Part I: Texas Instruments TI-82 Graphics Calculator

I.1 Getting started with the TI-82

I.1.1 Basics: Press the ON key to begin using your TI-82 calculator. If you need to adjust the display contrast, first press 2nd, then press and hold ▲ (the *up* arrow key) to lighten or ▼ (the *down* arrow key) to darken. As you press and hold ▲ or ▼, an integer between 0 (lightest) and 9 (darkest) appears in the upper right corner of the display. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF.

Check the TI-82's settings by pressing MODE. If necessary, use the arrow keys to move the blinking cursor to a setting you want to change. Press ENTER to select a new setting. To start, select the options along the left side of the MODE menu as illustrated in Figure I.1: normal display, floating decimals, radian measure, function graphs, connected lines, sequential plotting, and full screen display. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing CLEAR.

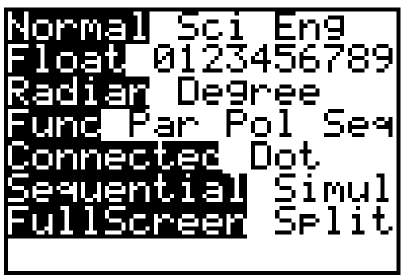


Figure I.1: MODE menu

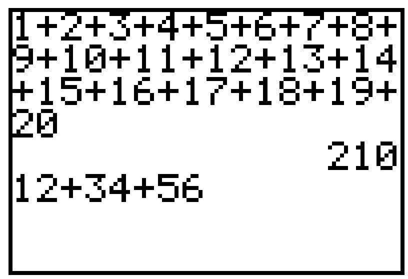


Figure I.2: Home screen

I.1.2 Editing: One advantage of the TI-82 is that up to 8 lines are visible at one time, so you can *see* a long calculation. For example, type this sum (see Figure I.2):

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20

Then press ENTER to see the answer.

Often we do not notice a mistake until we see how unreasonable an answer is. The TI-82 permits you to redisplay an entire calculation, edit it easily, then execute the *corrected* calculation.

Suppose you had typed 12 + 34 + 56 as in Figure I.2 but had *not yet* pressed ENTER, when you realize that 34 should have been 74. Simply press ◄ (the *left* arrow key) as many times as necessary to move the blinking cursor left to 3, then type 7 to write over it. On the other hand, if 34 should have been 384, move the cursor back to 4, press 2nd INS (the cursor changes to a blinking underline) and then type 8 (inserts at the cursor position and the other characters are pushed to the right). If the 34 should have been 3 only, move the cursor to 4 and press DEL to delete it.

Technology Tip: To move quickly to the *beginning* on an expression you are currently editing, press ▲ (the *up* arrow key); to jump to the *end* of that expression, press ▼ (the *down* arrow key).

Even if you had pressed ENTER, you may still edit the previous expression. Press 2nd and then ENTRY to *recall* the last expression that was entered. Now you can change it. In fact, the TI-82 retains many prior entries in a "last entry" storage area. Press 2nd ENTRY repeatedly until the previous line you want replaces the current line.

Technology Tip: When you need to evaluate a formula for different values of a variable, use the editing feature to simplify the process. For example, suppose you want to find the balance in an investment account if there is now $5000 in the account and interest is compounded annually at the rate of 8.5%. The formula for the balance is $P\left(1 + \frac{r}{n}\right)^{nt}$, where P = principal, r = rate of interest (expressed as a decimal), n = number of times interest is compounded each year, and t = number of years. In our example, this becomes $5000(1 + .085)^t$. Here are the keystrokes for finding the balance after t = 3, 5, and 10 years (results are shown in Figure I.3).

Years	*Keystrokes*	*Balance*
3	5000 (1 + .085) ^ 3 ENTER	$6386.45
5	2nd ENTRY ◀ 5 ENTER	$7518.28
10	2nd ENTRY ◀ 10 ENTER	$11,304.92

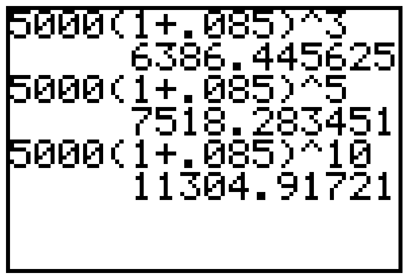


Figure I.3: Editing expressions

Then to find the balance from the same initial investment but after 5 years when the annual interest rate is 7.5%, press these keys to change the last calculation above: 2nd ENTRY ◀ DEL ◀ 5 ◀ ◀ ◀ ◀ ◀ 7 ENTER.

I.1.3 Key Functions: Most keys on the TI-82 offer access to more than one function, just as the keys on a computer keyboard can produce more than one letter ("g" and "G") or even quite different characters ("5" and "%"). The primary function of a key is indicated on the key itself, and you access that function by a simple press on the key.

To access the *second* function to the *left* above a key, first press 2nd (the cursor changes to a blinking ↑) and *then* press the key. For example, to calculate $\sqrt{25}$, press 2nd $\sqrt{\ }$ 25 ENTER.

When you want to use a letter of other characters printed to the *right* above a key, first press ALPHA (the cursor changes to a blinking **A**) and then the key. For example, to use the letter K in a formula, press ALPHA K. If you need several letters in a row, press 2nd A-LOCK, which is like **CAPS LOCK** on a computer keyboard, and then press all the letters you want. Remember to press ALPHA when you are finished and want to restore the keys to their primary functions.

I.1.4 Order of Operations: The TI-82 performs calculations according to the standard algebraic rules. Working outwards from inner parentheses, calculations are performed from left to right. Powers and roots are evaluated first, followed by multiplications and divisions, and then additions and subtractions.

Note that the TI-82 distinguishes between *subtraction* and the *negative sign*. If you wish to enter a negative number, it is necessary to use (-) key. For example, you would evaluate $-5 - (4 \cdot -3)$ by pressing (-) 5 – (4 × (-) 3) ENTER to get 7.

Enter these expressions to practice using your TI-82.

Expression	*Keystrokes*	*Display*
$7 - 5 \cdot 3$	7 – 5 × 3 ENTER	-8
$(7 - 5) \cdot 3$	(7 – 5) × 3 ENTER	6
$120 - 10^2$	120 – 10 x^2 ENTER	20
$(120 - 10)^2$	(120 – 10) x^2 ENTER	12100
$\frac{24}{2^3}$	24 ÷ 2 ^ 3 ENTER	3
$\left(\frac{24}{2}\right)^3$	(24 ÷ 2) ^ 3 ENTER	1728
$(7 - -5) \cdot -3$	(7 – (-) 5) × (-) 3 ENTER	-36

I.1.5 Algebraic Expressions and Memory: Your calculator can evaluate expressions such as $\frac{N(N+1)}{2}$ *after* you have entered a value for N. Suppose you want $N = 200$. Press 200 STO ▶ ALPHA N ENTER to store the value of 200 in memory location N. Whenever you use N in an expression, the calculator will substitute the value 200 until you make a change by storing *another* number in N. Next enter the expression $\frac{N(N+1)}{2}$ by typing ALPHA N (ALPHA N + 1) ÷ 2 ENTER. For $N = 200$, you will find that $\frac{N(N+1)}{2} = 20{,}100$.

The contents of any memory location may be revealed by typing just its letter name and then ENTER. And the TI-82 retains memorized values even when it is turned off, so long as its batteries are good.

I.1.6 Repeated Operations with ANS: The result of your *last* calculation is always stored in memory location ANS and replaces any previous result. This makes it easy to use the answer from one computation in another computation. For example, press 30 + 15 ENTER so that 45 is the last result displayed. Then press 2nd ANS ÷ 9 ENTER and get 5 because $45 \div 9 = 5$.

With a function like division, you press the ÷ key *after* you enter an argument. For such functions, whenever you would start a new calculation with the previous answer followed by pressing the function key, you may press just the function key. So instead of 2nd ANS ÷ 9 in the previous example, you could have pressed simply ÷ 9 to achieve the same result. This technique also works for these functions: + – × x^2 ^ x^{-1}.

