Part IV: Texas Instruments TI-89

IV.1 Getting started with the TI-89

> In this guide, the key with the green diamond symbol inside a green border will be indicated by ◆, the key with the white arrow pointing up inside a white border (the *shift* key) will be indicated by ⇧, and the key with the white arrow (the *backspace* key) pointing to the left will be indicated by ←.
>
> There are 5 blue keys below the calculator screen labeled F1 through F5. These *function keys* have different effects depending on the screen that is currently showing. The effect or menu of the function keys corresponding to a screen are shown across the top of the display.

IV.1.1 Basics: Press the ON key to begin using your TI-89. If you need to adjust the display contrast, first press and hold ◆, then press − (the minus key) to lighten or + (the plus key) to darken. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF.

Check our TI-89's settings by pressing MODE. If necessary, use the arrow keys to move the blinking cursor to a setting you want to change. You can also use F1 to go to page 1, F2 to go to page 2, and F3 to go to page 3 of the MODE menu. To change a setting, use ▼ to get to the setting that you want to change, then press ▶ to see the options available. Use ▲ or ▼ to highlight the setting you want and press ENTER to select the setting. To start with, select the options shown in Figures IV.1, IV.2, and IV.3: function graphs, main folder, floating decimals with 10 digits displayed, radian measure, normal exponential format, real numbers, rectangular vectors, pretty print, full screen display, home screen showing, approximate calculation mode, decimal base, international system of units (metric measurement), and English. Note that some of the lines on page 2 and page 3 of the MODE menu are not readable. These lines pertain to options that are not set as above. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing HOME or 2nd QUIT. Some of the current settings are shown on the status line of the home screen.

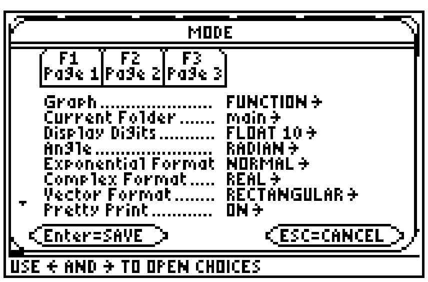


Figure IV.1: MODE menu, page 1

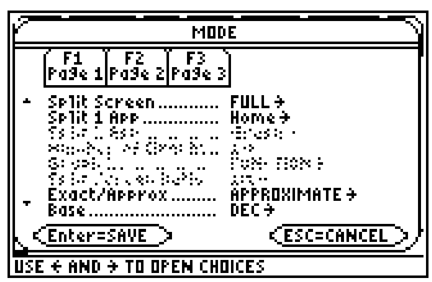


Figure IV.2: MODE menu, page 2

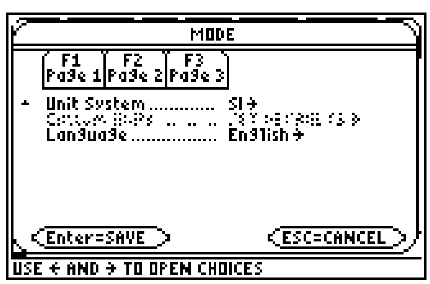


Figure IV.3: MODE menu, page 3

IV.1.2: Editing: One advantage of the TI-89 is that you can use the arrow keys to scroll in order to see a long calculation. For example, type this sum (Figure IV.4):

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20

Then press ENTER to see the answer. The sum is too long for both the entry line and the history area. The direction(s) in which the line extends off the screen is indicated by an ellipsis at the end of the entry line and arrows (◀ or ▶) in the history area. You can scroll through the entire calculation by using ▲ or ▼ to put the cursor on the appropriate line and then using ◀ or ▶ to move the cursor to the part of the calculation that you wish to see.

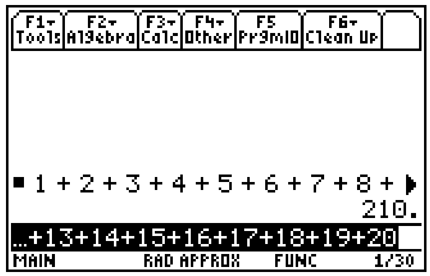


Figure IV.4: Home screen

Often we do not notice a mistake until we see how unreasonable an answer is. The TI-89 permits you to redisplay an entire calculation, edit it easily, then execute the *corrected* calculation.

Suppose you had typed 12 + 34 + 56 as in Figure IV.5 but had *not yet* pressed ENTER, when you realize that 34 should have been 74. Simply press ◀ as many times as necessary to move the blinking cursor line until it is to the immediate right of the 3, press ⇐ to delete the 3, and then type 7. On the other hand if 34 should have been 384, move the cursor until it is between the 3 and the 4 and then type 8. If the 34 should have been 3 only, move the cursor to the right of the 4, and press ⇐ to delete the 4.

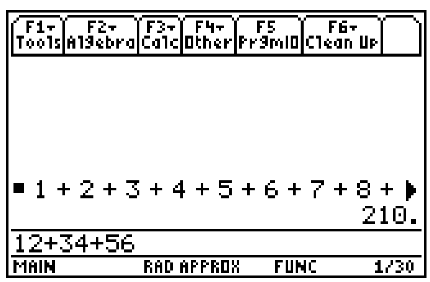


Figure IV.5: Editing a calculation

Technology Tip: The TI-89 has two different inputting modes: *insert* and *overtype*. The default mode is insert mode, in which the cursor is a blinking vertical line and new text will be inserted at the cursor's position and other characters are pushed to the right. In the overtype mode, the cursor is a blinking square and the characters that you type replace the existing characters. To change from one mode to another, press 2nd INS. The TI-89 remains in whatever the last input mode was, even after being turned off.

Even if you had pressed ENTER, you may still edit the previous expression. Immediately after you press ENTER your entry remains on the entry line. Pressing ◀ moves the cursor to the beginning of the line, while pressing ▶ puts the cursor at the end of the line. Now the expression can be edited as above. To edit a previous expression that is no longer on the entry line, press 2nd and then ENTRY to *recall* the prior expression. Now you can change it. In fact, the TI-89 retains as many entries as the current history area holds in a “last entry” storage area, including entries that have scrolled off the screen. Press 2nd ENTRY repeatedly until the previous line you want is on the entry line. (The number of entries that the history area can hold may be changed, see your user’s manual for more information.)

To clear the entry line, press CLEAR while the cursor is on that line. To clear previous entry/answer pairs from the history area, use ▲ or ▼ to move the cursor to either the entry or the answer and press CLEAR (both the entry and the answer will be deleted from the display). To clear the entire history area, press F1 *[Tools]* 8 *[Clear Home]*, although this will not clear the entry line.

Technology Tip: When you need to evaluate a formula for different values of a variable, use the editing feature to simplify the process. For example, suppose you want to find the balance in an investment account if there is now $5000 in the account and interest is compounded annually at the rate of 8.5%. The formula for the balance is $P = \left(1 + \frac{r}{n}\right)^{nt}$, where P = principal, r = rate of interest (expressed as a decimal), n = number of times interest is compounded each year, and t = number of years. In our example, this becomes $5000(1 + .085)^t$. Here are the keystrokes for finding the balance after t = 3, 5, and 10 years (results are shown in Figure IV.6).

Years	*Keystrokes*	*Balance*
3	5000 (1 + .085) ^ 3 ENTER	$6386.45
5	▶ ← 5 ENTER	$7518.28
10	▶ ← 10 ENTER	$11,304.92

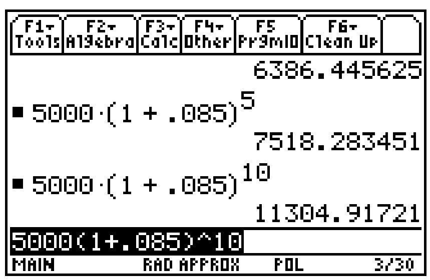


Figure IV.6: Editing expressions

Then to find the balance from the same initial investment but after 5 years when the annual interest rate is 7.5%, press the following keys to change the last calculation above: ▶ ← ← 5 ◀ ◀ ◀ ◀ ← 7 ENTER. You could also use the CLEAR key to erase everything to the right of the current location of the cursor. Then, changing the calculation from 10 years at the annual interest rate of 8.5% to 5 years at the annual interest rate of 7.5% is then done by pressing ▶ ◀ ◀ CLEAR 5 ◀ ◀ ◀ ◀ ← 7 ENTER.

IV.1.3 Key Functions: Most keys on the TI-89 offer access to more than one function, just as the keys on a computer keyboard can produce more than one letter (“g” and “G”) or even quite different characters (“5” and “%”). The primary function of a key is indicated on the key itself, and you access that function by a simple press on the key.

To access the *second* function indicated in yellow or to the *left* above a key, first press 2nd (“2nd” appears on the status line) and *then* press the key. For example, to calculate $\sqrt{25}$ press 2nd √ 25) ENTER.

Technology Tip: The TI-89 automatically places a left parenthesis, (, after many functions and operators (including 2nd LN, ◆ e^x, 2nd SIN, 2nd COS, 2nd TAN, and 2nd $\sqrt{\ }$). If a right parenthesis is not entered, the TI-89 will respond with an error message indicating that the right parenthesis is missing.

When you want to use a function printed in green or to the *right* above a key, first press ◆ ("◆" appears on the status line) and then press the key. For example, if you are in exact calculation mode and want to find the approximate value of $\sqrt{45}$ press 2nd $\sqrt{\ }$ 45) ◆ ≈.

The TI-89 can produce both upper and lower case letters. When you want to use a lower case letter printed in purple or to the *right* above a key, first press alpha (a lower case "a" appears on the status line) and press the key. For example, to use the letter k in a formula, press alpha K. If you need several letters in a row, press 2nd a-lock, and then press all the letters you want. Remember to press alpha when you are finished and want to restore the keys to their primary functions. To type upper case letters, press ⇧ and then press the letter. To lock in upper case letters, press ⇧ alpha (an upper case "A" appears on the status line). To restore the keys to their primary functions, press alpha.

Technology Tip: There are separate keys for the commonly used letters X, Y, Z, and T. A simple press of the key will produce a lower case letter while pressing ⇧ and then the key will produce an upper case letter.

IV.1.4 Order of Operations: The TI-89 performs calculations according to the standard algebraic rules. Working outwards from inner parentheses, calculations are performed from left to right. Powers and roots are evaluated first, followed by multiplications and divisions, and then additions and subtractions.

Note that the TI-89 distinguishes between *subtraction* and the *negative* sign. If you wish to enter a negative number, it is necessary to use the (-) key. For example, you would calculate $-5 - (-4 \cdot -3)$ by pressing (-) 5 − (4 × (-) 3) ENTER to get 7.

Enter these expressions to practice using your TI-89.

Expression	*Keystrokes*	*Display*
$7 - 5 \cdot 3$	7 − 5 × 3 ENTER	-8
$(7 - 5) \cdot 3$	(7 − 5) × 3 ENTER	6
$120 - 10^2$	120 − 10 ^ 2 ENTER	20
$(120 - 10)^2$	(120 − 10) ^ 2 ENTER	12100
$\frac{24}{2^3}$	24 ÷ 2 ^ 3 ENTER	3
$\left(\frac{24}{2}\right)^3$	(24 ÷ 2) ^ 3 ENTER	1728
$(7 - -5) \cdot -3$	(7 − (-) 5) × (-) 3 ENTER	-36

IV.1.5 Algebraic Expressions and Memory: Your calculator can evaluate expressions such as $\frac{N(N+1)}{2}$ *after* you have entered a value for N. Suppose you want $N = 200$. Press 200 STO ▶ alpha N ENTER to store the value 200 in memory location N. Whenever you use N in an expression, the calculator will substitute the value 200 until you make a change by storing another number in N. Next, enter the expression $\frac{N(N+1)}{2}$ by typing alpha N × (alpha N + 1) ÷ 2 ENTER. For $N = 200$, you find that $\frac{N(N+1)}{2} = 20{,}100$. Note that there is no distinction made between upper and lower case letters in this case.

The contents of any memory location may be revealed by typing just its letter name and then ENTER. And the TI-89 retains memorized values even when it is turned off, so long as its batteries are good.

IV.1.6 Repeated Operations with ANS: As many entry/answer pairs as the history area shows are stored in memory. The last result displayed can be entered on the entry line by pressing 2nd ANS, while the last entry computed is entered on the entry line by pressing 2nd ENTRY. This makes it easy to use the answer from one computation in another computation. For example, press 30 + 15 ENTER so that 45 is the last result displayed. Then press 2nd ANS ÷ 9 ENTER and get 5 because 45 ÷ 9 = 5.

The answer locations are indexed by ans(#), where # indicates the number of the answer. The pairs are numbered with the most recent computation as 1. So, the number of a pair changes with each successive computation that is entered. The number of an entry or answer can be found by using ▲ to scroll up to the entry or answer. The number, which is the same for both the entry and the answer, is shown on the status line.

To use an earlier answer or entry in a computation, to calculate, say 15 times answer 3 plus 75, press 15 × 2nd a-lock A N S alpha (3) + 75 ENTER.

With a function like division, you press ÷ after you enter an argument. For such functions, whenever you would start a new calculation with the previous answer followed by pressing the function key, you may press just the function key. So instead of 2nd ANS ÷ 9 in the previous example, you could have pressed simply ÷ 9 to achieve the same result. This technique also works for these functions: + – × ^.

Here is a situation where this is especially useful. Suppose a person makes $5.85 per hour and you are asked to calculate earnings for a day, a week, and a year. Execute the given keystrokes to find the person's incomes during these periods (results are shown in Figure IV.7).

Pay Period	*Keystrokes*	*Earnings*
8-hour day	5.85 × 8 ENTER	$46.80
5-day week	× 5 ENTER	$234
52-week year	× 52 ENTER	$12,168

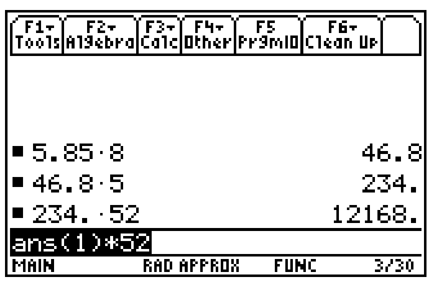


Figure IV.7: ANS variable

IV.1.7 The MATH Menu: Operators and functions associated with a scientific calculator are available either immediately from the keys of the TI-89 or by the 2nd keys. You have direct access to common arithmetic operations (2nd √, ^), trigonometric functions (2nd SIN, 2nd COS, 2nd TAN), and their inverses (◆ SIN^{-1} , ◆ COS^{-1} , ◆ TAN^{-1}), exponential and logarithmic functions (2nd e^x, 2nd LN), and a famous constant (2nd π).

A significant difference between the TI-89 graphing calculators and most scientific calculators is that TI-89 requires the argument of a function *after* the function, as you would see in a formula written in your textbook. For example, on the TI-89 you calculate $\sqrt{16}$ by pressing the keys 2nd √ 16) in that order.

Here are keystrokes for basic mathematical operations. Try them for practice on your TI-89.

Expression	*Keystrokes*	*Display*
$\sqrt{3^2 + 4^2}$	2nd √ 3 ^ 2 + 4 ^ 2) ENTER	5
$2\frac{1}{3}$	2 + 3 ^ (-) 1 ENTER	2.333333333
ln 200	2nd LN 200) ENTER	5.298317367
$2.34 \cdot 10^5$	2.34 × 10 ^ 5 ENTER	234000

Technology Tip: Note that if you had set the calculation mode to either AUTO or EXACT (page 2 of the MODE menu), the TI-89 would display $\frac{7}{3}$ for $2\frac{1}{3}$ and ln 200 for ln 200. So, you can use either fractions and exact numbers or decimal approximations. The AUTO mode will give exact rational results whenever all of the numbers entered are rational, and decimal approximations for other results.

Additional mathematical operations and functions are available from the MATH menu. Press 2nd MATH to see the various sub-menus (Figure IV.8). Press 1 *[Number]* or just ENTER to see the options available under the Number sub-menu (Figure IV.9). You will learn in your mathematics textbook how to apply many of them. As an example, calculate the remainder of 437 when divided by 49 by pressing 2nd MATH 1 *[Number]* then *either* alpha A *[remain(]* or ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ENTER; finally press 437 , 49) ENTER to see 45. To leave the MATH menu (or any other menu) and take no other action, press 2nd QUIT or just ESC.

