate, when will the river be 1 foot above its flood stage?

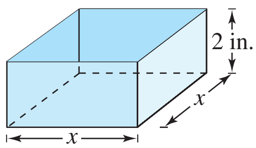
188. ***Floor Space*** The floor of a one-story building is 14 feet longer than it is wide. The building has 1632 square feet of floor space.

(a) Draw a diagram that gives a visual representation of the floor space. Represent the width as w and show the length in terms of w.

(b) Write a quadratic equation in terms of w.

(c) Find the length and width of the floor of the building.

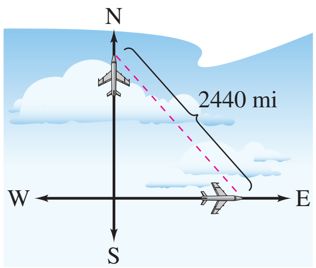
189. ***Packaging*** An open box with a square base (see figure) is to be constructed from 84 square inches of material. The height of the box is 2 inches. What are the dimensions of the box? (*Hint:* The surface area is $S = x^2 + 4xh$.)



190. ***Geometry*** The hypotenuse of an isosceles right triangle is 5 centimeters long. How long are its sides?

191. ***Geometry*** An equilateral triangle has a height of 10 inches. How long is one of its sides? (*Hint:* Use the height of the triangle to partition the triangle into two congruent right triangles.)

192. ***Flying Speed*** Two planes leave simultaneously from Chicago's O'Hare Airport, one flying due north and the other due east (see figure). The northbound plane is flying 50 miles per hour faster than the eastbound plane. After 3 hours, the planes are 2440 miles apart. Find the speed of each plane.



193. ***Voting Population*** The total voting-age population P (in millions) in the United States from 1990 to 2002 can be modeled by

$$P = \frac{182.45 - 3.189t}{1.00 - 0.026t}, \quad 0 \le t \le 12$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: U.S. Census Bureau)

(a) In which year did the total voting-age population reach 200 million?

(b) Use the model to predict when the total voting-age population will reach 230 million. Is this prediction reasonable? Explain.

194. ***Airline Passengers*** An airline offers daily flights between Chicago and Denver. The total monthly cost C (in millions of dollars) of these flights is $C = \sqrt{0.2x + 1}$ where x is the number of passengers (in thousands). The total cost of the flights for June is 2.5 million dollars. How many passengers flew in June?

195. ***Demand*** The demand equation for a video game is modeled by $p = 40 - \sqrt{0.01x + 1}$ where x is the number of units demanded per day and p is the price per unit. Approximate the demand when the price is \$37.55.

196. ***Demand*** The demand equation for a high definition television set is modeled by

$$p = 800 - \sqrt{0.01x + 1}$$

where x is the number of units demanded per month and p is the price per unit. Approximate the demand when the price is \$750.

Synthesis

True or False? **In Exercises 197–200, determine whether the statement is true or false. Justify your answer.**

197. The equation $x(3 - x) = 10$ is a linear equation.

198. If $(2x - 3)(x + 5) = 8$, then either $2x - 3 = 8$ or $x + 5 = 8$.

199. An equation can never have more than one extraneous solution.

200. When solving an absolute value equation, you will always have to check more than one solution.

201. ***Think About It*** What is meant by *equivalent equations*? Give an example of two equivalent equations.

202. ***Writing*** Describe the steps used to transform an equation into an equivalent equation.

203. To solve the equation $2x^2 + 3x = 15x$, a student divides each side by x and solves the equation $2x + 3 = 15$. The resulting solution $(x = 6)$ satisfies the original equation. Is there an error? Explain.

204. Solve $3(x + 4)^2 + (x + 4) - 2 = 0$ in two ways.

(a) Let $u = x + 4$, and solve the resulting equation for u. Then solve the u-solution for x.

(b) Expand and collect like terms in the equation, and solve the resulting equation for x.

(c) Which method is easier? Explain.

Think About It **In Exercises 205–210, write a quadratic equation that has the given solutions. (There are many correct answers.)**

205. -3 and 6

206. -4 and -11

207. 8 and 14

208. $\frac{1}{6}$ and $-\frac{2}{5}$

209. $1 + \sqrt{2}$ and $1 - \sqrt{2}$

210. $-3 + \sqrt{5}$ and $-3 - \sqrt{5}$

In Exercises 211 and 212, consider an equation of the form $x + |x - a| = b$, where a and b are constants.

211. Find a and b when the solution of the equation is $x = 9$. (There are many correct answers.)

212. ***Writing*** Write a short paragraph listing the steps required to solve this equation involving absolute values and explain why it is important to check your solutions.

213. Solve each equation, given that a and b are not zero.

(a) $ax^2 + bx = 0$

(b) $ax^2 - ax = 0$