Here is a situation where this is especially useful. Suppose a person makes $5.85 per hour and you are asked to calculate earnings for a day, a week, and a year. Execute the given keystrokes to find the person's incomes during these periods (results are shown in Figure I.4.):

Pay period	*Keystrokes*	*Balance*
8-hour day	5.85 × 8 ENTER	$46.80
5-day week	× 5 ENTER	$234
52-week year	× 52 ENTER	$12,168

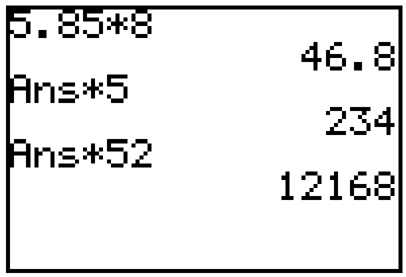


Figure I.4: ANS variable

I.1.7 The MATH Menu: Operators and functions associated with a scientific calculator are available either immediately from the keys of the TI-82 or by the 2nd keys. You have direct key access to common arithmetic operations (x^2, 2nd $\sqrt{\ }$, x^{-1}, ^, 2nd ABS), trigonometric functions (SIN, COS, TAN) and their inverses (2nd SIN^{-1}, 2nd COS^{-1}, 2nd TAN^{-1}), exponential and logarithmic functions (LOG, 2nd 10^x, LN, 2nd e^x), and a famous constant (2nd π).

A significant difference between the TI-82 graphing calculators and most scientific calculators is that the TI-82 requires the argument of a function *after* the function, as you would see a formula written in your textbook. For example, on the TI-82 you calculate $\sqrt{16}$ by pressing the keys 2nd $\sqrt{\ }$ 16 in that order.

Here are keystrokes for basic mathematical operations. Try them for practice on your TI-82.

Expression	*Keystrokes*	*Display*
$\sqrt{3^2 + 4^2}$	2nd $\sqrt{\ }$ (3 x^2 + 4 x^2) ENTER	5
$2\frac{1}{3}$	2 + 3 x^{-1} ENTER	2.333333333
$\lvert -5 \rvert$	2nd ABS (-) 5 ENTER	5
log 200	LOG 200 ENTER	2.301029996
$2.34 \cdot 10^5$	2.34 × 2nd 10^x 5 ENTER	234000

Additional mathematical operations and functions are available from the MATH menu. Press MATH to see the various options (Figure I.5). You will learn in your mathematics textbook how to apply many of them. As an example, calculate $\sqrt[3]{7}$ by pressing MATH and then *either* 4 [$\sqrt[3]{\ }$] *or* ▼ ▼ ▼ ENTER; finally press 7 ENTER to see 1.912931183. To leave MATH menu and take no other action, press 2nd QUIT or just CLEAR.

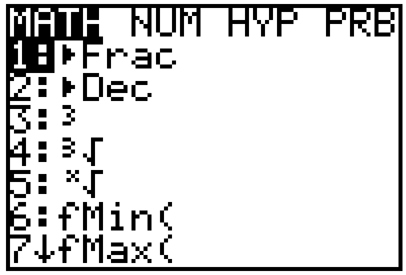


Figure I.5: MATH menu

The *factorial* of a nonnegative integer is the *product* of *all* the integers from 1 up to the given integer. The symbol for factorial is the exclamation point. So 4! (pronounced *four factorial*) is $1 \cdot 2 \cdot 3 \cdot 4 = 24$. You will learn more about applications of factorials in your textbook, but for now use the TI-82 to calculate 4! The factorial command is located in the MATH menu's PRB sub-menu. to compute 4!, press these keystrokes: 4 MATH ◀ 4 ENTER *or* 4 MATH ◀ ▼ ▼ ▼ ENTER ENTER.

Note that you can select a sub-menu from the MATH menu by pressing either ◀ or ▶. It is easier to press ◀ once than to press ▶ three times to get to the PRB sub-menu.

I.2 Functions and Graphs

I.2.1 Evaluating Functions: Suppose you received a monthly salary of $1975 plus a commission of 10% of sales. Let x = your sales in dollars; then your wages W in dollars are given by the equation $W = 1975 + .10x$.

If your January sales were $2230 and your February sales were $1865, what was your income during those months?

Here's one method to use your TI-82 to perform this task. Press the Y= key at the top of the calculator to display the function editing screen (Figure I.6). You may enter as many as ten different functions for the TI-82 to use at one time. If there is already a function Y_1, press ▲ or ▼ as many times as necessary to move the cursor to Y_1 and then press CLEAR to delete whatever was there. Then enter the expression $1975 + .10x$ by pressing these keys: 1975 + .10 X,T,θ. (The X,T,θ key lets you enter the variable X easily without having to use the ALPHA key.) Now press 2nd QUIT to return to the main calculations screen.

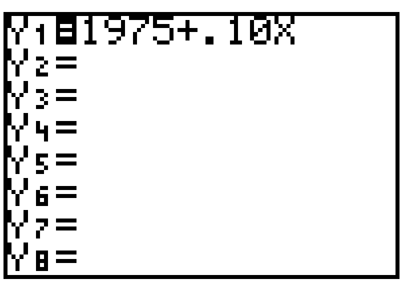


Figure I.6: Y= screen

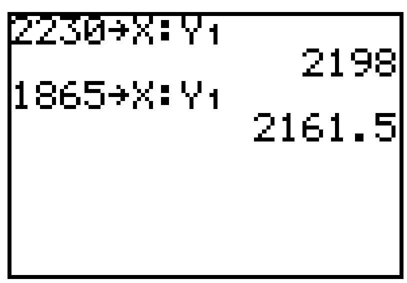


Figure I.7: Evaluating a function

Assign the value 2230 to the variable x by using these keystrokes (see Figure I.7): 2230 STO ► X,T,θ. Then press 2nd : to allow another expression to be entered on the same command line. Next press the following keystrokes to evaluate Y_1 and find January's wages: 2nd Y-VARS 1 *[Function]* 1 *[Y_1]* ENTER. It is not necessary to repeat all these steps to find the February wages. Simply press 2nd ENTRY to recall the entire previous line, change 2230 to 1865, and press ENTER. Each time the TI-82 evaluates the function Y_1, it uses the *current* value of x.

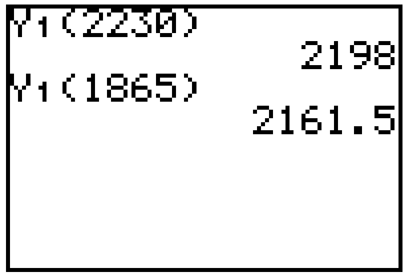


Figure I.8: Function notation

Like your textbook, the TI-82 uses standard function notation. So, to evaluate $Y_1(2230)$ when $Y_1(x) = 1975 + .10x$, press 2nd Y-VARS 1 1 (2230) ENTER (see Figure I.8). Then to evaluate $Y_1(1865)$, press 2nd ENTRY to recall the last line, change 2230 to 1865, and press ENTER.

You may also have the TI-82 make a table of values for the function. Press 2nd TblSet to set up the table (Figure I.9). Move the blinking cursor onto Ask beside Indpnt:, then press ENTER. This configuration permits you to input values for x one at a time. Now press 2nd TABLE, enter 2230 in the X column, and press ENTER (see Figure I.10). Continue to enter additional values for X and the calculator automatically completes the table with corresponding values of Y_1. Press 2nd QUIT to leave the TABLE screen.

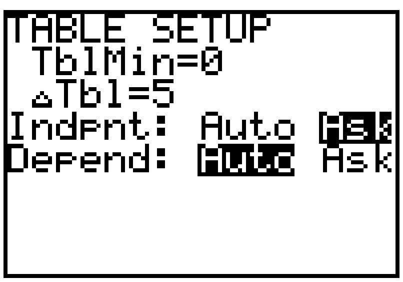


Figure I.9: TblSet screen

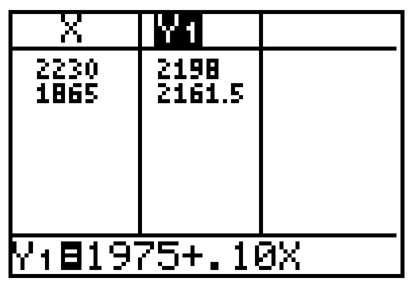


Figure I.10: Table of values

For a table containing values for $x = 1, 2, 3, 4, 5$, and so on, set TblMin = 1 to start at $x = 1$, ΔTbl = 1 to increment in steps of 1, and both Indpnt and Depend to Auto.