Note that you can select a function or a sub-menu from the current menu by pressing either ▼ until the desired item is highlighted and then ENTER, or by pressing the number or letter corresponding to the function or sub-menu. It is easier to press alpha A than to press ▼ nine times to get the remain(function.

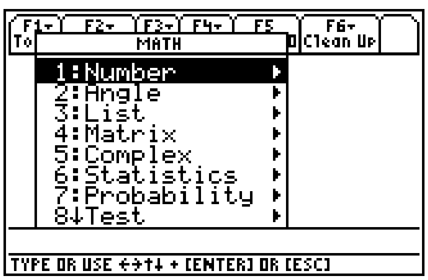


Figure IV.8: MATH menu

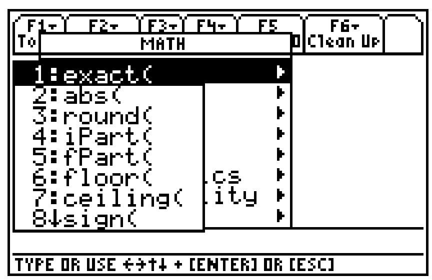


Figure IV.9: Number sub-menu

The *factorial* of a nonnegative integer is the *product* of *all* the integers from 1 up to the given integer. The symbol for factorial is the exclamation point. So 4! (pronounced *four factorial*) is $1 \cdot 2 \cdot 3 \cdot 4 = 24$. You will learn more about applications of factorials in your textbook, but for now use the TI-89 to calculate 4!. The factorial command is located in the MATH menu's Probability sub-menu. To compute 4!, press these keystrokes: 4 2nd MATH 7 *[Probability]* 1 *[!]* ENTER.

On the TI-89 it is possible to do calculations with complex numbers. To enter the imaginary number *i*, press 2nd *i*. For example, to divide $2 + 3i$ by $4 - 2i$, press (2 + 3 2nd *i*) ÷ (4 − 2 2nd *i*) ENTER. The result is $0.1 + 0.8i$ (Figure IV.10).

To find the complex conjugate of $4 + 5i$ press 2nd MATH 5 *[Complex]* 1 *[conj(]* 4 + 5 2nd *i*) ENTER (Figure IV.10).

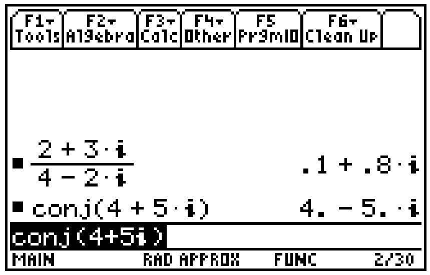


Figure IV.10: Complex number calculations

The TI-89 can also solve for the real and complex solutions of an equation. This is done by using the cSolve(function which is found in the Algebra sub-menu of the MATH menu.

The format of cSolve(is cSolve(*expression*, *variable)*. For example, to find the zeros of $f(x) = x^3 - 4x^2 + 14x - 20$, press 2nd MATH 9 *[Algebra]* alpha A *[Complex]* 1 *[cSolve(]*. To complete the computation, press X ^ 3 − 4 X ^ 2 + 14 X − 20 = 0 , X) ENTER. The TI-89 will display real and complex solutions of the equation, as shown in Figure IV.11.

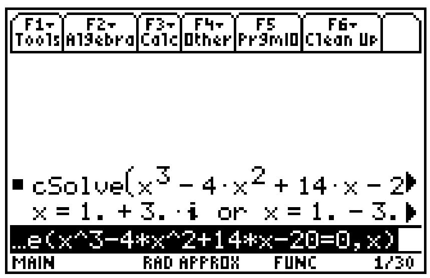


Figure IV.11: cSolve function

Note that this calculation and the answer are too long for both the entry line and the history area. You can scroll through the entire calculation or answer by using ▲ or ▼ to put the cursor on the appropriate line and then using ◀ or ▶ to move the cursor to the part of the calculation or answer that you wish to see.

IV.2 Functions and Graphs

IV.2.1 Evaluating Functions: Suppose you receive a monthly salary of \$1975 plus a commission of 10% of sales. Let $x =$ your sales in dollars; then your wages W in dollars are given by the equation $W = 1975 + .10x$. If your January sales were \$2230 and your February sales were \$1865, what was your income during those months?

Here's one method to use your TI-89 to perform this task. Press ◆ Y= (above F1 key) or APPS 2 *[Y= Editor]* to display the function editing screen (Figure IV.12). You may enter as many as 99 different functions for the TI-89 to use at one time. If there is already a function y1, press ▲ or ▼ as many times as necessary to move the cursor to y1 and then press CLEAR to delete whatever was there. Then enter the expression $1975 + .10x$ by pressing these keys: 1975 + .10 X ENTER. Now press HOME.

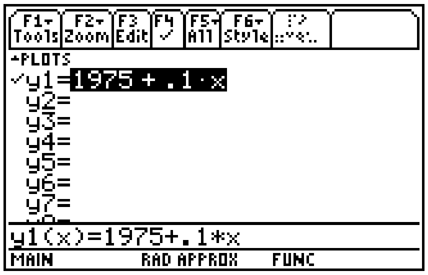


Figure IV.12: Y= screen

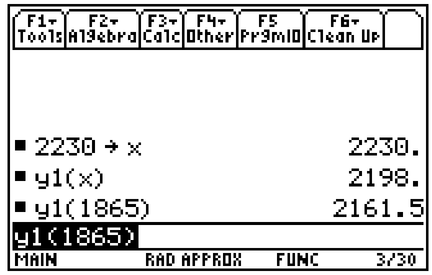


Figure IV.13: Evaluating a function

Assign the value 2230 to the variable x by using these keystrokes (see Figure IV.13): 2230 STO ▶ X ENTER. Then press the following keystrokes to evaluate y1 and find January's wages: Y 1 (X) ENTER, completes the calculation. It is not necessary to repeat all these steps to find the February wages. Simply press ▶ to begin editing the previous entry, change X to 1865, and press ENTER (see Figure IV.13).

You may also have TI-89 make a table of values for the function. Press ◆ TblSet to set up the table (Figure IV.14). Move the blinking cursor down to the fourth line beside Independent, then press ▶ and 2 *[ASK]* ENTER. This configuration permits you to input values for x one at a time. Now press ◆ TABLE or APPS 5 *[Table]*, enter 2230 in the x column, and press ENTER (see Figure IV.15). Press ▼ to move to the next line and continue to enter additional values for x. The TI-89 automatically completes the table with the corresponding values of y1. Press 2nd QUIT to leave the TABLE screen.

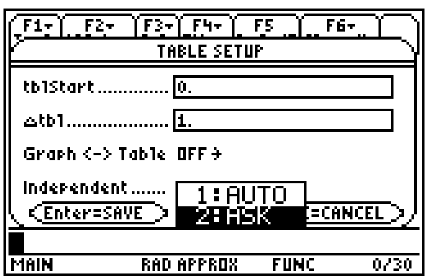


Figure IV.14: TABLE SETUP screen

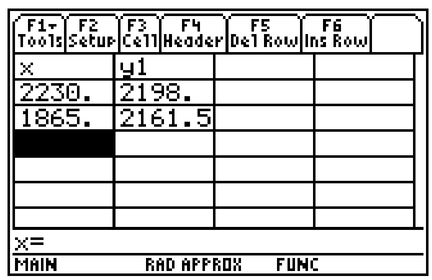


Figure IV.15: Table of values

Technology Tip: The TI-89 *requires* multiplication to be expressed between variables, so xxx does not mean x^3, rather it is a new variable named xxx. So, you must use either ×'s between the x's or ^ for powers of x. Of course, expressed multiplication is not required between a constant and a variable. See your TI-89 manual for more information about the allowed usage of implied multiplication.

IV.2.2 Functions in a Graph Window: Once you have entered a function in the Y= screen of the TI-89, just press ◆ GRAPH to see its graph. The ability to draw a graph contributes substantially to our ability to solve problems.

For example, here is how to graph $y = -x^3 + 4x$. First press ◆ Y= and delete anything that may be there by moving with the arrow keys to y1 or to any of the other lines and pressing CLEAR wherever necessary. Then, with the cursor on the (now cleared) top line (y1), press (-) X ^ 3 + 4 X ENTER to enter the function (as in Figure IV.16). Now press ◆ GRAPH and the TI-89 changes to a window with the graph of $y = -x^3 + 4x$ (Figure IV.18).

While the TI-89 is calculating coordinates for a plot, it displays the word BUSY on the status line.

Technology Tip: If you would like to see a function in the Y= menu and its graph in a graph window, both at the same time, press MODE to open the MODE menu and press F2 to go to the second page. The cursor will be next to Split Screen. Select either TOP-BOTTOM or LEFT-RIGHT by pressing ▶ and 2 or 3, respectively. Now the 2 lines below the Split 1 App line have become readable, because these options apply only when the calculator is in the split screen mode. The Split 1 App will automatically be the screen you were on prior to pressing MODE. You can choose what you want the top or left-hand screen to show by moving down to the Split 1 App line, pressing ▶ and the number of the application you want in that window. The Split 2 App determines what is shown in the bottom or right-hand window. Press ENTER to confirm your choices and your TI-89's screen will now be divided either horizontally or vertically (as you choose). Figure IV.16 show the graph and the Y= screen with the settings shown in Figure IV.17. The split screen is also useful when you need to do some calculations as you trace along a graph. In split screen mode, one side of the screen will be more heavily outlined. This is the active screen, i.e., the screen that you can currently modify. You can change which side is active by using 2nd to access the symbol above the APPS key. For now, restore the TI-89 to Full screen.

Technology Tip: Note that if you set one part of your screen to contain a table and the other to contain a graph, the table will not necessarily correspond to the graph unless you use ◆ TblSet to generate a new table based on the functions being graphed (as in Section IV.2.1).

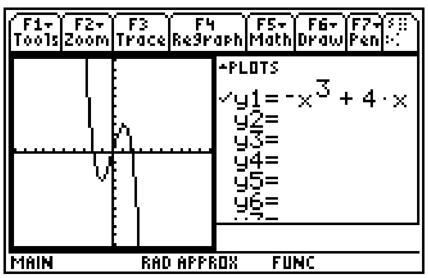


Figure IV.16: Split screen: LEFT-RIGHT

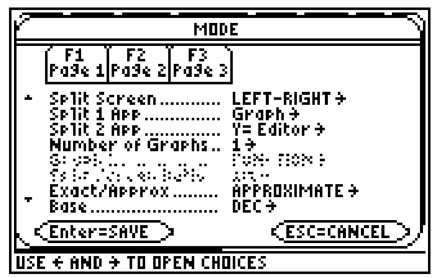


Figure IV.17: MODE settings for Figure IV.16

Your graph window may look like the one in Figure IV.18 or it may be different. Because the graph of $y = -x^3 + 4x$ extends infinitely far left and right and also infinitely far up and down, the TI-89 can display only a piece of the actual graph. This displayed rectangular part is called a *viewing window*. You can easily change the viewing window to enhance your investigation of a graph.

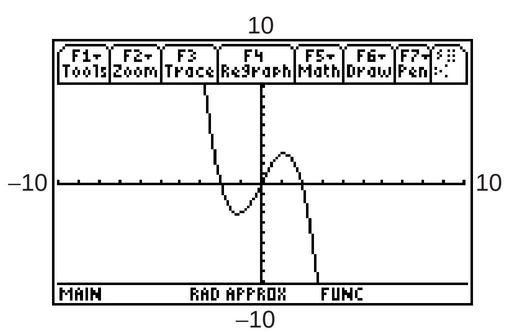


Figure IV.18: Graph of $y = -x^3 + 4x$

The viewing window in Figure IV.18 shows the part of the graph that extends horizontally from -10 to 10 and vertically from -10 to 10. Press ◆ WINDOW to see information about your viewing window. Figure IV.19 shows the WINDOW screen that corresponds to the viewing window in Figure IV.18. This is the *standard* viewing window for the TI-89.

The variables xmin and xmax are the minimum and maximum x-values of the viewing window; ymin and ymax are the minimum and maximum y-values.

xscl and yscl set the spacing between the tick marks on the axes.

xres sets pixel resolution (1 through 10) for function graphs.

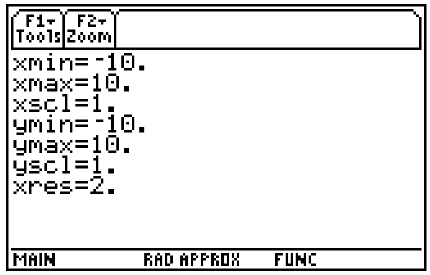


Figure IV.19: Standard WINDOW

Technology Tip: Small xres values improve graph resolution, but may cause the TI-89 to draw graphs more slowly.

Use ▲ and ▼ to move up and down from one line to another in this list; pressing the ENTER key will move down the list. Press CLEAR to delete the current value and then enter a new value. You may also edit the entry as you would edit an expression. Remember that a minimum *must* be less than the corresponding maximum or the TI-89 will issue an error message. Also, remember to use the (-) key, not − (which is subtraction), when you want to enter a negative value. Figures IV.18-19, IV.20-21, and IV.22-23 show different WINDOW screens and the corresponding viewing window for each one.

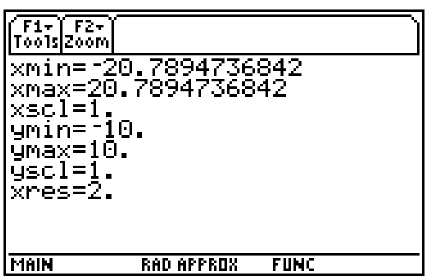


Figure IV.20: Square WINDOW

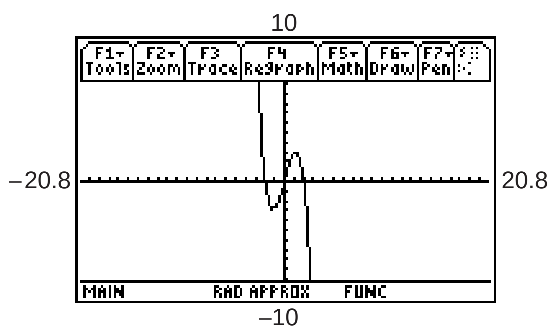


Figure IV.21: Graph of $y = -x^3 + 4x$

To initialize the viewing window quickly to the standard viewing window (Figure IV.19), press F2 *[Zoom]* 6 *[ZoomStd]*. To set the viewing window quickly to a square window (Figure IV.20), press F2 5 *[ZoomSqr]*. More information about square windows is presented later in Section IV.2.4.

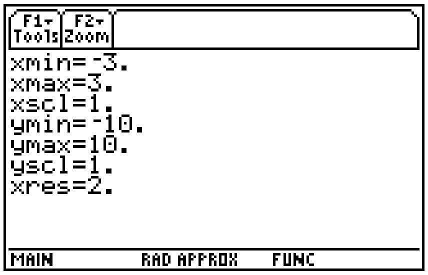


Figure IV.22: Custom WINDOW

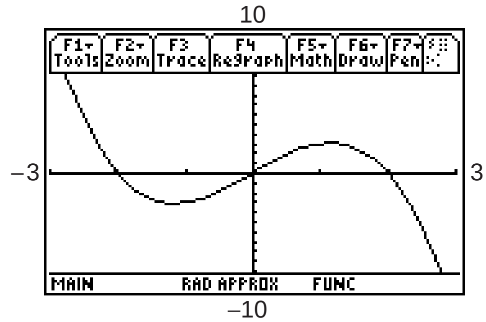


Figure IV.23: Graph of $y = x^3 + 4x$

Sometimes you may wish to display grid points corresponding to tick marks on the axes. This and other graph format options may be changed while you are viewing the graph by pressing F1 to get the Tools menu (Figure IV.24) and then pressing 9 *[Format]* to display the Format menu (Figure IV.25) or by pressing ◆ | as indicated on the Tools menu in Figure IV.24. Move the blinking cursor to Grid; press ▶ 2 *[ON]* ENTER to redraw the graph. Figure IV.26 shows the same graph as in Figure IV.23 but with the grid turned on.