Technology Tip: The TI-82 does not require multiplication to be expressed between variables, so xxx means x^3. It is often easier to press two or three x's together than to search for the square key or the powers key. Of course, expressed multiplication is also not required between a constant and variable. So, to enter $2x^3 + 3x^2 - 4x + 5$ in the TI-82, you might save keystrokes and press just these keys: 2 X,T,θ X,T,θ X,T,θ + 3 X,T,θ X,T,θ − 4 X,T,θ + 5.

I.2.2 Functions in a Graph Window: Once you have entered a function in the Y= screen of the TI-82, just press GRAPH to see its graph. The ability to draw a graph contributes substantially to our ability to solve problems.

For example, here is how to graph $y = -x^3 + 4x$. First press Y= and delete anything that may be there by moving with the arrow keys to Y_1 or to any of the other lines and pressing CLEAR wherever necessary. Then, with the cursor on the top line Y_1, press (-) X,T,θ MATH 3 + 4 X,T,θ to enter the function (as in Figure I.11). Now press GRAPH and the TI-82 changes to a window with the graph of $y = -x^3 + 4x$ (Figure I.12).

While the TI-82 is calculating coordinates for a plot, it displays a busy indicator at the top right of the graph window.

Technology Tip: If you would like to see a function in the Y= menu and its graph in a graph window, both at the same time, open the MODE menu, move the cursor down to the last line, and select Split screen. Your TI-82's screen is now divided horizontally (see Figure I.11), with an upper graph window and a lower window that can display the home screen or an editing screen. The split screen is also useful when you need to do some calculations as you trace along a graph. For now, restore the TI-82 to FullScreen.

Your graph window may look like the one in Figure I.12 or it may be different. Because the graph of $y = -x^3 + 4x$ extends infinitely far left and right and also infinitely far up and down, the TI-82 can display only a piece of the actual graph. This displayed rectangular part is called a *viewing window*. You can easily change the viewing window to enhance your investigation of a graph.

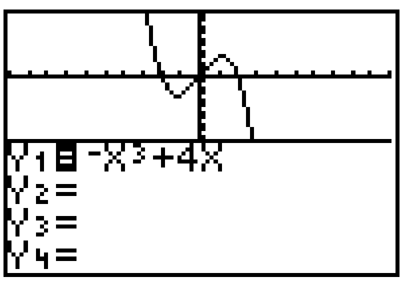


Figure I.11: Split screen: Y= below

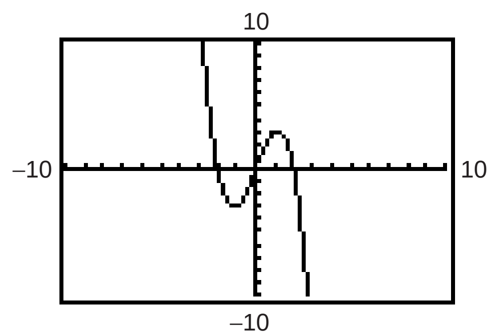


Figure I.12: Graph of $y = -x^3 + 4x$

The viewing window in Figure I.12 shows the part of the graph that extends horizontally from −10 to 10 and vertically from −10 to 10. Press WINDOW to see information about your viewing window. Figure I.13 shows the WINDOW screen that corresponds to the viewing window in Figure I.12. This is the *standard* viewing window for the TI-82.

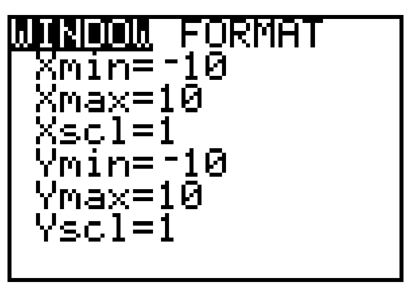


Figure I.13: Standard WINDOW

The variables Xmin and Xmax are the minimum and maximum *x*-values of the viewing window. Ymin and Ymax are its minimum and maximum *y*-values.

Xscl and Yscl set the spacing between tick marks on the axes.

Use the arrow keys ▲ and ▼ to move up and down from one line to another in this list; pressing the ENTER key will move down the list. Press CLEAR to delete the current value and then enter a new value. You may also edit the entry as you would edit an expression. Remember that a minimum *must* be less than the corresponding maximum or the TI-82 will issue an error message. Also, remember to use the (-) key, not – (which is subtraction), when you want to enter a negative value. Figures I.12–13, I.14–15, and I.16–17 show different WINDOW screens and the corresponding viewing window for each one.

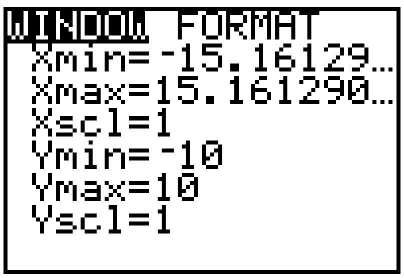


Figure I.14: Square window

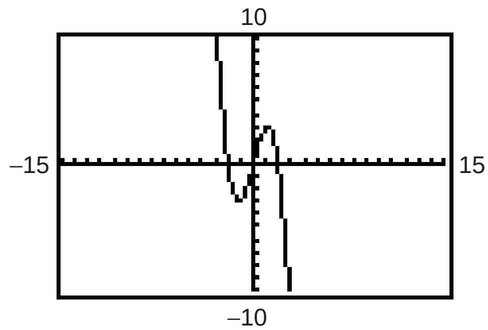


Figure I.15: Graph of $y = -x^3 + 4x$

To initialize the viewing window quickly to the standard viewing window (Figure I.13), press ZOOM 6 *[ZStandard]*. To set the viewing window quickly to a square window (Figure I.14), press ZOOM 5 *[ZSquare]*. More information about square windows is presented later in Section I.2.4.

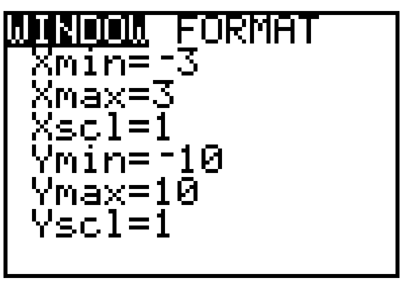


Figure I.16: Custom window

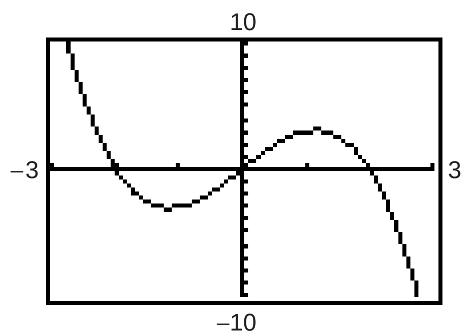


Figure I.17: Graph of $y = -x^3 + 4x$

Sometimes you may wish to display grid points corresponding to tick marks on the axes. This and other graph format options may be changed by pressing WINDOW ▶ to display the FORMAT menu (Figure I.18). Use arrow keys to move the blinking cursor to GridOn; press ENTER and then GRAPH to redraw the graph. Figure I.19 shows the same graph as in Figure I.17 but with the grid turned on. In general, you'll want the grid turned *off*, so do that now by pressing WINDOW ▶, use the arrow keys to move the blinking cursor to GridOff, and press ENTER and CLEAR.