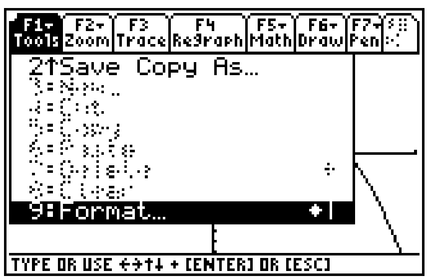


Figure IV.24: Tools menu

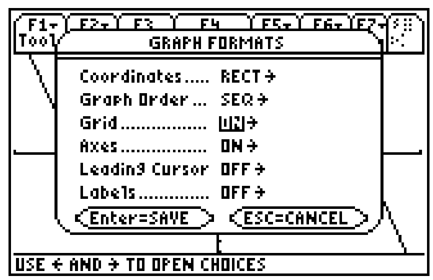


Figure IV.25: Format menu

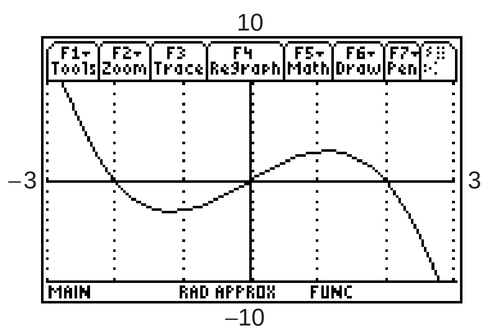


Figure IV.26: Grid turned on for $y = -x^3 + 4x$

In general, you'll want the grid turned *off*, so do that now by pressing ◆ | and turning the Grid option to OFF, then pressing ENTER.

IV.2.3 Graphing Step and Piecewise-Defined Functions: The greatest integer function, written $[\![x]\!]$, gives the greatest *integer* less than or equal to a number x. On the TI-89, the greatest integer function is called floor(and is located under the Number sub-menu of the MATH menu (Figures IV.8-9). So, calculate $[\![6.78]\!] = 6$ by pressing 2nd MATH 1 6 *[floor(]* 6.78) ENTER.

To graph $y = [\![x]\!]$, go into the Y= menu, move beside y1 and press CLEAR 2nd MATH 1 6 X) ENTER ◆ GRAPH. Figure IV.27 show this graph in a viewing window from -5 to 5 in both directions.

The true graph of the greatest integer function is a step graph, like the one in Figure IV.28. For the graph of $y = [\![x]\!]$, a segment should not be drawn between every pair of successive points. You can change this graph from a Line to a Dot graph on the TI-89 by going to the Y= screen, moving the cursor up until this function is selected (highlighted) and then pressing 2nd F6 *[Style]*. This opens the Graph Style menu. Move the cursor down to the second line and press ENTER or press 2; to have the selected graph plotted in Dot style. Now press ◆ GRAPH to see the result.

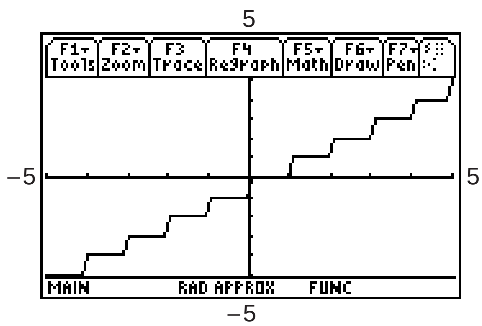


Figure IV.27: Line graph of $y = [\![x]\!]$

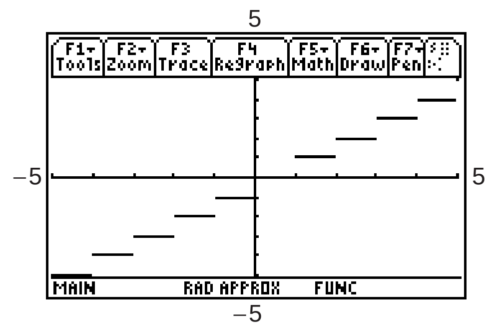


Figure IV.28: Dot graph of $y = [\![x]\!]$

Technology Tip: When graphing functions in the Dot style, it improves the appearance of the graph to set xres to 1. Figure IV.28 was graphed with xres $= 1$. Also, the default graph style is Line, so you have to set the style to Dot each time you wish to graph a function in Dot mode.

The TI-89 can graph piecewise-defined functions by using the when(function. The when(function is not on any of the keys but can be found in the CATALOG or typed from the keys. The format of the when(function is when(*condition*, *trueResult*, *falseResult*, *unknownResult*) where the *falseResult* and *unknownResult* are optional arguments.

For example, to graph the function $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$ (using Dot graph), you want to graph $x^2 + 2$ when the condition $x < 0$ is true and graph $x - 1$ when the condition is false. First, clear any existing functions in the Y= screen. Then move to the y1 line and press 2nd a-lock W H E N alpha (X 2nd < 0 , X ^ 2 + 2 , X − 1) ENTER (Figure IV.29). Then press ◆ GRAPH to display the graph. Figure IV.30 shows this graph in a viewing window from -5 to 5 in both directions. This was done in Dot style, because the TI-89 will (incorrectly) connect the two sides of the graph at $x = 0$ if the function is graphed in Line style.

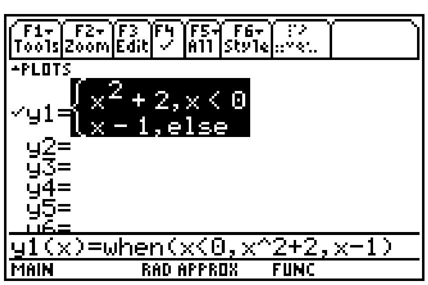


Figure IV.29: Piecewise-defined function

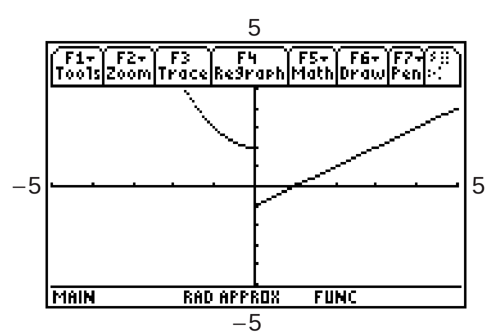


Figure IV.30: Piecewise-defined graph

Other *test* functions, such as ≤, ≥, and ≠ as well as logic operators can be found on the Test sub-menu of the MATH menu.

IV.2.4 Graphing a Circle: Here is a useful technique for graphs that are not functions but can be "split" into a top part and a bottom part, or into multiple parts. Suppose you wish to graph the circle whose equation is $x^2 + y^2 = 36$. First solve for y and get an equation for the top semicircle, $y = \sqrt{36 - x^2}$, and for the bottom semicircle, $y = -\sqrt{36 - x^2}$. Then graph the two semicircles simultaneously.

Use the following keystrokes to draw the circle's graph. First clear any existing functions on the Y= screen. Enter $\sqrt{36 - x^2}$ as y1 and $-\sqrt{36 - x^2}$ as y2 (see Figure IV.31) by pressing 2nd √ 36 − X ^ 2) ENTER (-) 2nd √ 36 − X ^ 2) ENTER. Then press ◆ GRAPH to draw them both (Figure IV.32).

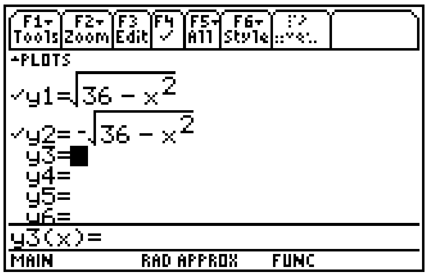


Figure IV.31: Two semicircles

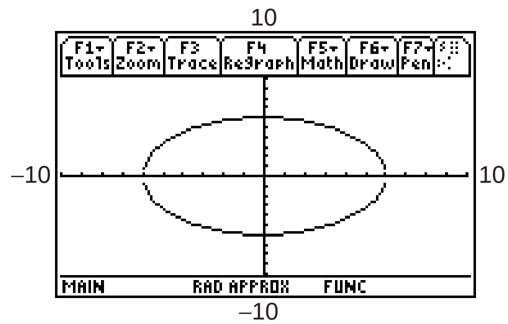


Figure IV.32: Circle's graph – standard WINDOW

If your range were set to the standard viewing window, your graph would look like Figure IV.32. Now this does *not* look like a circle, because the units along the axes are not the same. This is where the square viewing window is important. Press F2 5 and see a graph that appears more circular.

Technology Tip: Another way to get a square graph is to change the range variables so that the value of ymax − ymin is approximately $\frac{38}{79}$ times xmax − xmin. For example, see the WINDOW in Figure IV.33 to get the corresponding graph in Figure IV.34. This method works because the dimensions of the TI-89's display are such that the ratio of vertical to horizontal is approximately $\frac{38}{79}$.

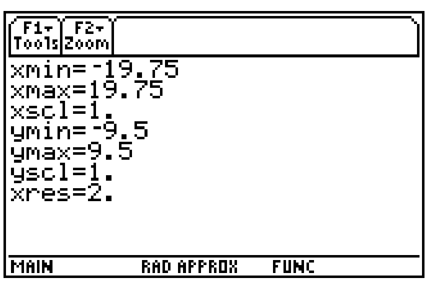


Figure IV.33: $\frac{\text{vertical}}{\text{horizontal}} = \frac{19}{39.5} = \frac{38}{79}$

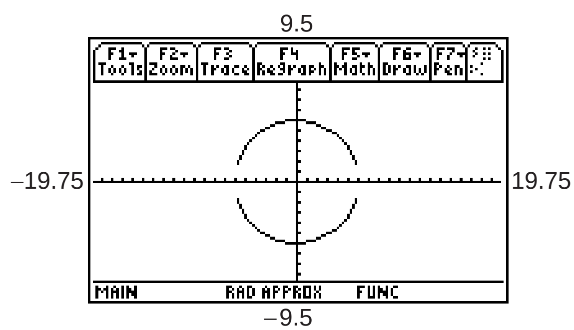


Figure IV.34: A "square" circle

The two semicircles in Figure IV.34 do not connect because of an idiosyncrasy in the way the TI-89 plots a graph.

Back when you entered $-\sqrt{36 - x^2}$ as y2, you could have entered -y1 as y2 and saved some keystrokes. Try this by going into the Y= screen and pressing ▲ to move the cursor up to y2. Then press CLEAR (-) Y 1 (X) ENTER. The graph should be as before.

IV.2.5 Trace: Graph the function $y = -x^3 + 4x$ from Section IV.2.2 using the standard viewing window. (Remember to clear any other functions in the Y= screen.) Press any of the cursor directions ▲ ▼ ▶ ◀ and see the cursor move from the center of the viewing window. The coordinates of the cursor's location are displayed at the bottom of the screen, as in Figure IV.35, in floating decimal format. This cursor is called a *free-moving cursor* because it can move from dot to dot *anywhere* in the graph window.

Remove the free-moving cursor and its coordinates from the window by pressing ◆ GRAPH, CLEAR, ESC, or ENTER. Press the cursor directions again and the free-moving cursor will reappear at the same point you left it.

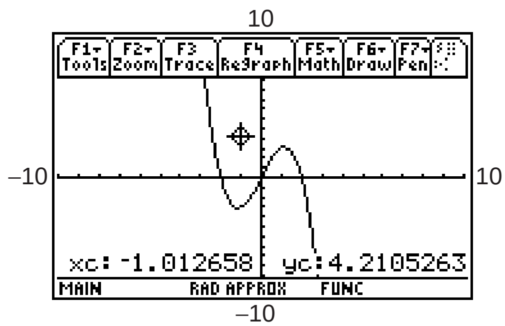


Figure IV.35: Free-moving cursor

Press F3 *[Trace]* to enable the left ◀ and right ▶ directions to move the cursor from point to point along the graph of the function. The cursor is no longer free-moving, but is now constrained to the function. The coordinates that are displayed belong to points on the function's graph, so the *y*-coordinate is the calculated value of the function at the corresponding *x*-coordinate (Figure IV.36).

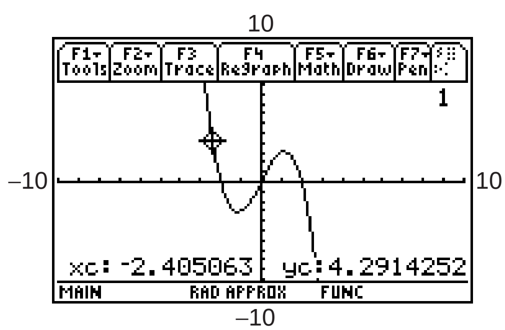


Figure IV.36: Trace

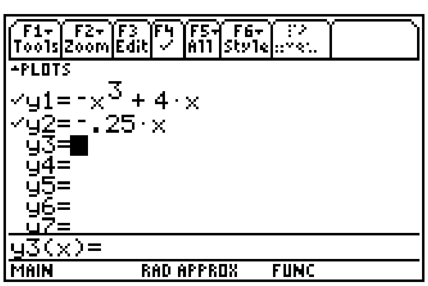


Figure IV.37: Two functions

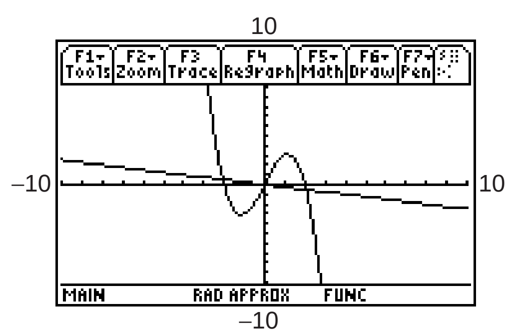


Figure IV.38: $y = -x^3 + 4x$ and $y = -.25x$

Now plot a second function, $y = -.25x$, along with $y = -x^3 + 4x$. Press ◆ Y= and enter $-.25x$ for y2, then press ◆ GRAPH to see both functions.

Notice that in Figure IV.37 there are checkmarks ✓ to the left of *both* y1 and y2. This me that *both* functions will be graphed, as shown in Figure IV.38. In the Y= screen, move the cursor onto y1 and press F4 *[✓]*. The checkmark left of y1 should disappear (Figure IV.39). Now press ◆ GRAPH and see that only y2 is plotted (Figure IV.40).

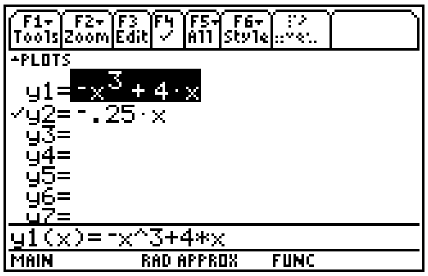


Figure IV.39: Only y2 active

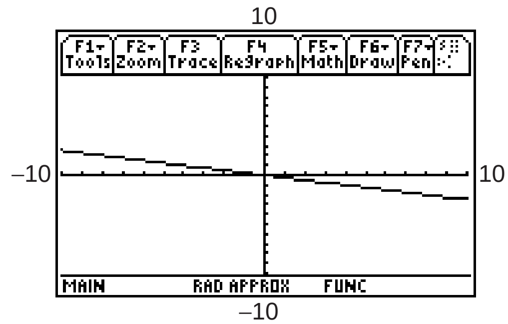


Figure IV.40: Graph of $y = -.25x$

Many different functions can be stored in the Y= list and any combination of them may be graphed simultaneously. You can make a function active or inactive for graphing by pressing F4 when the function is highlighted to add a checkmark (activate) or remove the checkmark (deactivate). Now go back to the Y= screen and do what is needed in order to graph y1 but not y2.

Now activate both functions so that both graphs are plotted. Press F3 and the cursor appears first on the graph of $y = -x^3 + 4x$ because it is higher up on the Y= list. You know that the cursor is on this function, y1, because of the numeral 1 that is displayed in the upper right corner of the screen. Press the up ▲ or down ▼ direction to move the cursor vertically to the graph of $y = -.25x$. Now the numeral 2 is displayed in the upper right corner of the screen. Next press the left and right arrow keys to trace along the graph of $y = -.25x$. When more than one function is plotted, you can move the trace cursor vertically from one graph to another with the ▲ and ▼ directions.

Technology Tip: Trace along the graph of $y = -.25x$ and press and hold either the ◀ or ▶ direction. The cursor becomes larger and pulses as it moves along the graph. Eventually you will reach the left or right edge of the window. Keep pressing the direction and the TI-89 will allow you to continue to trace by panning the viewing window. Check the WINDOW screen to see that the xmin and xmax are automatically updated.

If you trace along the graph of $y = -x^3 + 4x$, the cursor will eventually move above or below the viewing window. The cursor's coordinates on the graph will still be displayed, though the cursor itself can no longer be seen. When you are tracing along a graph, press ENTER and the window will quickly pan over so that the cursor's position on the function is centered in a new viewing window. This feature is especially helpful when you trace near or beyond the edge of the current viewing window.