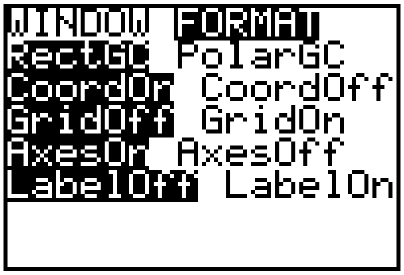


Figure I.18: WINDOW FORMAT menu

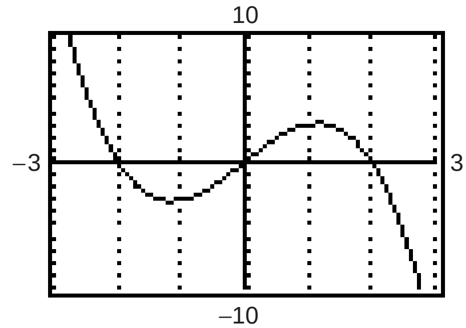


Figure I.19: Grid turned on for $y = -x^3 + 4x$

I.2.3 Graphing Step and Piecewise-Defined Functions: The greatest integer function, written $[\![x]\!]$, gives the greatest *integer* less than or equal to a number x. On the TI-82, the greatest integer function is called Int and is located under the NUM sub-menu of the MATH menu (see Figure I.5). So, calculate $[\![6.78]\!] = 6$ by pressing MATH ▶ 4 6.78 ENTER.

To graph $y = [\![x]\!]$, go into the Y= menu, move beside Y_1, and press CLEAR MATH ▶ 4 X,T,θ GRAPH. Figure I.20 shows this graph in a viewing window from −5 to 5 in both directions.

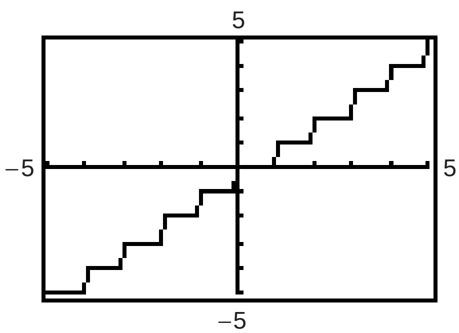


Figure I.20: Connected graph of $y = [\![x]\!]$

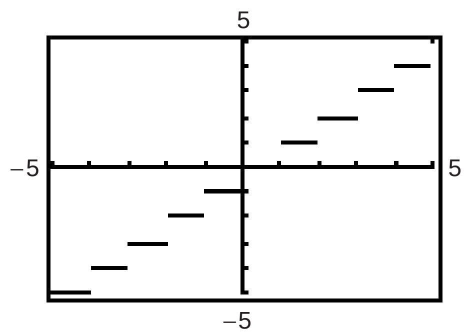


Figure I.21: Dot graph of $y = [\![x]\!]$

The true graph of the greatest integer function is a step graph, like the one in Figure I.21. For the graph of $y = [\![x]\!]$, a segment should *not* be drawn between every pair of successive points. You can change from Connected line to Dot graph on the TI-82 by opening the MODE menu. Move the cursor down to the fifth line; select whichever graph type you require; press ENTER to put it into effect, and GRAPH to see the result.

Make sure to change your TI-82 back to Connected line, because most of the functions that you will be graphing should be viewed this way.

The TI-82 can graph piecewise-defined functions by using the options in the TEST menu (Figure I.22) that is displayed by pressing 2nd TEST. Each TEST function returns the value 1 if the statement is true, and the value 0 if the statement is false.

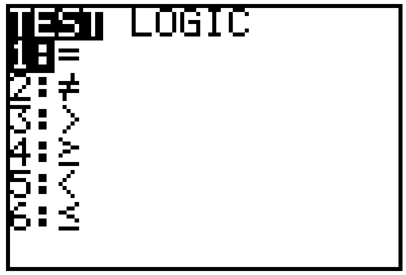


Figure I.22: 2nd TEST menu

For example, to graph the function $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$ (using Dot graph) enter the following keystrokes Y= (X,T,θ x^2 + 1) (X,T,θ 2nd TEST 5 0) + (X,T,θ – 1) (X,T,θ 2nd TEST 4 0) (Figure I.23). Then change the mode to Dot graph and press GRAPH to display the graph. Figure I.24 shows this graph in a viewing window from -5 to 5 in both directions.

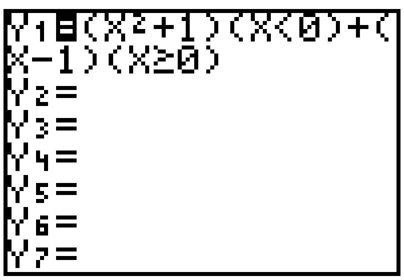


Figure I.23: Piecewise-defined function

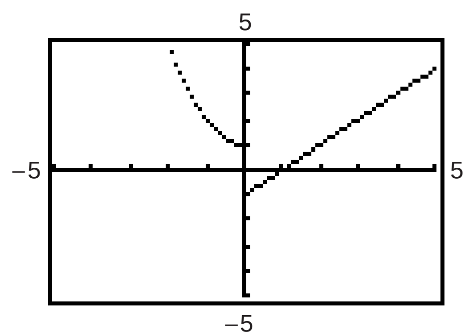
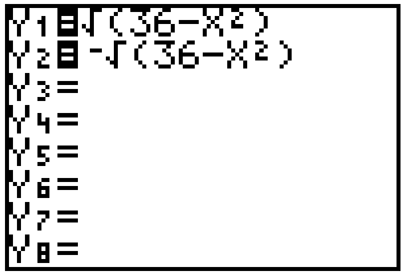


Figure I.24: Piecewise-defined graph

I.2.4 Graphing a Circle: Here is a useful technique for graphs that are not functions, but that can be "split" into a top part and a bottom part, or into multiple parts. Suppose you wish to graph the circle whose equation is $x^2 + y^2 = 36$. First solve for y and get an equation for the top semicircle, $y = \sqrt{36 - x^2}$, and for the bottom semicircle, $y = -\sqrt{36 - x^2}$. Then graph the two semicircles simultaneously.

Use the following keystrokes to draw the circle's graph. Enter $\sqrt{36 - x^2}$ as Y_1 and $-\sqrt{36 - x^2}$ as Y_2 (see Figure I.25) by pressing Y= CLEAR 2nd $\sqrt{\ }$ (36 $-$ X,T,θ x^2) ENTER CLEAR (-) 2nd $\sqrt{\ }$ (36 $-$ X,T,θ x^2). Then press GRAPH to draw them both.



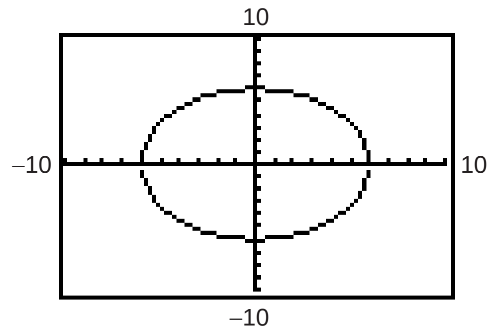
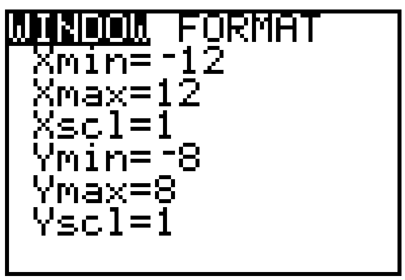


Figure I.25: Two semicircles

Figure I.26: Circle's graph – standard WINDOW

If your range were set to the standard viewing window, your graph would look like Figure I.26. Now this does *not* look like a circle, because the units along the axes are not the same. This is where the square viewing window is important. Press ZOOM 5 and see a graph that appears more circular.



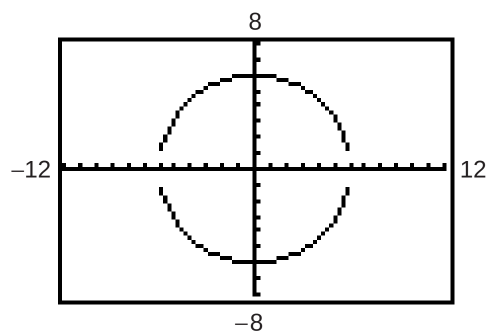


Figure I.27: $\frac{\text{vertical}}{\text{horizontal}} = \frac{16}{24} = \frac{2}{3}$

Figure I.28: A "square" circle

Technology Tip: Another way to get a square graph is to change the range variables so that the value of Ymax - Ymin is approximately $\frac{2}{3}$ times Xmax - Xmin. For example, see the WINDOW in Figure I.27 and the corresponding graph in Figure I.28. This method works because the dimensions of the TI-82's display are such that the ratio of vertical to horizontal is approximately $\frac{2}{3}$.

The two semicircles in Figure I.28 do not connect because of an idiosyncrasy in the way the TI-82 plots a graph.

Back when you entered $\sqrt{36 - x^2}$ as Y_1 and $-\sqrt{36 - x^2}$ as Y_2, you could have entered -Y_1 as Y_2 and saved some keystrokes. Try this by going back to the Y= menu and pressing ▼ to move the cursor down to Y_2. Then press CLEAR (-) 2nd Y-VARS 1 1. The graph should be just as it was before.

I.2.5 TRACE: Graph $y = -x^3 + 4x$ from Section I.2.2 in the standard viewing window. (Remember to clear any other functions in the Y= screen.) Press any of the arrow keys ▲, ▼, ◀, ▶ and see the cursor move from the center of the viewing window. The coordinates of the cursor's location are displayed at the bottom of the screen, as in Figure I.29, in floating decimal format. This cursor is called a *free-moving cursor* because it can move from dot to dot *anywhere* in the graph window.

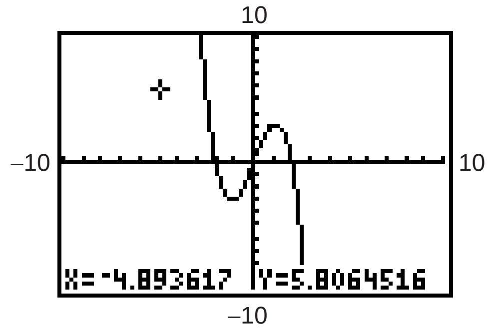


Figure I.29: Free-moving cursor

Remove the free-moving cursor and its coordinates from the window by pressing GRAPH, CLEAR, or ENTER. Press an arrow key again and the free-moving cursor will reappear at the same point you left it.

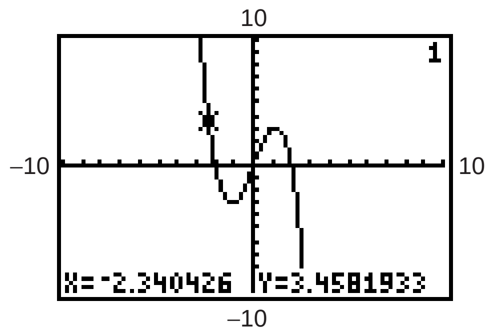


Figure I.30: TRACE

Press TRACE to enable the left ◀ and right ▶ arrow keys to move the cursor from point to point along the graph of the function. The cursor is no longer free-moving, but is now constrained to the function. The coordinates that are displayed belong to points on the function's graph, so the *y*-coordinate is the calculated value of the function at the corresponding *x*-coordinate (Figure I.30).

Now plot a second function, $y = -.25x$, along with $y = -x^3 + 4x$. Press Y=, move the cursor to the Y_2 line, and enter $-.25x$, then press GRAPH to see both functions.

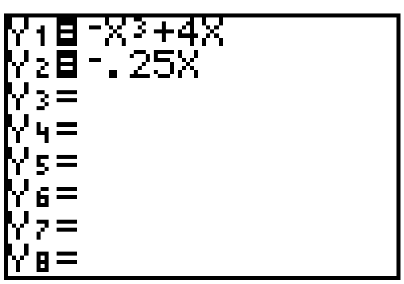


Figure I.31: Two functions

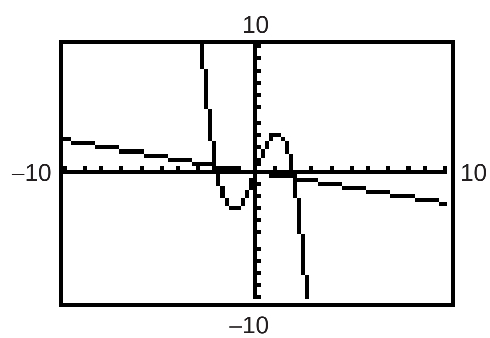


Figure I.32: $y = -x^3 + 4x$ and $y = -.25x$

Note in Figure I.31 that the equal signs next to Y_1 and Y_2 are *both* highlighted. This means *both* functions will be graphed as shown in Figure I.32. In the Y= screen, move the cursor directly on top of the equal sign next to Y_1 and press ENTER. This equal sign should no longer be highlighted (see Figure I.33). Now press GRAPH and see that only Y_2 is plotted (Figure I.34).

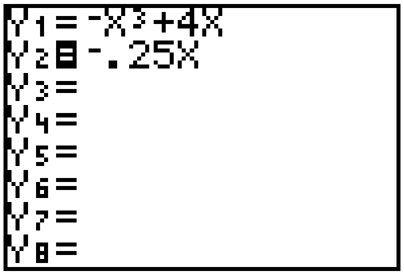


Figure I.33: Y= screen with only Y_2 active

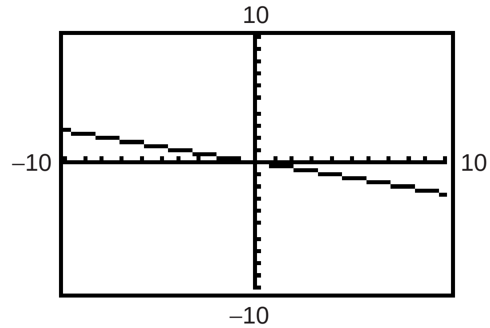


Figure I.34: Graph of $y = -.25x$

Many different functions can be stored in the Y= list and any combination of them may be graphed simultaneously. You can make a function active or inactive for graphing by pressing ENTER on its equal sign to highlight (activate) or remove the highlight (deactivate). Now go back to the Y= screen and do what is needed in order to graph Y_1 but not Y_2.

Now activate both functions so that both graphs are plotted. Press TRACE and the cursor appears first on the graph of $y = -x^3 + 4x$ because it is higher up in the Y= list. You know that the cursor is on this function, Y_1, because of the numeral 1 that is displayed in the upper right corner of the screen (see Figure I.30). Press the up ▲ or down ▼ arrow key to move the cursor vertically to the graph of $y = -.25x$. Now the numeral 2 is displayed in the upper right corner of the screen. Next press the left and right arrow keys to trace along the graph of $y = -.25x$. When more than one function is plotted, you can move the trace cursor vertically from one graph to another with the ▲ and ▼ keys.

Technology Tip: Trace along the graph of $y = -.25x$ and press and hold either ◀ or ▶. Eventually you will reach the left or right edge of the window. Keep pressing the arrow key and the TI-82 will allow you to continue the trace by panning the viewing window. Check the WINDOW screen to see that Xmin and Xmax are automatically updated.

If you trace along the graph of $y = -x^3 + 4x$, the cursor will eventually move *above* or *below* the viewing window. The cursor's coordinates on the graph will still be displayed, though the cursor itself can no longer be seen. When you are tracing along a graph, press ENTER and the window will quickly pan over so that the cursor's position on the function is centered in a new viewing window. This feature is especially helpful when you trace near or beyond the edge of the current viewing window.

The TI-82's display has 95 horizontal columns of pixels and 63 vertical rows. So when you trace a curve across a graph window, you are actually moving from Xmin to Xmax in 94 equal jumps, each called Δx. You would calculate the size of each jump to be $\Delta x = \frac{\text{Xmax} - \text{Xmin}}{94}$. Sometimes you may want the jumps to be friendly numbers like 0.1 or 0.25 so that, when you trace along the curve, the x-coordinates will be incremented by such a convenient amount. Just set your viewing window for a particular increment Δx by making $\text{Xmax} = \text{Xmin} + 94 \cdot \Delta x$. For example, if you want $\text{Xmin} = -5$ and $\Delta x = 0.3$, set $\text{Xmax} = -5 + 94 \cdot 0.3 = 23.2$. Likewise, set $\text{Ymax} = \text{Ymin} + 62 \cdot \Delta y$ if you want the vertical increment to be some special Δy.