The TI-89's display has 159 horizontal columns of pixels and 99 vertical rows. So, when you trace a curve across a graph window, you are actually moving from xmin to xmax in 158 equal jumps, each called Δx. You would calculate the size of each jump to be $\Delta x = \frac{\text{xmax} - \text{xmin}}{158}$. Sometimes you may want the jumps to be friendly numbers like 0.1 or 0.25 so that, when you trace along the curve, the x-coordinates will be incremented by such a convenient amount. Just set your viewing window for a particular increment Δx by making $\text{xmax} = \text{xmin} + 158 \cdot \Delta x$. For example, if you want $\text{xmin} = -5$ and $\Delta x = 0.3$, set $\text{xmax} = -5 + 158 \cdot 0.3 = 42.4$. Likewise, set $\text{ymax} = \text{ymin} + 98 \cdot \Delta y$ if you want the vertical increment to be some special Δy.

To center your window around a particular point, say (h, k), and also have a certain Δx, set xmin $= h - 79 \cdot \Delta x$ and make xmax $= h + 79 \cdot \Delta x$. Likewise, make ymin $= k - 49 \cdot \Delta y$ and make ymax $= k + 49 \cdot \Delta y$. For example, to center a window around the origin $(0, 0)$, with both horizontal and vertical increments of 0.25, set the range so that xmin $= 0 - 79 \cdot 0.25 =$ -19.75, xmax $= 0 + 79 \cdot 0.25 = 19.75$, ymin $= 0 - 49 \cdot 0.25 = -12.25$, and ymax $= 0 + 49 \cdot 0.25 = 12.25$.

See the benefit by first graphing $y = x^2 + 2x + 1$ in a standard viewing window. Trace near its y-intercept, which is $(0, 1)$, and move towards its x-intercept, which is $(-1, 0)$. Then press F2 4 *[ZoomDec]* and trace again near the intercepts.

IV.2.6 Zoom: Plot again the two graphs for $y = -x^3 + 4x$ and $y = -.25x$. There appears to be an intersection near $x = 2$. The TI-89 provides several ways to enlarge the view around this point. You can change the viewing window directly by pressing ◆ WINDOW and editing the values of xmin, xmax, ymin, and ymax. Figure IV.42 shows a new viewing window for the range displayed in Figure IV.41. The cursor has been moved near the point of intersection; move your cursor closer to get the best approximation possible for the coordinates of the intersection.

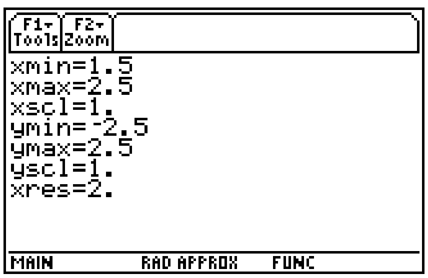


Figure IV.41: New WINDOW

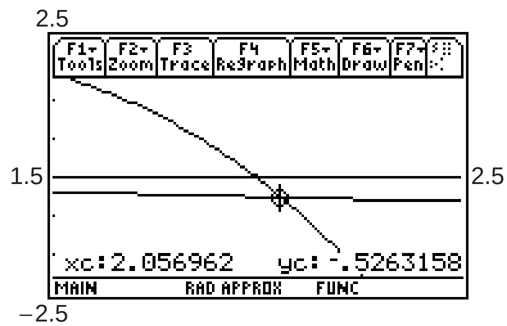


Figure IV.42: Closer view

A more efficient method for enlarging the view is to draw a new viewing window with the cursor. Start again with a graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$ in a standard viewing window (press F2 6 for the standard viewing window).

Now imagine a small rectangular box around the intersection point, near $x = 2$. Press F2 1 *[ZoomBox]* (Figure IV.43) to draw a box to define this new viewing window. Use the arrow keys to move the cursor, whose coordinates are displayed at the bottom of the window, to one corner of the new viewing window you imagine.

Press ENTER to fix the corner where you moved the cursor; it changes shape and becomes a blinking square (Figure IV.44). Use the arrow keys again to move the cursor to the diagonally opposite corner of the new window (Figure IV.45). Note that you can press and hold ▲ or ▼ with ◀ or ▶ for this. If this box looks all right to you, press ENTER. The rectangular area you have enclosed will now enlarge to fill the graph window (Figure IV.46).

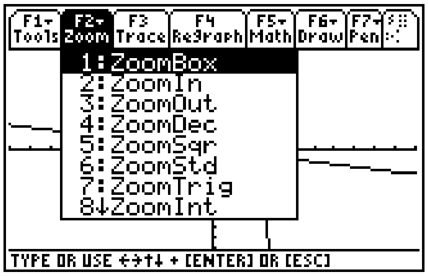


Figure IV.43: Zoom menu

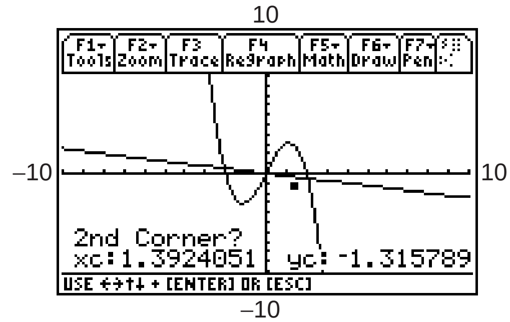


Figure IV.44: One corner selected

You may cancel the zoom any time before you press this last ENTER. Press F2 once more and start over. Press ESC or ◆ GRAPH to cancel the zoom, or press 2nd QUIT to cancel the zoom and return to the home screen.

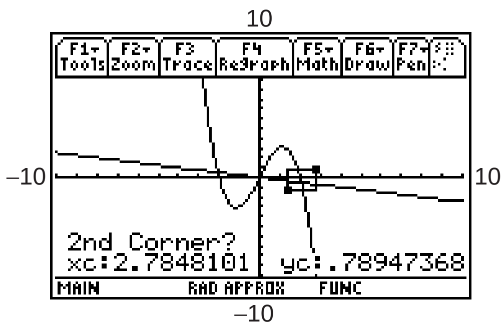


Figure IV.45: Box drawn

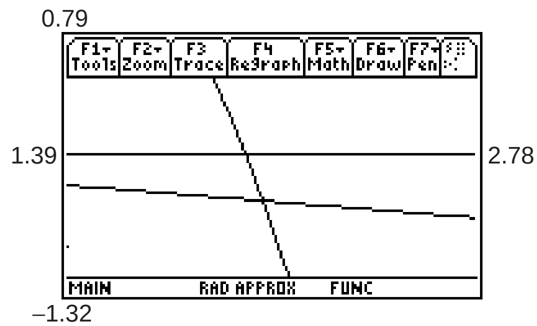


Figure IV.46: New viewing window

You can also quickly magnify a graph around the cursor's location. Return once more to the standard viewing window for the graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$. Press F2 2 *[ZoomIn]* and then move the cursor as close as you can to the point of intersection near $x = 2$ (see Figure IV.47). Then press ENTER and the calculator draws a magnified graph, centered at the cursor's position (Figure IV.48). The range variables are changed to reflect this new viewing window. Look in the WINDOW menu to verify this.

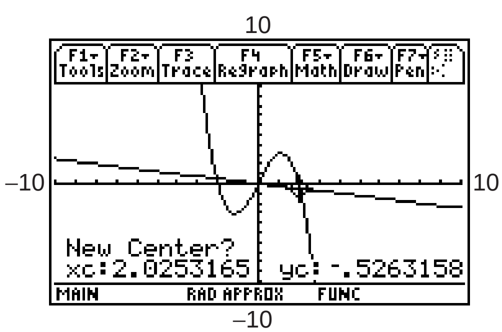


Figure IV.47: Before a zoom in

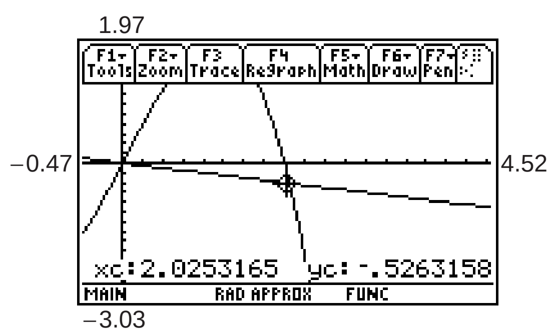


Figure IV.48: After a zoom in

As you see in the Zoom menu (Figure IV.43), the TI-89 can zoom in (press F2 2) or zoom out (press F2 3). Zoom out to see a larger view of the graph, centered at the cursor position. You can change the horizontal and vertical scale of the magnification by pressing F2 alpha C *[SetFactors...]* see Figure IV.49) and editing xFact and yFact, the horizontal and vertical magnification factors. (The zFact is only used when dealing with three-dimensional graphs.)

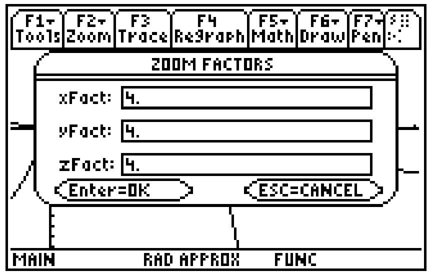


Figure IV.49: ZOOM FACTORS menu

The default zoom factor is 4 in both directions. It is not necessary for xFact and yFact to be equal. Sometimes, you may prefer to zoom in one direction only, so the other factor should be set to 1. Press ESC to leave the ZOOM FACTORS menu and go back to the graph. (Pressing 2nd QUIT will take you back to the home screen.)

Technology Tip: The TI-89 remembers the window it displayed before a zoom. So, if you should zoom in too much and lose the curve, press F2 alpha B *[Memory]* 1 *[ZoomPrev]* to go back to the window before. If you want to execute a series of zooms but then return to a particular window, press F2 alpha B 2 *[ZoomSto]* to store the current window's dimensions. Later, press F2 alpha B 3 *[ZoomRcl]* to recall the stored window.

IV.2.7 Value: Graph $y = -x^3 + 4x$ in the standard viewing window (Figure IV.18). The TI-89 can calculate the value of this function for any given x (between the xmin and xmax values).

Press F5 *[Math]* to display the Math menu (see Figure IV.50), then press 1 *[Value]*. The graph of the function is displayed and you are prompted to enter a value for x. Press 1 ENTER. The x-value you entered and its corresponding y-value are shown at the bottom of the screen and the cursor is located at the point $(1, 3)$ on the graph (see Figure IV.51).

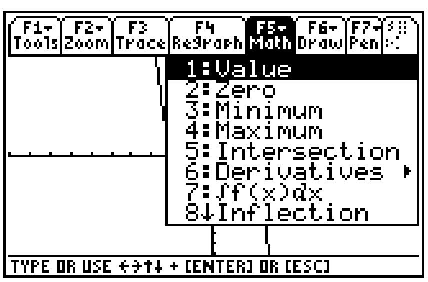


Figure IV.50: Math menu

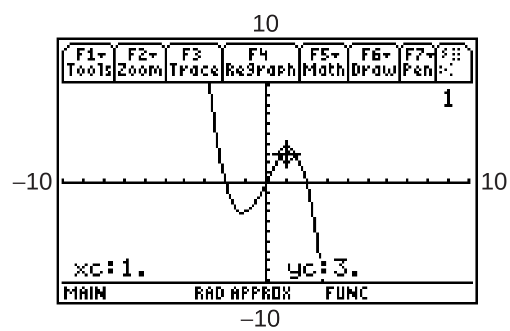


Figure IV.51: Finding a value

Note that if you have more than one graph on the screen, the upper left corner of the TI-89 screen will display the numeral corresponding to the equation of the function in the Y= list whose value is being calculated. Press ▲ or ▼ to move the cursor vertically between functions at the entered x-value.

IV.2.8 Relative Minimums and Maximums: Graph $y = -x^3 + 4x$ once again in the standard viewing window. This function appears to have a relative minimum near $x = -1$ and a relative maximum near $x = 1$. You may zoom and trace to approximate these extreme values.

First trace along the curve near the relative minimum. Notice by how much the x-values and y-values change as you move from point to point. Trace along the curve until the y-coordinate is as *small* as you can get it, so that you are as close as possible to the relative minimum, and zoom in (press F2 2 ENTER or use a zoom box). Now trace again along the curve and, as you move from point to point, see that the coordinates change by smaller amounts than before. Keep zooming and tracing until you find the coordinates of the relative minimum point as accurately as you need them, approximately $(-1.15, -3.08)$.

Follow a similar procedure to find the relative maximum. Trace along the curve until the y-coordinate is as *great* as you can get it, so that you are as close as possible to the relative maximum, and zoom in. The relative maximum point on the graph of $y = -x^3 + 4x$ is approximately $(1.15, 3.08)$.

The TI-89 can automatically find the relative maximum and relative minimum points. While viewing the graph, press F5 to display the Math menu (Figure IV.50). Choose 3 *[Minimum]* to calculate the minimum value of the function and 4 *[Maximum]* for the maximum. You will be prompted to trace the cursor along the graph first to a point *left* of the minimum/maximum (press ENTER to set this *lower bound*). Note the arrow near the top of the display marking the lower bound (as in Figure IV.52).

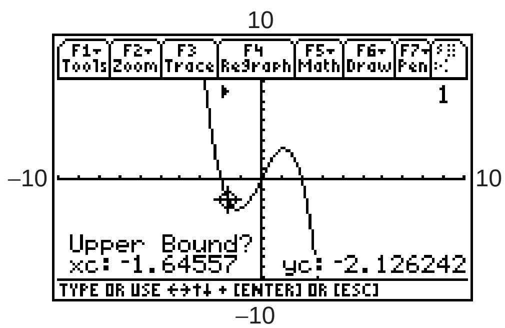


Figure IV.52: Finding a minimum

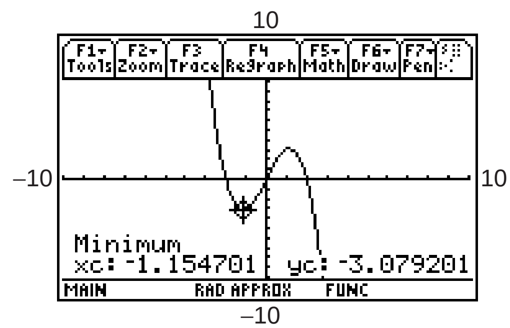


Figure IV.53: Relative minimum on $y = -x^3 + 4x$

Now move to a point *right* of the minimum/maximum and set an *upper bound* by pressing ENTER. The coordinates of the relative minimum/maximum point will be displayed (see Figure IV.53). Good choices for the lower and upper bounds can help the TI-89 work more efficiently and quickly.

Note that if you have more than one graph on the screen, the upper right corner of the TI-89 screen will show the number of the function whose minimum/maximum is being calculated.

IV.2.9 Inverse Functions: The TI-89 draws the inverse function of a one-to-one function. Graph $y = x^3 + 1$ as y1 in the standard viewing window (see Figure IV.54). Next, press 2nd F6 *[Draw]* to display the Draw menu (see Figure IV.55), then press 3 *[DrawInv]*. You are automatically returned to the home screen. Press X ^ 3 + 1 ENTER (see Figure IV.56). These keystrokes instruct the TI-89 to draw the inverse function of $y = x^3 + 1$. The original function and its inverse function will be displayed (see Figure IV.57). Note that the calculator must be in function mode in order to use DrawInv.