To center your window around a particular point, (h, k), and also have a certain Δx, set $\text{Xmin} = h - 47 \cdot \Delta x$ and $\text{Xmax} = h + 47 \cdot \Delta x$. Likewise, make $\text{Ymin} = k - 31 \cdot \Delta y$ and $\text{Ymax} = k + 31 \cdot \Delta y$. For example, to center a window around the origin $(0, 0)$, with both horizontal and vertical increments of 0.25, set the range so that $\text{Xmin} = 0 - 47 \cdot 0.25 = -11.75$, $\text{Xmax} = 0 + 47 \cdot 0.25 = 11.75$, $\text{Ymin} = 0 - 31 \cdot 0.25 = -7.75$, and $\text{Ymax} = 0 + 31 \cdot 0.25 = 7.75$.

See the benefit by first graphing $y = x^2 + 2x + 1$ in a standard viewing window. Trace near its y-intercept, which is $(0, 1)$, and move towards its x-intercept, which is $(-1, 0)$. Then press ZOOM 4 *[ZDecimal]* and trace again near the intercepts.

I.2.6 ZOOM: Plot again the two graphs for $y = -x^3 + 4x$ and for $y = -.25x$. There appears to be an intersection near $x = 2$. The TI-82 provides several ways to enlarge the view around this point. You can change the viewing window directly by pressing WINDOW and editing the values of Xmin, Xmax, Ymin, and Ymax. Figure I.36 shows a new viewing window for the range displayed in Figure I.35. The cursor has been moved near the point of intersection; move your cursor closer to get the best approximation possible for the coordinates in the intersection.

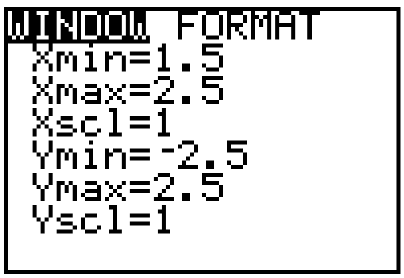


Figure I.35: New WINDOW

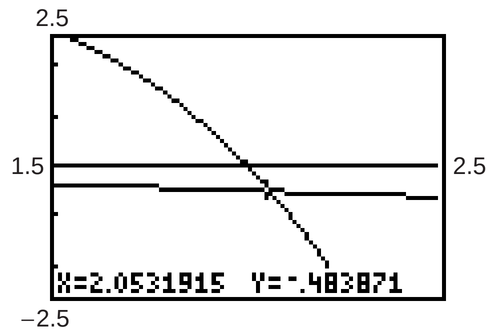


Figure I.36: Closer view

A more efficient method for enlarging the view is to draw a new viewing window with the cursor. Start again with a graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$ in a standard viewing window (press ZOOM 6 for the standard window).

Now imagine a small rectangular box around the enter section point, near $x = 2$. Press ZOOM 1 *[ZBox]* (Figure I.37) to draw a box to define this new viewing window. Use the arrow keys to move the cursor, whose coordinates are displayed at the bottom of the window, to one corner of the new viewing window you imagine.

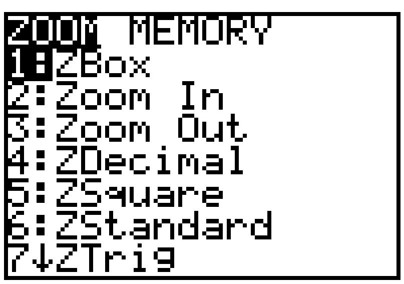


Figure I.37: ZOOM menu

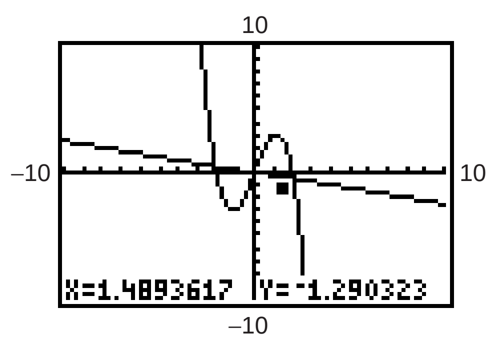


Figure I.38: One corner selected

Press ENTER to fix the corner where you have moved the cursor; it changes shape and becomes a blinking square (Figure I.38). Use the arrow keys again to move the cursor to the diagonally opposite corner of the new rectangle (Figure I.39), then press ENTER. The rectangular area you have enclosed will now enlarge to fill the graph window (Figure I.40).

You may cancel the zoom any time *before* you press this last ENTER. Press ZOOM once more and start over. Press CLEAR or GRAPH to cancel the zoom, or press 2nd QUIT to cancel the zoom and return to the home screen.

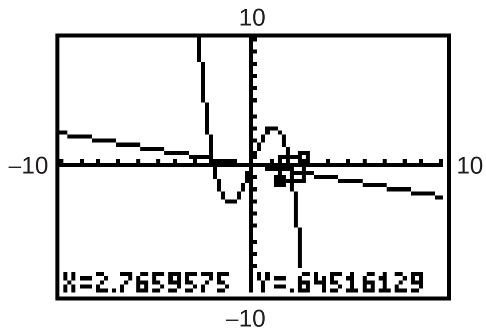


Figure I.39: Box drawn

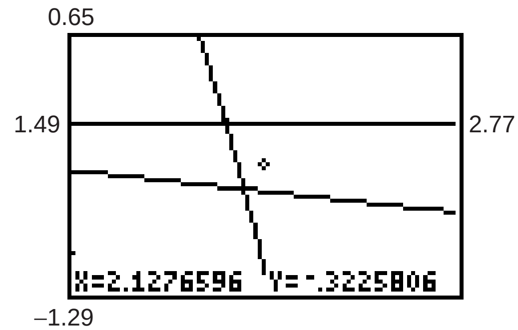


Figure I.40: New viewing window

You can also quickly magnify a graph around the cursor's location. Return once more to the standard viewing window for the graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$. Press ZOOM 2 *[Zoom In]* and then press arrow keys to move the cursor as close as you can to the point of intersection near $x = 2$ (see Figure I.41). Then press ENTER and the calculator draws a magnified graph, centered at the cursor's position (Figure I.42). The range variables are changed to reflect this new viewing window. Look in the WINDOW menu to verify this.

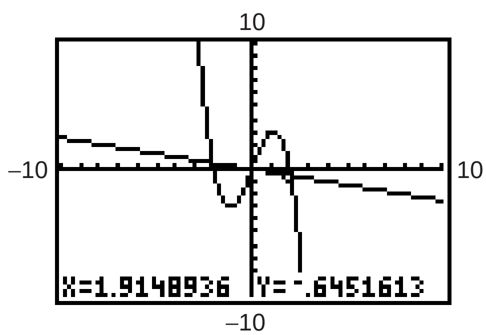


Figure I.41: Before a zoom in

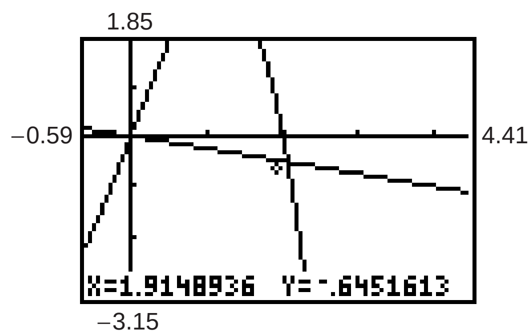


Figure I.42: After a zoom in

As you see in the ZOOM menu (Figure I.37), the TI-82 can Zoom In (press ZOOM 2) or Zoom Out (press ZOOM 3). Zoom out to see a larger view of the graph, centered at the cursor position. You can change the horizontal and vertical scale of the magnification by pressing ZOOM ▶ 4 *[SetFactors...]* (see figure I.43) and editing XFact and YFact, the horizontal and vertical magnification factors (see Figure I.44).