To clear the graph of the inverse function, press 2nd F6 1 *[ClrDraw].*

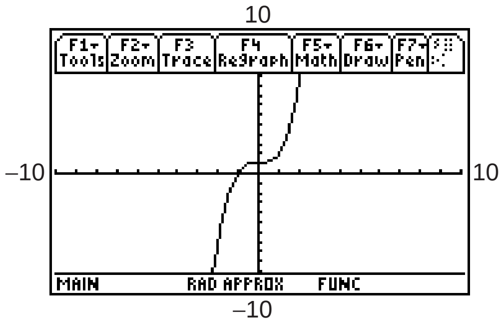


Figure IV.54: Graph of $y = x^3 + 1$

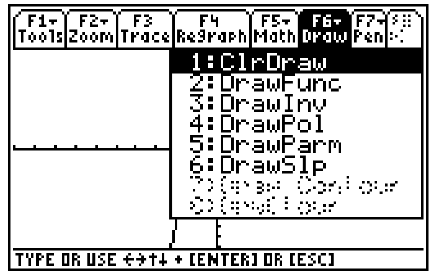


Figure IV.55: Draw menu

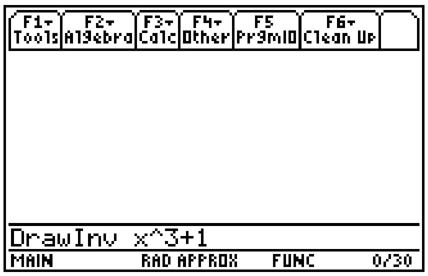


Figure IV.56: DrawInv

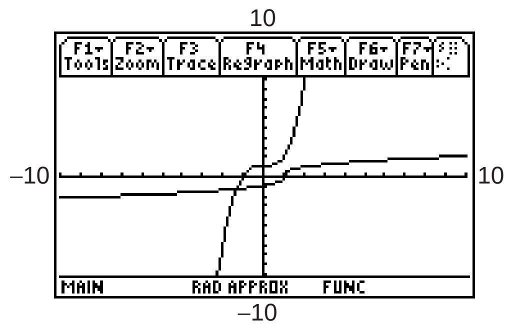


Figure IV.57: Graph of $y = x^3 + 1$ and its inverse function

IV.2.10 Tangent Lines: Once again, graph $y = x^3 + 1$ in the standard viewing window (see Figure IV.54). The TI-89 can draw the tangent line to a graph of a function at a specified point.

Press F5 alpha A *[Tangent]*. You are prompted to enter a value for x. So, press ◀ or ▶ to select a point or enter a value for x (see Figure IV.58). Press 1 ENTER. The graph of the original function and the tangent line to the graph at $x = 1$ will be displayed (see Figure IV.59). Note that the equation of the tangent line is displayed at the bottom of the screen.

To clear the tangent line, press 2nd F6 1.

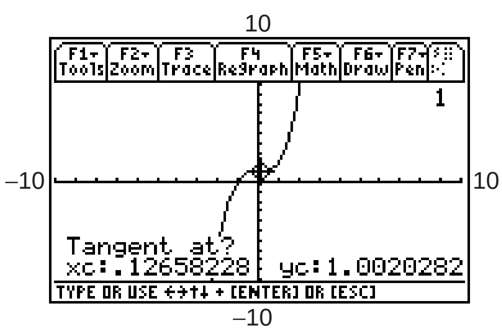


Figure IV.58: Tangent

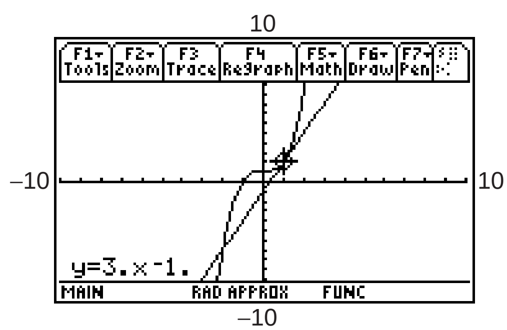


Figure IV.59: Graph of $y = x^3 + 1$ and tangent line at $x = 1$

IV.3 Solving Equations and Inequalities

IV.3.1 Intercepts and Intersections: Tracing and zooming are also used to locate an x-intercept of a graph, where a curve crosses the x-axis. For example, the graph of $y = x^3 - 8x$ crosses the x-axis three times (see Figure IV.60). After tracing over to the x-intercept point that is farthest to the left, zoom in (Figure IV.61). Continue this process until you have located all three intercepts with as much accuracy as you need. The three x-intercepts of $y = x^3 - 8x$ are approximately -2.828, 0, and 2.828.

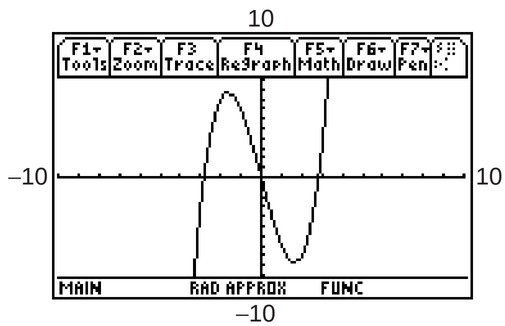


Figure IV.60: Graph of $y = x^3 - 8x$

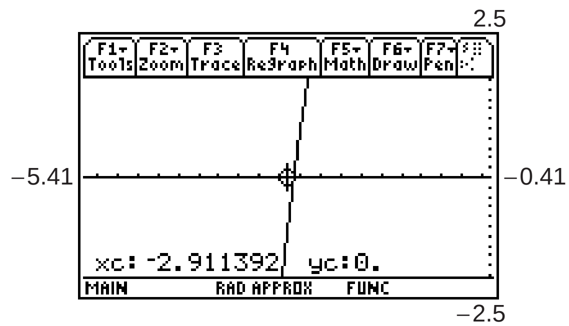


Figure IV.61: Near an x-intercept of $y = x^3 - 8x$

Technology Tip: As you zoom in, you may also wish to change the spacing between tick marks on the x-axis so that the viewing window shows scale marks near the intercept point. Then the accuracy of your approximation will be such that the error is less than the distance between two tick marks. Change the x-scale on the TI-89 from the WINDOW menu. Move the cursor down to xscl and enter an appropriate value.

The x-intercept of a function's graph is a *zero* of the function, so while viewing the graph, press F5 (Figure IV.50) and choose 2 *[Zero]* to find a zero of this function. Set a lower bound and upper bound as described in Section IV.2.8. The TI-89 shows the coordinates of the point and indicates that it is a zero (Figure IV.62).

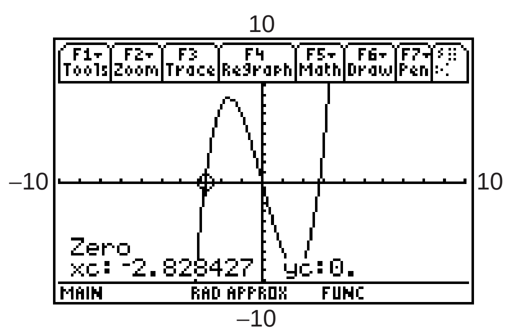


Figure IV.62: A zero of $y = x^3 - 8x$

Trace and Zoom are especially important for locating the intersection points of two graphs, say the graphs of $y = -x^3 + 4x$ and $y = -.25x$. Trace along one of the graphs until you arrive close to an intersection point. Then press ▲ or ▼ to jump to the other graph. Notice that the x-coordinate does not change, but the y-coordinate is likely to be different (Figures IV.63 and IV.64).

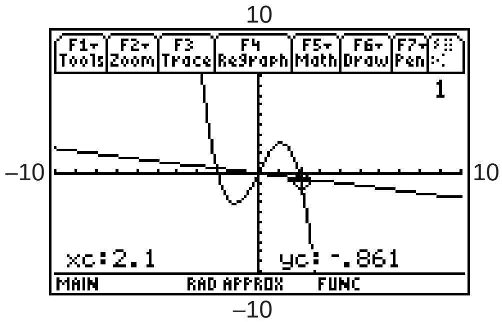


Figure IV.63: Trace on $y = -x^3 + 4x$

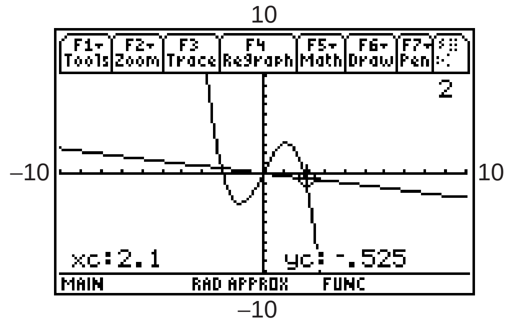


Figure IV.64: Trace on $y = -.25x$

When the two *y*-coordinates are as close as they can get, you have come as close as you now can to the point of intersection. So, zoom in around the intersection point, then trace again until the two *y*-coordinates are as close as possible. Continue this process until you have located the point of intersection with as much accuracy as necessary. The points of intersection are approximately $(-2.062, 0.515)$, $(0, 0)$, and $(2.062, -0.515)$.

You can also find the point of intersection of two graphs by pressing F5 5 *[Intersection]*. Trace with the cursor first along one graph near the intersection and press ENTER; then trace with the cursor along the other graph and press ENTER. Marks $^+$ are placed on the graphs at these points. Then set the lower and upper bounds for the *x*-coordinate of the intersection point and press ENTER again. Coordinates of the intersection will be displayed at the bottom of the window. More will be said about the Intersection feature in Section IV.3.3.

IV.3.2 Solving Equations by Graphing: Suppose you need to solve the equation $24x^3 - 36x + 17 = 0$. First graph $y = 24x^3 - 36x + 17$ in a window large enough to exhibit *all* its *x*-intercepts, corresponding to all the equation's real zeros (roots). Then use Zoom and Trace, or the TI-89's zero finder, to locate each one. In fact, this equation has just one real solution, $x \approx -1.414$.

Remember that when an equation has more than one *x*-intercept, it may be necessary to change the viewing window a few times to locate all of them.

The TI-89 has a solve(function. To use this function, you must be in the home screen. To solve the equation $24x^3 - 36x + 17 = 0$, press 2nd MATH 9 1 *[solve(]* 24 X ^ 3 − 36 X + 17 = 0 , X) ENTER. The TI-89 displays the value of the zero (Figure IV.65). Note that any letter could have been used for the variable. This is the reason that you must indicate to the TI-89 that the variable being used is X.

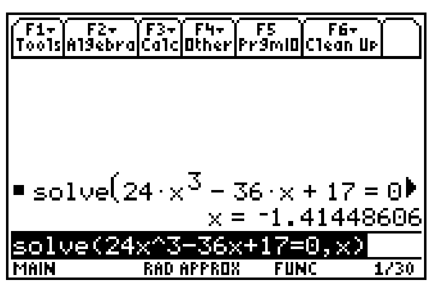


Figure IV.65: solve(function

Technology Tip: To solve an equation like $24x^3 + 17 = 36x$, you may first rewrite it in general form, $24x^3 - 36x + 17 = 0$, and proceed as above. However, the solve(function does not require that the function be in general form. You may also graph the two functions $y = 24x^3 + 17$ and $y = 36x$, then zoom and trace to locate their point of intersection.

IV.3.3 Solving Systems by Graphing: The solutions to a system of equations correspond to the points of intersection of their graphs (Figure IV.66). For example, to solve the system $y = 2x + 5$ and $y = -2x + 1$, first graph them together. Then use Zoom and Trace or the Intersection option in the Math menu, to locate their point of intersection, which is $(-1, 3)$ (see Figure IV.67).

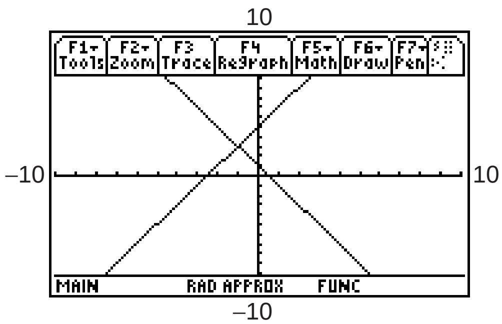


Figure IV.66: Solving a system of equations

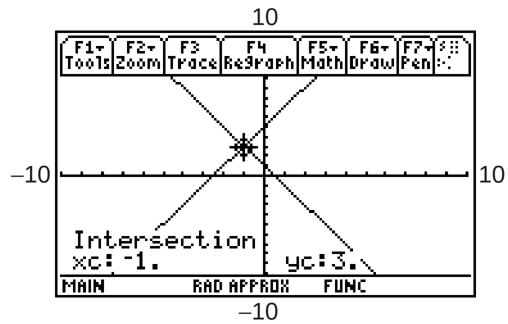


Figure IV.67: The point of intersection is $(-1, 3)$.

The solutions of the system of two equations $y = 2x + 5$ and $y = -2x + 1$ correspond to the solutions of the single equation $2x + 5 = -2x + 1$, which simplifies to $4x + 4 = 0$. So, you may also graph $y = 4x + 4$ and find its x-intercept to solve the system or use the solve(function.

IV.3.4 Solving Inequalities by Graphing: Consider the inequality $1 - \frac{3x}{2} \geq x - 4$. To solve it with your TI-89, graph the two functions $y = 1 - \frac{3x}{2}$ and $y = x - 4$ (Figure IV.68). First locate their point of intersection, at $x = 2$. The inequality is true when the graph of $y = 1 - \frac{3x}{2}$ lies *above* the graph of $y = x - 4$, and that occurs when $x < 2$. So the solution is $x \leq 2$, or $(-\infty, 2]$.

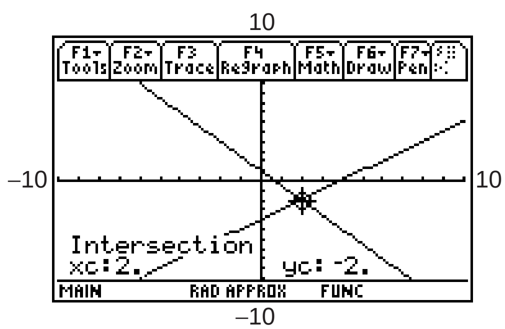


Figure IV.68: Solving $1 - \frac{3x}{2} \geq x - 4$

The TI-89 is capable of shading the region above or below a graph, or between two graphs. For example, to graph $y \geq x^2 - 1$, first enter the function $y = x^2 - 1$ as y1. Then, highlight y1 and press 2nd F6 7 *[Above]* (see Figure IV.69). These keystrokes instruct the TI-89 to shade the region above $y = x^2 - 1$. Press ◆ GRAPH to see the graph. The region above the graph will be shaded using the default shading option of vertical lines, as in Figure IV.70.

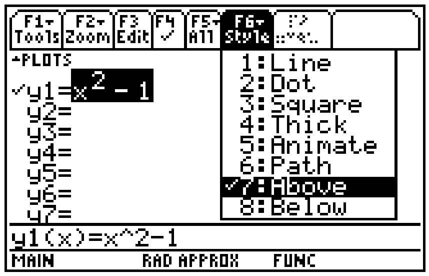


Figure IV.69: Shade Above style

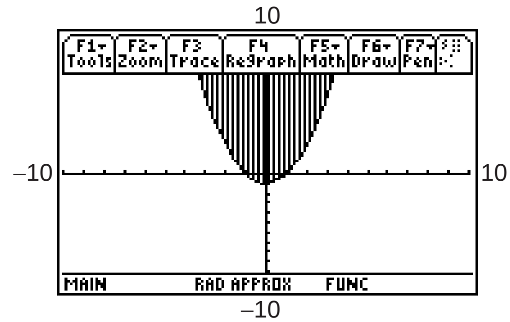


Figure IV.70 Graph of $y \geq x^2 - 1$

Now use shading to solve the previous inequality, $1 - \frac{3x}{2} \geq x - 4$. The solution is the region which is *below* the graph of $y = 1 - \frac{3x}{2}$ and *above* $y = x - 4$. First graph both equations. Then, from the graph screen, press F5 alpha C *[Shade]*. The TI-89 will prompt for the function that you want to have the shading above. Use ▲ or ▼ to move the cursor to the graph of $y = x - 4$, then press ENTER. The TI-89 will then prompt for the function that you want to have the shading below, so use ▲ or ▼ to move the cursor to the graph of $y = 1 - \frac{3x}{2}$ and press ENTER. The TI-89 will then prompt for the *lower bound* then the *upper bound*, which are the left and right edges, respectively, of the extent of the shading. If you do not enter a lower or upper bound, the values of xmin and xmax will be used. So, in this case, press ENTER twice to set the lower and upper bounds. The shaded area extends left from $x = 2$, so the solution to $1 - \frac{3x}{2} \geq x - 4$ is $x \leq 2$, or $(-\infty, 2]$ (see Figure IV.71).

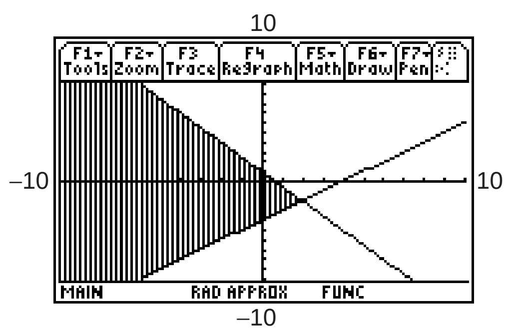


Figure IV.71: Graph of $1 - \frac{3x}{2} \geq x - 4$

IV.4 Trigonometry

IV.4.1 Degrees and Radians: The trigonometric functions can be applied to angles measured either in radians or degrees, but you should take care that the TI-89 is configured for whichever measure you need. Press MODE to see the current settings. Press ▼ three times and move down to the fourth line of the first page of the MODE menu where angle measure is selected. Then press ▶ to display the options. Use ▲ or ▼ to move from one option to the other. Either press the number corresponding to the measure or, when the measure is highlighted, press ENTER to select it. Then press ENTER to confirm your selection and leave the MODE menu.

It's a good idea to check the angle measure setting before executing a calculation that depends on a particular measure. You may change a mode setting at any time and not interfere with pending calculations. From the home screen, try the following keystrokes to see this in actions

Expression	*Keystrokes*	*Display*
sin 45°	MODE ▼ ▼ ▼ ▶ ▼ ENTER ENTER 2nd SIN 45) ENTER	.7071067812
sin π°	2nd SIN 2nd π) ENTER	.0548036651
sin π	MODE ▼ ▼ ▼ ▶ ▲ ENTER ENTER 2nd SIN 2nd π) ENTER	0
sin 45	2nd SIN 45) ENTER	.8509035245
sin $\frac{\pi}{6}$	2nd SIN 2nd π ÷ 6) ENTER	.5

The first line of keystrokes sets the TI-89 in degree mode and calculates the sine of 45 *degrees*. While the calculator is still in degree mode, the second line of keystrokes calculates the sine of π *degrees*, approximately 3.1415°. The third line changes to radian mode just before calculating the sine of π *radians*. The fourth line calculates the sine of 45 *radians*. Finally, the fifth line calculates the sine of $\frac{\pi}{6}$ *radians* (the calculator remains in radian mode).

The TI-89 makes it possible to mix degrees and radians in a calculation. Execute these keystrokes to calculate $\tan 45° + \sin\frac{\pi}{6}$ as shown in Figure IV.72: 2nd TAN 45 2nd MATH 2 *[Angle]* 1 *[°]*) + 2nd SIN 2nd π ÷ 6) 2nd MATH 2 2 *[ʳ]* ENTER. Do you get 1.5 whether your calculator is in *either* degree mode *or* in radian mode?

The degree sign can also be entered by pressing 2nd |, which saves keystrokes. There is no corresponding key for the radian symbol.

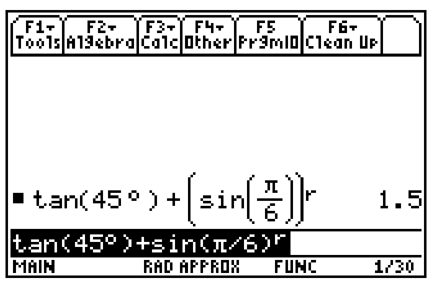


Figure IV.72: Angle measure

Technology Tip: The automatic left parenthesis that the TI-89 places after functions such as sine, cosine, and tangent (as noted in Section IV.1.3) *can* affect the outcome of calculations. In the previous example, the degree sign must be *inside* of the parentheses so that when the TI-89 is in radian mode, it calculates the tangent of 45 degrees, rather than converting the tangent of 45 radians into an equivalent number of degrees. Also, the parentheses around the fraction $\frac{\pi}{6}$ are required so that when the TI-89 is in radian mode, it converts $\frac{\pi}{6}$ into radians rather than converting merely the 6 to radians. Experiment with the placement of parentheses to see how they affect the result of computation.

IV.4.2 Graphs of Trigonometric Functions: When you graph a trigonometric function, you need to pay careful attention to the choice of graph window and to your angle measure configuration. For example, graph $y = \frac{\sin 30x}{30}$ in the standard viewing window in radian mode. Trace along the curve to see where it is. Zoom in to a better window, or use the period and amplitude to establish better WINDOW values.

Technology Tip: Because $\pi \approx 3.1$, when in radian mode, set xmin $= 0$ and xmax $= 6.3$ to cover the interval from 0 to 2π.

Next graph $y = \tan x$ in the standard window first, then press F2 7 *[ZoomTrig]* to change to a special window for trigonometric functions in which the xscl is $\frac{\pi}{2} \approx 1.5708$ or 90° and the vertical range is from -4 to 4. The TI-89 plots consecutive points and then connects them with a segment, so the graph is not exactly what you should expect. You may wish to change the plot style from Line to Dot (see Section IV.2.3) when you plot the tangent function.

IV.5 Scatter Plots

IV.5.1 Entering Data: The table shows the total prize money (in millions of dollars) awarded at the Indianapolis 500 race from 1995 to 2003. (*Source:* Indy Racing League)

Year	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*
Prize (in millions)	$8.06	$8.11	$8.61	$8.72	$9.05	$9.48	$9.61	$10.03	$10.15

We'll now use the TI-89 to construct a scatter plot that represents these points and to find a linear model that approximates the given data.

The TI-89 holds data in *lists*. You can create as many list names as your TI-89 memory has space to store. Before entering this new data, clear the data in the lists that you want to use. To delete a list press 2nd VAR-LINK. This will display a list of folders showing the variables defined in each folder. Highlight the name of the list that you wish to delete and press F1 *[Manage]* 1 *[Delete]* ENTER. The TI-89 will ask you to confirm the deletion by pressing ENTER once more.

Now press APPS 6 *[Data/Matrix Editor]* 3 *[New...]* ▼ ▼ P R I Z E ENTER to open a new variable called PRIZE (Figure IV.73). Press ENTER to then begin entering the variable values, with the years going in column c1. Instead of entering the full year, let $x = 5$ represent 1995, $x = 6$ represent 1996, and so on. Here are the keystrokes for the first three years: 5 ENTER 6 ENTER 7 ENTER and so on, then press ▶ to move to the next list. Move up to the first row and press 8.06 ENTER 8.11 ENTER 8.61 ENTER and so on (see Figure IV.74).

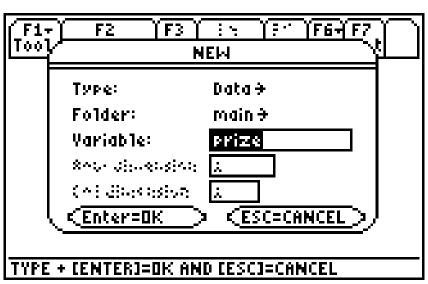


Figure IV.73: Entering a new variable

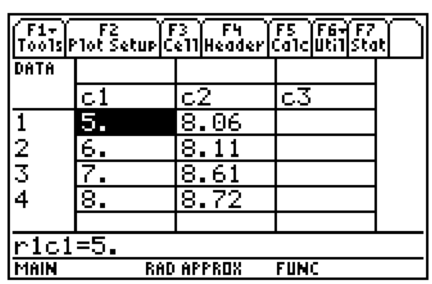


Figure IV.74: Entering data points

You may edit statistical data in almost the same way you edit expressions in the home screen. ← will delete the *entire cell*, not just the character or value to the left of the cursor. So, move the cursor to any value you wish to change, then type the correction. To insert or delete a data point, move the cursor over the data point (cell) you wish to add or delete. To insert a cell, move to the cell *below* the place where you want to insert the new cell and press 2nd F6 *[Util]* 1 *[Insert]* 1 *[cell]* and a new empty cell is open.

IV.5.2 Plotting Data: First check the MODE screen (Figure IV.1) to make sure that you are in FUNCTION graphing mode. With the data points showing, press F2 *[Plot Setup]* to display the Plot Setup screen. If no other plots have been entered, Plot 1 is highlighted by default. Press F1 *[Define]* to select the options for the plot. Use ▲, ▼, and ENTER to select the Plot Type as Scatter and the Mark as a Box. Press alpha C 1 to set the independent variable, *x*, and press alpha C 2 to set the dependent variable, *y*, as shown in Figure IV.75, then press ENTER to save the options and press ◆ GRAPH to graph the data points. (Make sure that you have cleared or turned off any functions in the Y= screen, or those functions will be graphed simultaneously.) Figure IV.76 shows this plot in a window from 0 to 15 horizontally and vertically. You may now press F3 *[Trace]* to move from data point to data point.

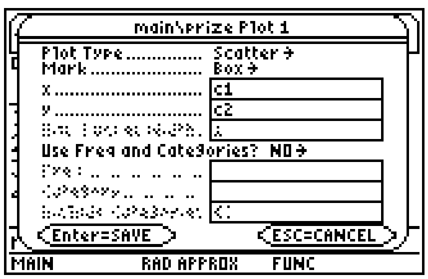


Figure IV.75: Plot 1 menu

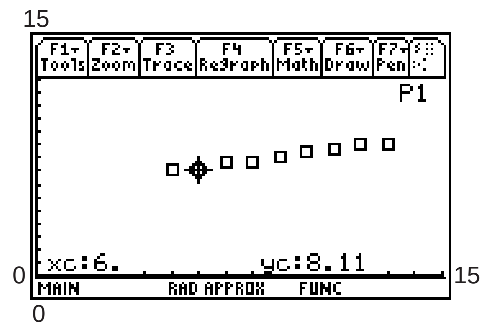


Figure IV.76: Scatter plot

To draw the scatter plot in a window adjusted automatically to include all the data you entered, press F2 9 *[ZoomData]*.

When you no longer want to see the scatter plot press APPS 6 1 *[Current]* F2, highlight Plot 1 and use F4 *[✓]* to deselect Plot 1 or press ◆ Y=, move the cursor up to highlight Plot 1, and press F4 *[✓]*. The TI-89 still retains all the data you entered.

IV.5.3 Regression Line: The TI-89 calculates slope and *y*-intercept for the line that best fits all the data. After the data points have been entered, while still in the Data/Matrix Editor, press F5 *[Calc]*. For the Calculation Type, choose 5 *[LinReg]* and set the *x* variable to c1 and the *y* variable to c2. In order to have the TI-89 graph the regression equation, set Store RegEQ to as y1(x) as shown in Figure IV.77. Press ENTER and the TI-89 will calculate a linear regression model with the slope named a and the *y*-intercept named b (Figure IV.78). The *correlation coefficient* (corr) measures how well the linear regression equation fits with the data. The closer the absolute value of the correlation coefficient is to 1, the better the fit; the closer the absolute value of the correlation coefficient is to 0, the worse the fit. The TI-89 displays both the correlation coefficient and the coefficient of determination (R^2).

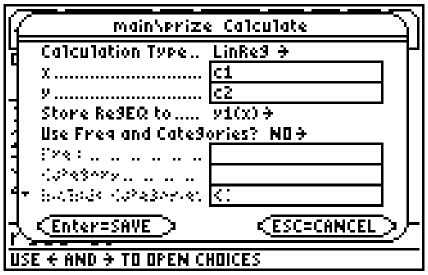


Figure IV.77: Linear regression: Calculate dialog box

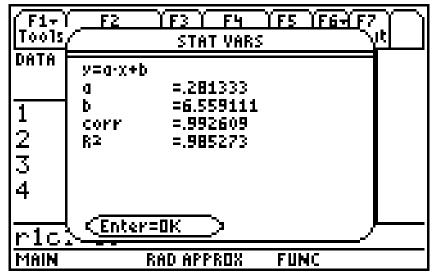


Figure IV.78: Linear regression model

Press ENTER to accept the regression equation and close the STAT VARS screen. To see both the data points and the regression line (Figure IV.79), go to the Plot Setup screen and select Plot 1, then press ◆ GRAPH to display the graph.

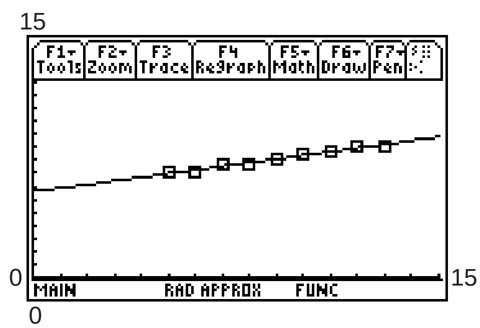


Figure IV.79: Linear regression line

IV.5.4 Other Regression Models: After data points have been entered, you can choose from nine different regression models. They are all located in the Calc menu of the Data/Matrix Editor.

IV.6 Matrices

IV.6.1 Making a Matrix: The TI-89 can display and use as many different matrices as the memory will hold. Here's how to store this 3×4 $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & 4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$ matrix in your calculator.

From the home screen, press APPS 6 3. Set the Type to Matrix, the Variable to a (this is the "name" of the matrix), the Row dimension to 3 and the Col dimension to 4 (Figure IV.80). Press ENTER to accept these values.

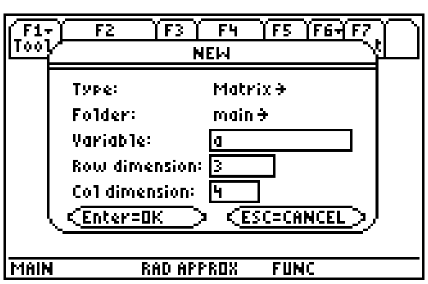


Figure IV.80: Data/Matrix Editor

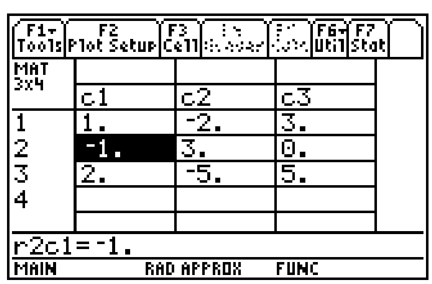


Figure IV.81: Editing a matrix

The display will show the matrix as a grid with zeros in the rows and columns specified in the definition of the matrix.

Use the cursor pad or press ENTER repeatedly to move the cursor to a matrix element you want to change. If you press ENTER, you will move right across a row and then back to the first column of the next row. The lower left of the screen shows the cursor's current location within the matrix. The element in the second row and first column in Figure IV.81 is highlighted, so the lower left of the window is r2c1 = -1 showing that element's current value. Enter all the elements of matrix a; pressing ENTER after entering each value.

When you are finished, leave the matrix editing screen by pressing 2nd QUIT or HOME to return to the home screen.

IV.6.2 Matrix Math: From the home screen, you can perform many calculations with matrices. To see matrix a, press alpha A ENTER (Figure IV.82).

Perform the scalar multiplication 2a by pressing 2 alpha A ENTER. The resulting matrix is displayed on the screen. To create matrix b as 2a press 2 alpha A STO ► alpha B ENTER (Figure IV.83), or if you do this immediately after calculating 2a, press only STO ► alpha B ENTER. The calculator will display the matrix.

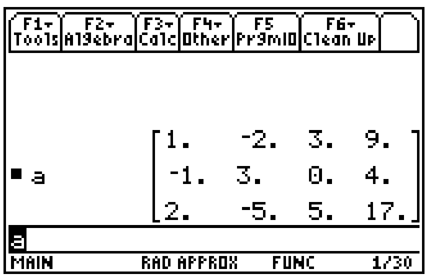


Figure IV.82: Matrix a

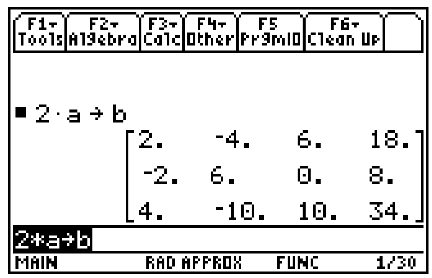


Figure IV.83: Matrix b

To add two matrices, say a and b, create b (with the same dimensions as a) and then press alpha A + alpha B ENTER. Subtraction is performed in a similar manner.

Now create a matrix called c with dimensions of 2×3 and enter the matrix $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -5 & -1 \end{bmatrix}$ as c. For matrix multiplication of c by a, press alpha C × alpha A ENTER. If you tried to multiply a by c, your TI-89 would notify you of an error because the dimensions of the two matrices do not permit multiplication in this way.

IV.6.3 Row Operations: Here are the keystrokes necessary to perform elementary row operations on a matrix. Your textbook provides a more careful explanation of the elementary row operations and their uses.

To interchange the second and third rows of the matrix that was defined in Figure IV.82, press 2nd MATH 4 *[Matrix]* alpha J *[Row ops]* 1 *[rowSwap(]* alpha A , 2 , 3) ENTER (see Figure IV.84). The format of this command is rowSwap(*matrix1*, *index1*, *index2*).

To add row 2 and row 3 and *store* the results in row 3, press 2nd MATH 4 alpha J 2 *[rowAdd(]* alpha A , 2 , 3) ENTER. The format of this command is rowAdd(*matrix1*, *index1*, *index2*).