The default zoom factor is 4 in both directions. It is not necessary for XFact and YFact to be equal. Sometimes, you may prefer to zoom in one direction only, so the other factor should be set to 1. As usual, press 2nd QUIT to leave the ZOOM menu.

Figure I.43: ZOOM MEMORY menu

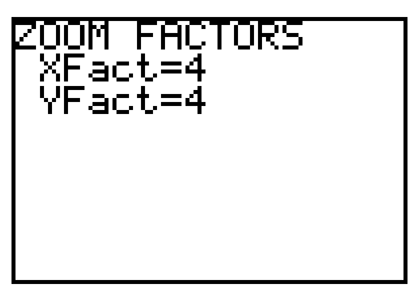


Figure I.44: ZOOM MEMORY SetFactors...

Technology Tip: The TI-82 remembers the window it displayed before a zoom. So, if you should zoom in too much and lose the curve, press ZOOM ▶ 1 *[ZPrevious]* to go back to the window before. If you want to execute a series of zooms but then return to a particular window, press ZOOM ▶ 2 *[ZoomSto]* to store the current window's dimensions. Later, press ZOOM ▶ 3 *[ZoomRcl]* to recall the stored window.

I.2.7 Value: Graph $y = -x^3 + 4x$ in the standard viewing window (Figure I.12). The TI-82 can calculate the value of this function for any given x (between the Xmin and Xmax values).

Press 2nd CALC to display the CALCULATE menu (see Figure I.45), then press 1 *[value]*. The graph of the function is displayed and you are prompted to enter a value for x. Press 1 ENTER. The x-value you entered and its corresponding y-value are shown at the bottom of the screen and the cursor is located at the point $(1, 3)$ on the graph (see Figure I.46).

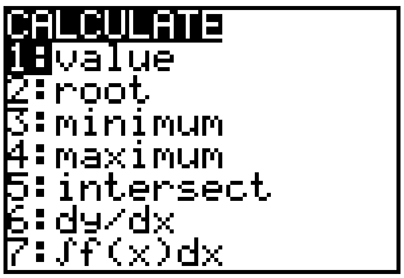


Figure I.45: CALCULATE menu

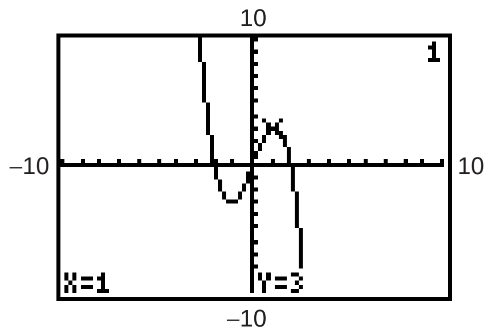


Figure I.46: Finding a value

Note that if you have more than one graph on the screen, the upper right corner of the TI-82 screen will display the numeral corresponding to the equation of the function in the Y= list whose value is being calculated. Press the up ▲ or down ▼ arrow key to move the cursor vertically between functions at the entered x-value.

I.2.8 Relative Minimums and Maximums: Graph $y = -x^3 + 4x$ once again in the standard viewing window (Figure I.12). This function appears to have a relative minimum near $x = -1$ and a relative maximum near $x = 1$. You may zoom and trace to approximate these extreme values.

First trace along the curve near the relative minimum. Notice by how much the x-values and y-values change as you move from point to point. Trace along the curve until the y-coordinate is as *small* as you can get it, so that you are as close as possible to the relative minimum, and zoom in (press ZOOM 2 ENTER or use a zoom box). Now trace again along the curve and, as you move from point to point, see that the coordinates change by smaller amounts than before. Keep zooming and tracing until you find the coordinates of the relative minimum point as accurately as you need them, approximately $(-1.15, -3.08)$.

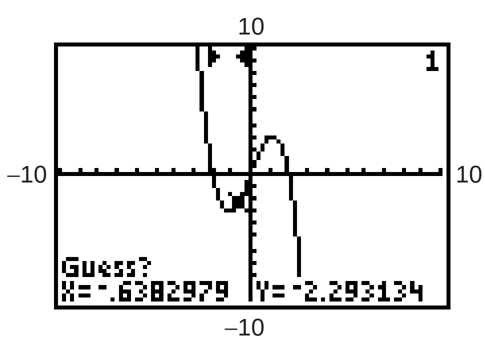


Figure I.47: Finding a minimum

Follow a similar procedure to find the relative maximum. Trace along the curve until the *y*-coordinate is as *great* as you can get it, so that you are as close as possible to the relative maximum, and zoom in. The relative maximum point on the graph of $y = -x^3 + 4x$ is approximately (1.15, 3.08).

The TI-82 can automatically find the relative minimum and relative maximum points. Press 2nd CALC to display the CALCULATE menu (Figure I.45). Choose 3 *[minimum]* to calculate the minimum value of the function and 4 *[maximum]* for the maximum. You will be prompted to trace the cursor along the graph first to a point *left* of the minimum/maximum (press ENTER to set this *lower bound*). Then move to a point *right* of the minimum/maximum and set an *upper bound* and press ENTER. Note the two arrows at the top of the display marking the lower and upper bounds (as in Figure I.47).

Next move the cursor along the graph between the two bounds and as close to the minimum/maximum as you can; this serves as a *guess* for the TI-82 to start its search. Good choices for the lower bound, upper bound, and guess can help the calculator work more efficiently and quickly. Press ENTER and the coordinates of the relative minimum/maximum point will be displayed (see Figure I.48).

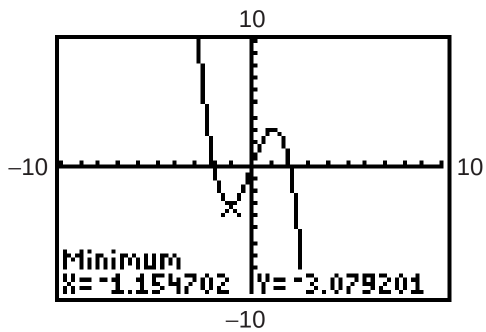


Figure I.48: Relative minimum on $y = -x^3 + 4x$

Note that if you have more than one graph on the screen, the upper right corner of the TI-82 screen will display the numeral corresponding to the equation of the function in the Y= list whose minimum/maximum is being calculated.

I.2.9 Inverse Functions: The TI-82 draws the inverse function of a one-to-one function. Graph $y = x^3 + 1$ as Y_1 in the standard viewing window (see Figure I.49). Next, press 2nd DRAW to display the DRAW menu. Use ▼ to move down and then choose 8 to draw the inverse function (see Figure I.50). Press 2nd Y-VARS 1 1 ENTER (see Figure I.51). These keystrokes instruct the TI-82 to draw the inverse function of Y_1. The original function and its inverse function will be displayed (see Figure I.52). Note that the calculator must be in function mode in order to use DrawInv.

To clear the graph of the inverse function, press 2nd DRAW 1 *[ClrDraw]*.

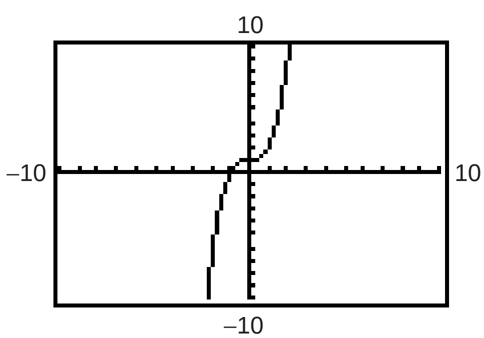


Figure I.49: Graph of $y = x^3 + 1$

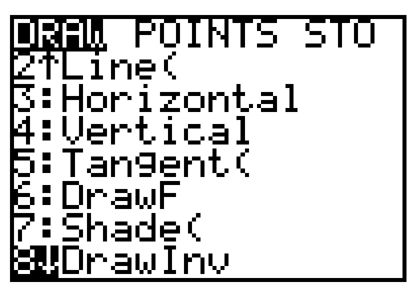
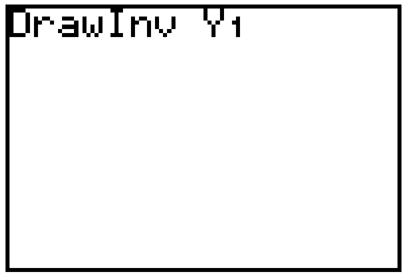