To multiply row 2 by -4, and store the results in row 2, thereby replacing row 2 with new values, press 2nd MATH 4 alpha J 3 *[mRow(]* (-) 4 , alpha A , 2) ENTER. The format of this command is mRow(*expression*, *matrix1*, *index*).

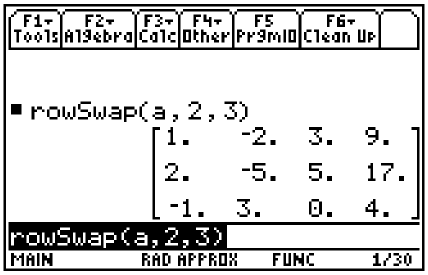


Figure IV.84: Interchange rows 2 and 3

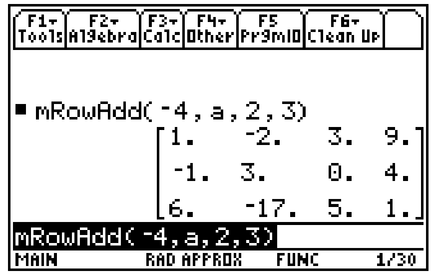


Figure IV.85: Add -4 times row 2 to row 3

To multiply row 2 by -4 and *add* the results to row 3, thereby replacing row 3 with new values, press 2nd MATH 4 alpha J 4 *[mRowAdd(]* (-) 4 , alpha A , 2 , 3) ENTER (see Figure IV.85). The format of this command is mRowAdd(*expression*, *matrix1*, *index1*, *index2*).

Technology Tip: Note that your TI-89 does *not* store a matrix obtained as the result of any row operations. So, when you need to perform several row operations in succession, it is a good idea to store the result of each one in a temporary place.

For example, use row operations to solve this system of linear equations: $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4. \\ 2x - 5y + 5z = 17 \end{cases}$

First enter this *augmented matrix* as a in your TI-89: $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$. Then return to the home screen and store this matrix as e (press alpha A STO ▶ alpha E ENTER) so you may keep the original in case you need to recall it. Here are the row operations and their associated keystrokes. At each step, the result is stored in e and replaces the previous matrix e. The last step of the row operations is shown in Figure IV.86.

Row Operation	*Keystrokes*
Add row 1 to row 2.	2nd MATH 4 alpha J 2 alpha E , 1 , 2) STO ▶ alpha E ENTER
Add -2 times row 1 to row 3.	2nd MATH 4 alpha J 4 (-) 2 , alpha E , 1 , 3) STO ▶ alpha E ENTER
Add row 2 to row 3.	2nd MATH 4 alpha J 2 alpha E , 2 , 3) STO ▶ alpha E ENTER
Multiply row 3 by $\frac{1}{2}$.	2nd MATH 4 alpha J 3 1 ÷ 2 , alpha E , 3) STO ▶ alpha E ENTER

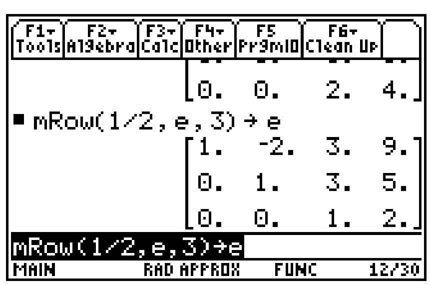


Figure IV.86: Row-echelon form of matrix after row operations

So, $z = 2$, $y = -1$, and $x = 1$.

Technology Tip: The TI-89 can produce a row-echelon form and the reduced row-echelon form of a matrix. The row-echelon form of matrix a is obtained by pressing 2nd MATH 4 3 *[ref(]* alpha A) ENTER (see Figure IV.87) and the reduced row-echelon form is obtained by pressing 2nd MATH 4 4 *[rref(]* alpha A ENTER (see Figure IV.88). Note that the row-echelon form of a matrix is not unique, so your calculator may not get exactly the same matrix as you do by using row operations. However, the matrix that the TI-89 produces will result in the same solution to the system.

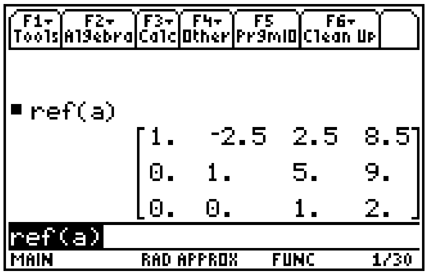


Figure IV.87: Row-echelon form

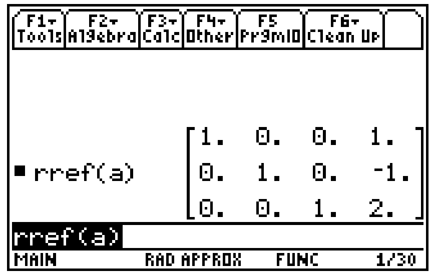


Figure IV.88: Reduced row-echelon form

IV.6.4 Determinants and Inverses: Enter this 3×3 matrix as a: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. Because this consists of the first three columns of the matrix a that was previously used, you can go to the matrix, move the cursor into the fourth column and press 2nd F6 *[Util]* 2 *[Delete]* 3 *[column]*. This will delete the column that the cursor is in. To calculate its determinant $\begin{vmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{vmatrix}$, go to the home screen and press 2nd MATH 4 2 *[det(]* alpha A) ENTER. You should find that the determinant is 2 as shown in Figure IV.89.

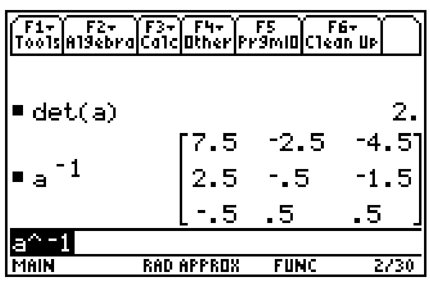


Figure IV.89: $|a|$ and a^{-1}

Because the determinant of the matrix is not zero, it has an inverse matrix. Press alpha A ^ (-) 1 ENTER to calculate the inverse of matrix a. The result is shown in Figure IV.89.

Now let's solve a system of linear equations by matrix inversion. Once again, consider $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$. The coefficient matrix for this system is the matrix $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$ which was entered as a in the previous example. Now enter the 3×1 matrix $\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$ as b. Because b was used before, when we stored 2a as b, press APPS 6 2 *[Open…]* ▶ 2 *[Matrix]* ▼ ▼ ▶ and use ▼ to move the cursor to b, then press ENTER twice to go to the matrix previously saved as b, which can be edited. Return to the home screen and press alpha A ^ (-) 1 × alpha B ENTER to calculate the solution matrix (Figure IV.90). The solution is still $x = 1$, $y = -1$, and $z = 2$.

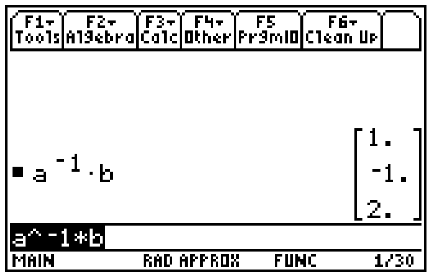


Figure IV.90: Solution matrix

IV.7 Sequences

IV.7.1 Iteration with the ANS key: The ANS feature enables you to perform *iteration*, the process of evaluating a function repeatedly. As an example, calculate $\frac{n-1}{3}$ for $n = 27$. Then calculate $\frac{n-1}{3}$ for $n =$ the answer to the previous calculation. Continue to use each answer as n in the *next* calculation. Here are keystrokes to accomplish this iteration on the TI-89 calculator (see the results in Figure IV.91). Notice that when you use ANS in place of n in a formula, it is sufficient to press ENTER to continue an iteration.

Iteration	*Keystrokes*	*Display*
1	27 ENTER	27
2	(2nd ANS − 1) ÷ 3 ENTER	8.666666667
3	ENTER	2.555555556
4	ENTER	.5185185185

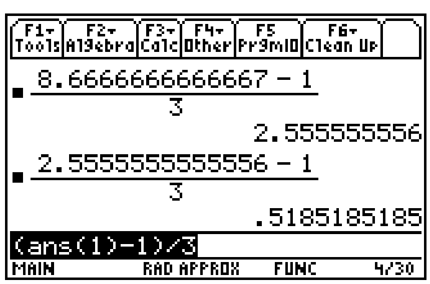


Figure IV.91: Iteration

Press ENTER several more times and see what happens with this iteration. You may wish to try it again with a different starting value.

IV.7.2 Terms of Sequences: Another way to display the terms of a sequence is to enter the sequence and the number of terms you want listed. For example, to find the first five terms of the sequence $u_n = -n + 4$, press 2nd MATH 3 *[List]* 1 *[seq(]* (-) alpha N + 4 , alpha N , 1 , 5 , 1) ENTER (see Figure IV.92). The format of this command is seq(*expression*, *variable*, *low*, *high*, *step*).

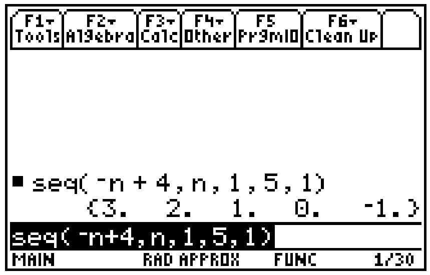


Figure IV.92: Terms of sequence $u_n = -n + 4$

IV.7.3 Arithmetic and Geometric Sequences: Use iteration with the ANS variable to determine the *n*th term of a sequence. For example, find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Enter the first term 7, then start the progression with the recursion formula, 2nd ANS + 4 ENTER. This yields the 2nd term, so press ENTER sixteen more times to find the 18th term, 75. For a *geometric* sequence whose common ratio is 4, start the progression with 2nd ANS × 4 ENTER.

You can also define the sequence recursively with the TI-89 by selecting SEQUENCE in the Graph type on the first page of the MODE menu (see Figure IV.1). Once again, let's find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Press MODE ▶ 4 *[SEQUENCE]* ENTER. Then press ◆ Y= to edit any of the TI-89's sequences, u1 through u99. Make u1 $= u1(n - 1) + 4$ and ui1 $= 7$ by pressing alpha U 1 (alpha N − 1) + 4 ENTER 7 ENTER (Figure IV.93). Press 2nd QUIT to return to the home screen. To find the 18th term of this sequence, calculate u1(18) by pressing alpha U 1 (18) ENTER (see Figure IV.94).

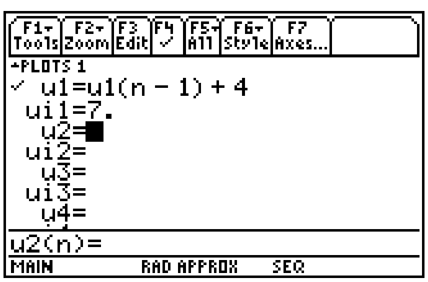


Figure IV.93: Sequence Y= menu

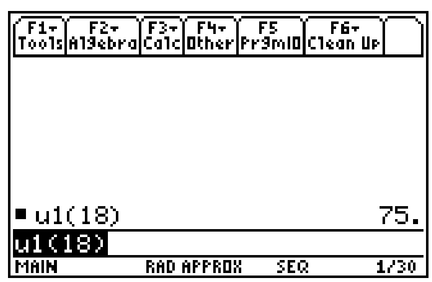


Figure IV.94: Sequence mode

Of course, you could also use the *explicit* formula for the *n*th term of an arithmetic sequence, $t_n = a + (n - 1)d$. First enter values for the variables *a*, *d*, and *n*, then evaluate the formula by pressing alpha A + (alpha N − 1) alpha D ENTER. For a geometric sequence whose *n*th term is given by $t_n = a \cdot r^{n-1}$, enter values for the variables *a*, *n*, and *r*, then evaluate the formula by pressing alpha A alpha R ^ (alpha N − 1) ENTER.

To use the explicit formula in sequence mode, make u1 $= 7 + (n - 1) \cdot 4$ by pressing ◆ Y= then using ▲ to move up to the u1 line and pressing CLEAR 7 + (N − 1) × 4 ENTER 2nd QUIT. Once more, calculate u1(18) by pressing alpha U 1 (18) ENTER.

IV.7.4 Finding Sums and Partial Sums of Sequences: You can find the sum of a sequence by combining the sum(feature with the seq(feature on the List sub-menu of the MATH menu. The format of the sum(command is sum(*list*). The format of the seq(command is seq(*expression*, *variable*, *low*, *high*, *step*) where the *step* argument is optional and the default is for integer values from *low* to *high*.

For example, suppose you want to find the sum $\sum_{n=1}^{12} 4(0.3)^n$. Press 2nd MATH 3 6 *[sum(]* 2nd MATH 3 1 4 (. 3) ^ alpha N , alpha N , 1 , 12)) ENTER (Figure IV.95). The seq(command generates a list, which the sum(command then sums. Note that any letter can be used for the variable in the sum, i.e., the N could just have easily been an A or a K.

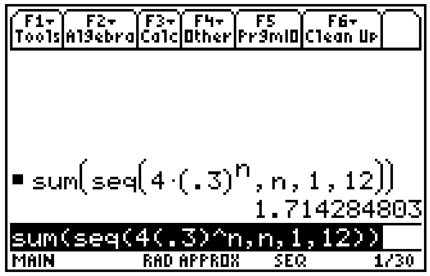


Figure IV.95: $\sum_{n=1}^{12} 4(0.3)^n$

Now calculate the sum starting at $n = 0$ by using ▶, ◀, and ⬅ to edit the range. You should obtain a sum of approximately 5.71284803.

The seq(feature can also be combined with the cumSum(feature to find partial sums of a series. The IV-33 format of the cumSum(command is cumSum(*list*).

For example, suppose you want to find the first four partial sums of the series $\sum_{n=1}^{4} 3^{n+1}$. Press 2nd MATH 3 7 *[cumSum(]* 2nd MATH 3 1 3 ^ (alpha N + 1) , alpha N , 1 , 4)) ENTER (Figure IV.96).

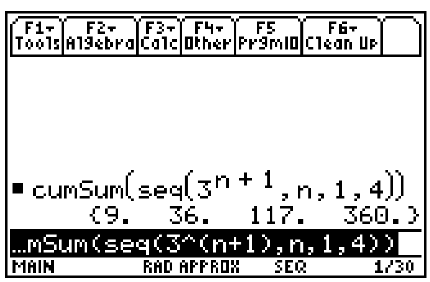


Figure IV.96: Partial sums of $\sum_{n=1}^{4} 3^{n+1}$

IV.8 Parametric and Polar Graphs

IV.8.1 Graphing Parametric Equations: The TI-89 plots up to 99 pairs of parametric equations as easily as it plots functions. In the first page of the MODE menu (Figure IV.1) change the Graph setting to PARAMETRIC. Be sure, if the independent parameter is an angle measure, that the angle measure in the MODE menu is set to whichever you need, RADIAN or DEGREE.

For example, here are the keystrokes needed to graph the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$. First check that angles are currently being measured in radians and change to parametric mode. Then press ◆ Y= (2nd COS T)) ^ 3 ENTER (2nd SIN T)) ^ 3 ENTER (Figure IV.97).

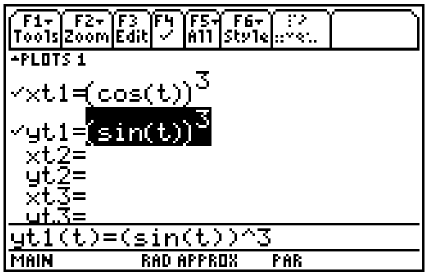


Figure IV.97: Parametric Y= menu

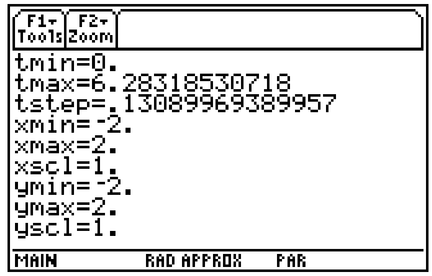


Figure IV.98: Parametric WINDOW menu

Press ◆ WINDOW to set the graphing window and to initialize the values of t. In the standard window, the values of t go from 0 to 2π in steps of $\frac{\pi}{24} \approx 0.1309$, with the view from -10 to 10 in both directions. In order to provide a better viewing window, press ENTER three times to move the cursor down, and set the window to extend from -2 to 2 in both directions (Figure IV.98). Press ◆ GRAPH to see the parametric graph (Figure IV.99).

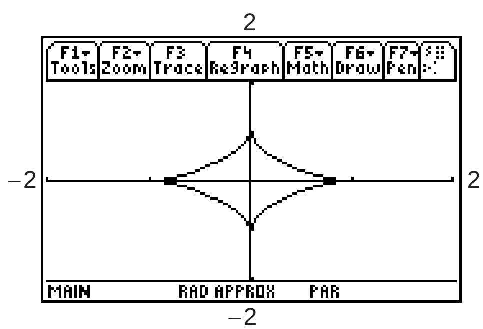
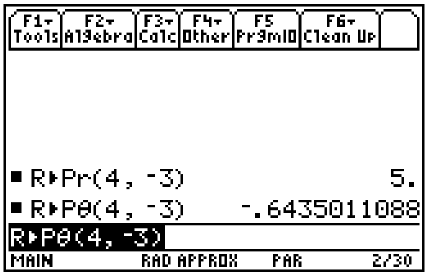


Figure IV.99: Parametric graph of $x = \cos^3 t$ and $y = \sin^3 t$

You may zoom and trace along parametric graphs just as you did with function graphs. However, unlike with function graphs, the cursor will not move to values outside of the t range, so ◀ will not work when t = 0, and ▶ will not work when t = 2π. As you trace along this graph, notice that the cursor moves in the counterclockwise direction as t increases.

IV.8.2 Rectangular-Polar Coordinate Conversion: The Angle sub-menu of the MATH menu provides a function for converting between rectangular and polar coordinate systems. These functions use the current angle measure setting, so it is a good idea to check the default angle measure before any conversion. Of course, you may override the current angle measure setting, as explained in Section IV.4.1. For the following examples, the TI-89 is set to radian mode.

Given the rectangular coordinates, $(x, y) = (4, -3)$, convert *from* these rectangular coordinates *to* polar coordinates (r, θ) by pressing 2nd MATH 2 5 *[R►Pr(]* 4 , (-) 3) ENTER to display the value of r. Now press 2nd MATH 2 6 *[R►Pθ(]* 4 , (-) 3) ENTER to display the value of θ (see Figure IV.100).



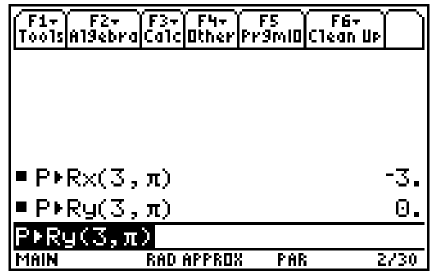


Figure IV.100: Rectangular to polar coordinates

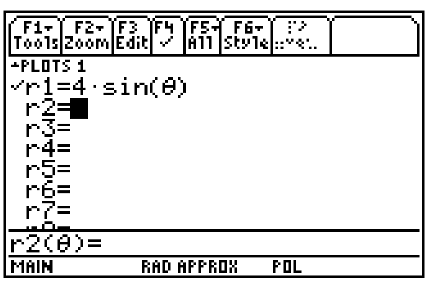
Figure IV.101: Polar to rectangular coordinates

Suppose $(r, \theta) = (3, \pi)$. To convert *from* these polar coordinates *to* rectangular coordinates (x, y), press 2nd MATH 2 3 *[P ►Rx(]* 3 , 2nd π) ENTER to display the *x*-coordinate. Now press 2nd MATH 2 4 *[P ►Ry(]* 3 , 2nd π) ENTER to display the *y*-coordinate (see Figure IV.101).

IV.8.3 Graphing Polar Equations: The TI-89 graphs polar functions in the form $r = f(\theta)$. In the Graph line of the first page of the MODE menu, select POLAR for polar graphs. You may now graph up to 99 polar functions at one time. Be sure that the angle measure has been set to whichever you need, RADIAN or DEGREE. Here we will use radian measure.

For example, to graph $r = 4 \sin \theta$, press ◆ Y= for the polar graph editing screen. Then enter the expression $4 \sin \theta$ for r1 by pressing 4 2nd SIN ◆ θ) ENTER (see Figure IV.102). The θ function is located to the right of the ^ key. Choose a good viewing window and an appropriate interval and increment for θ. In Figure IV.103, the viewing window is roughly "square" and extends from -12 to 12 horizontally and from -6 to 6 vertically.

Figure IV.103 shows *rectangular* coordinates of the cursor's location on the graph. You may sometimes wish to trace along the curve and see *polar* coordinates of the cursor's location. The first line of the Graph Format menu (Figure IV.25) has options for displaying the cursor's position in rectangular (RECT) or polar (POLAR) form.



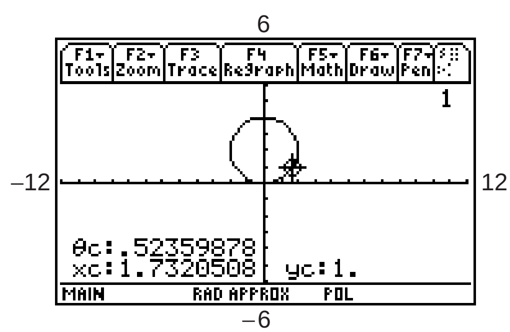


Figure IV.102: Polar Y= menu

Figure IV.103: Polar graph of $r = 4 \sin \theta$

IV.9 Probability and Statistics

IV.9.1 Random Numbers: The command rand(generates numbers. You will find this command in the Probability sub-menu of the MATH menu. Press 2nd MATH 7 4 *[rand(]*) ENTER to generate a random number between 0 and 1. Press ENTER to generate another number; keep pressing ENTER to generate more of them.

If you need a random number between, say, 0 and 10, then press 10 2nd MATH 7 4) ENTER. To get a random number between 5 and 15, press 5 + 10 2nd MATH 7 4) ENTER.

If you need the random number to be an *integer* between 1 and 10 (inclusive), press 2nd MATH 7 4 10) ENTER. For a random negative integer between -1 and -10 (inclusive), press 2nd MATH 7 4 (-) 10) ENTER.

IV.9.2 Permutations and Combinations: To calculate the number of permutations of 12 objects taken 7 at a time, ${}_{12}P_7$, press 2nd MATH 7 2 *[nPr(]* 12 , 7) ENTER. So, ${}_{12}P_7 = 3{,}991{,}680$, as shown in Figure IV.104.

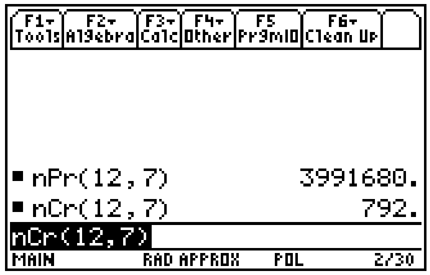


Figure IV.104: ${}_{12}P_7$ and ${}_{12}C_7$

For the number of combinations of 12 objects taken 7 at a time, ${}_{12}C_7$, press 2nd MATH 7 3 *[nCr(]* 12 , 7) ENTER. So, ${}_{12}C_7 = 792$, as shown in Figure IV.104.

IV.9.3 Probability of Winning: A state lottery is configured so that each player chooses six different numbers from 1 to 40. If these six numbers match the six numbers drawn by the State Lottery Commission, the player wins the top prize. There are ${}_{40}C_6$ ways for the six numbers to be drawn. If you purchase a single lottery ticket, your probability of winning is 1 in ${}_{40}C_6$. Press 1 ÷ 2nd MATH 7 3 40 , 6) ENTER to calculate your chances, but don't be disappointed.

IV.9.4 Sum of Data: The following data are a student's scores on 8 quizzes and 2 tests throughout an algebra course.

25, 20, 18, 89, 17, 24, 23, 22, 25, 93

To find the total points earned by the student, first enter the data using the TI-89's list editor. Press APPS 6 3 ▶ 3 *[List]* ▼ ▼ S C O R E S ENTER ENTER to open a new variable called SCORES (see Figure IV.105). Now begin entering the test scores as shown in Figure IV.106. Then press 2nd QUIT. To find the sum of the scores, press 2nd MATH 3 6 2nd a-lock S C O R E S alpha) ENTER. From Figure IV.107, the student earned 356 points throughout the algebra course.

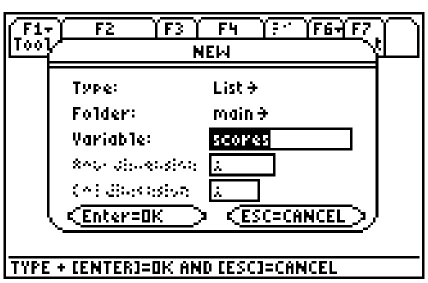


Figure IV.105: Entering a new variable

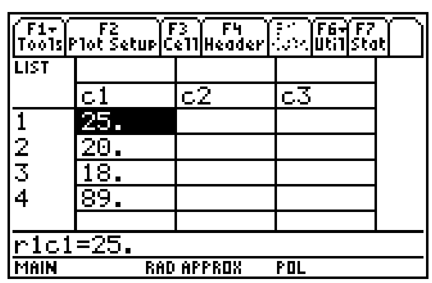


Figure IV.106: List editor

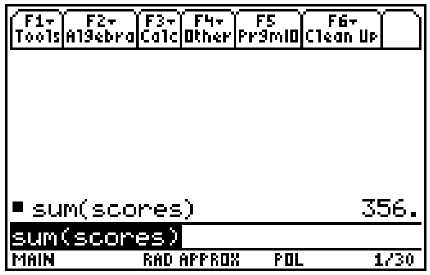


Figure IV.107: Sum

IV.9.5 Statistics: The following data are the high temperatures (in degrees Fahrenheit) recorded in Lincoln, Nebraska from October 1, 2003 to October 12, 2003 (*Source:* University of Nebraska-Lincoln)

65, 68, 74, 79, 83, 81, 80, 80, 79, 72, 67, 71

To find the mean and median of these temperatures, first enter the data using the TI-89's list editor. Press APPS 6 3 ► 3 ▼ ▼ T E M P S ENTER ENTER to open a new variable called TEMPS (see Figure IV.108). Now begin entering the temperatures as shown in Figure IV.109. Then press 2nd QUIT. To find the mean, press 2nd MATH 6 *[Statistics]* 4 *[mean(]* 2nd a-lock T E M P S alpha) ENTER and to find the median, press 2nd MATH 6 7 *[median(]* 2nd a-lock T E M P S alpha) ENTER (see Figure IV.110). So, the mean of the temperatures is approximately 75°F and the median is 76.5°F.

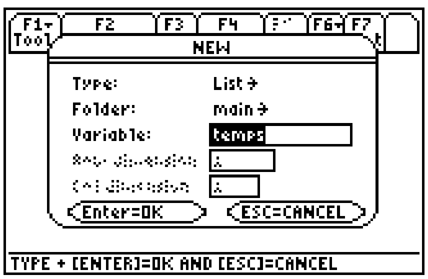


Figure IV.108: Entering a new variable

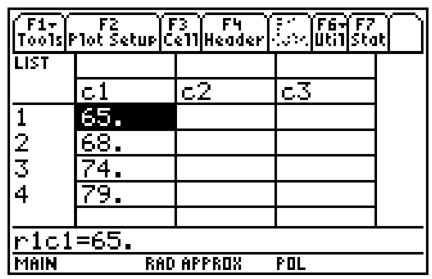


Figure IV.109: List editor

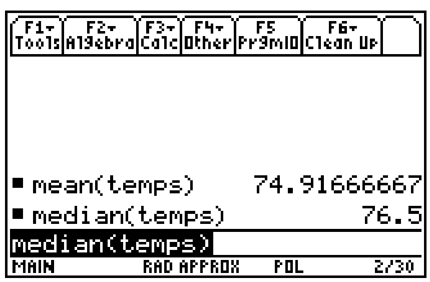


Figure IV.110: Mean and median

You can also find the mean and median of the above data by using the OneVar command found in the Calc menu of the Data/Matrix Editor. You can copy the data you entered in Figure IV.108 to a data list by opening the TEMPS list first. Then press F1 2 *[Save Copy As…]* ► 1 *[Data]* ▼ ▼ T E M P S alpha 2 ENTER ENTER (see Figure V.109). Note that you cannot name the data list TEMPS. Now, to use the OneVar command you must have the data list TEMPS2 open. Then press F5 *[Calc]*. For the Calculation Type, choose 1 *[OneVar]*, set the *x* variable to c1, and press ENTER ENTER. The TI-89 will calculate several different statistical values. The first line represents the mean of the data which is approximately 75°F (see Figure IV.112). The second line is the sum of the data, the third line is the sum of the squares of the data, the fourth line is the sample standard deviation of the data, the fifth line is the number of data values, the sixth line is the minimum value of the data, the seventh line is the first quartile of the data, and the eighth line is the median of the data which is 76.5°F (see Figure IV.113). The ninth line is the third quartile of the data and the tenth line is the maximum value of the data.

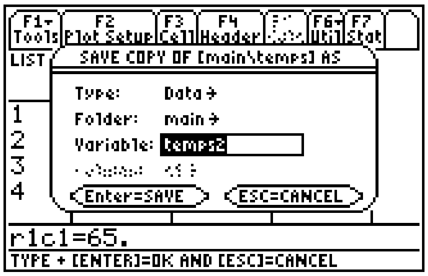


Figure IV.111: Saving a list as a data list

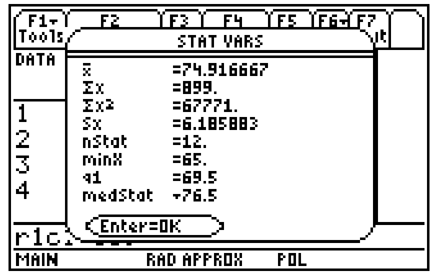


Figure IV.112: OneVar command

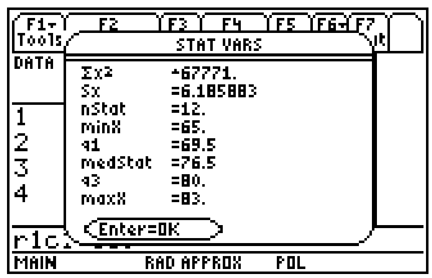


Figure IV.113: OneVar command

You can scroll through the list of statistical values by pressing ▲ or ▼.

IV.10 Programming

IV.10.1 Entering a Program: The TI-89 is a programmable calculator that can store sequences of commands for later replay. Press APPS 7 *[Program Editor]* to access the programming menu. The TI-89 has space for many programs, each called by a title you give it. To create a new program, start by pressing APPS 7 3 *[New…]*. Set the Type to Program and the Folder to main (unless you have another folder in which you want to store the program). Enter a descriptive title for the program in the Variable line. After you name the program, press ENTER ENTER to go to the program editor. The program name and the beginning and ending commands of the program are automatically displayed with the cursor on the first line after Prgm, the begin program command.

In the program, each line begins with a colon : supplied automatically by the calculator. Any command you could enter directly in the TI-89's home screen can be entered as a line in a program. There are also special programming commands.

You may interrupt programming input at any stage by pressing 2nd QUIT. To return later for more editing, press APPS 7 2 *[Open…]*, move the cursor down to the Variable list, highlight the program's name, and press ENTER ENTER.

You may remove a program from memory by pressing 2nd VAR-LINK, move the cursor to highlight the name of the program you want to delete, then press F1 *[Manage]* 1 *[Delete]* ENTER and then ENTER again to confirm the deletion from the calculator's memory.

Technology Tip: If your program uses one-letter variables such as, a, b, c, or d, note that any previous values for these variables, including matrices, will be replaced by the values used by the program. The TI-89 does not distinguish between A and a in these uses. Note that you will have to clear the variables (using 2nd VAR-LINK) in order to use these names again in the current folder. From the home screen, press F4 *[Other]* alpha B *[NewFold]* and type the name of the new folder. The work you do from that point on will be in the new folder, as indicated by the folder name in the lower left corner of the status line. You can change folders from the MODE menu or, from the home screen, by typing setFold(*foldername*), where *foldername* is the existing folder that you wish to be in.

IV.10.2 Executing a Program: To execute a program you entered, go to the home screen and type the name of the program, including the parentheses and then press ENTER to execute it. If you have forgotten its name, press 2nd VAR-LINK to list all the variables that exist. The programs will have PRGM after the name. You can execute the program from this screen by highlighting the name and then pressing ENTER. The screen will return to the home screen and you will have to enter the closing parenthesis) and press ENTER to execute the program.

If you need to interrupt a program during execution, press ON.

The instruction manual for your TI-89 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.