Figure I.50: DRAW menu



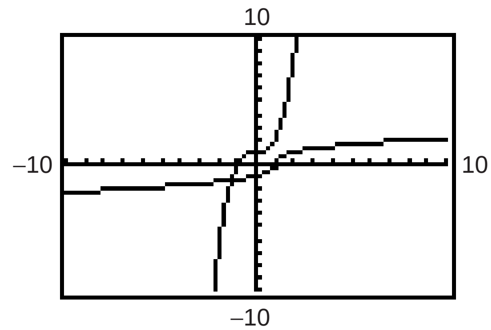
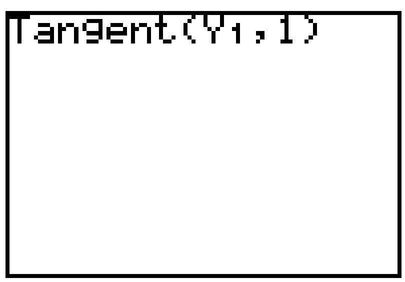


Figure I.51: DrawInv   Figure I.52: Graph of $y = x^3 + 1$ and its inverse function

I.2.10 Tangent Lines: Once again, graph $y = x^3 + 1$ in the standard viewing window (see Figure I.49). The TI-82 can draw the tangent line to a graph of a function at a specified point.

While on the home screen, press 2nd DRAW 5 *[Tangent(]* 2nd Y-VARS 1 1 , 1) ENTER (see Figure I.53). These keystrokes instruct the TI-82 to draw the tangent line to the graph of Y_1 at $x = 1$. The graph of the original function and the tangent line to the graph at $x = 1$ will be displayed (see Figure I.54).

To clear the tangent line, press 2nd DRAW 1.



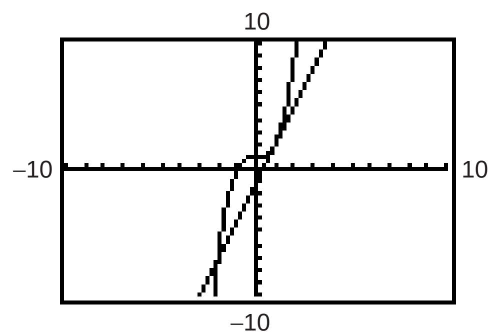
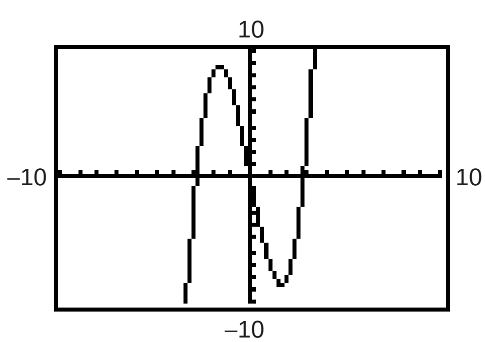


Figure I.53: Tangent   Figure I.54: Graph of $y = x^3 + 1$ and tangent line at $x = 1$

I.3 Solving Equations and Inequalities

I.3.1 Intercepts and Intersections: Tracing and zooming are also used to locate an x-intercept of a graph, where a curve crosses the x-axis. For example, the graph of $y = x^3 - 8x$ crosses the x-axis three times (see Figure I.55). After tracing over to the x-intercept point that is farthest to the left, zoom in (Figure I.56). Continue this process until you have located all three intercepts with as much accuracy as you need. The three x-intercepts of $y = x^3 - 8x$ are approximately -2.828, 0, and 2.828.



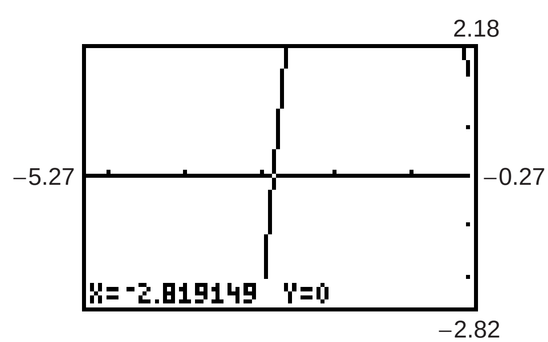


Figure I.55: Graph of $y = x^3 - 8x$ Figure I.56: Near an x-intercept of $y = x^3 - 8x$

Technology Tip: As you zoom in, you may also wish to change the spacing between tick marks on the x-axis so that the viewing window shows scale marks near the intercept point. Then the accuracy of your approximation will be such that the error is less than the distance between two tick marks. Change the x-scale on the TI-82 from the WINDOW menu. Move the cursor down to Xscl and enter an appropriate value.

The x-intercept of a function's graph is a *root* of the equation $f(x) = 0$. So press 2nd CALC to display the CALCULATE menu (Figure I.45) and choose 2 *[root]* to find a root of this function. Set a lower bound, upper bound, and guess as described in Section I.2.8. The TI-82 shows the coordinates of the point and indicates that it is a root (Figure I.57).

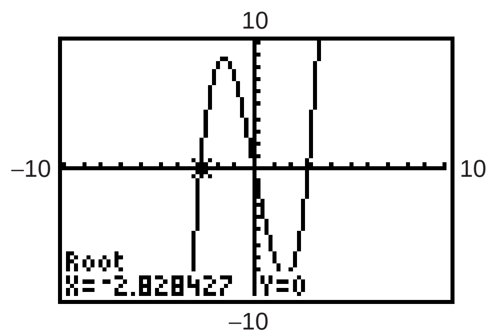


Figure I.57: A root of $y = x^3 - 8x$

TRACE and ZOOM are especially important for locating the intersection points of two graphs, say the graphs of $y = -x^3 + 4x$ and $y = -.25x$. Trace along one of the graphs until you arrive close to an intersection point. Then press ▲ or ▼ to jump to the other graph. Notice that the x-coordinate does not change, but the y-coordinate is likely to be different (see Figures I.58 and I.59).

When the two y-coordinates are as close as they can get, you have come as close as you now can to the point of intersection. so zoom in around the intersection point, then trace again until the two y-coordinates are as close as possible. Continue this process until you have located the point of intersection with as much accuracy as necessary. The points of intersection are approximately $(-2.062, 0.515)$, $(0, 0)$, and $(2.062, 0.515)$.

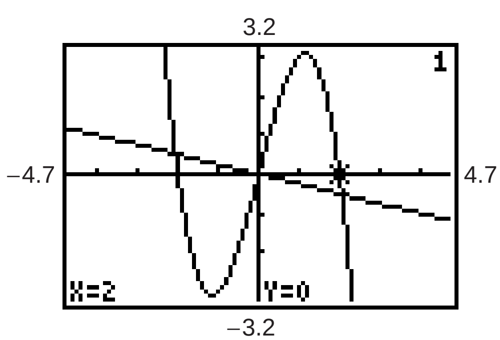


Figure I.58: Trace on $y = -x^3 + 4x$

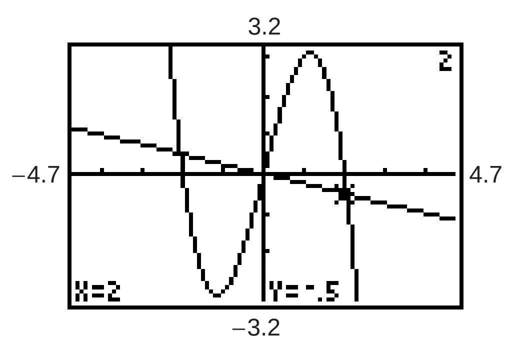


Figure I.59: Trace on $y = -.25x$

You can also find the point of intersection of two graphs by pressing 2nd CALC 5 *[intersect]*. Trace with the cursor first along one graph near the intersection and press ENTER; then trace with the cursor along the other graph and press ENTER. Marks + are placed on the graphs at these points. Finally, move the cursor near the point of intersection and press ENTER again. Coordinates of the intersection will be displayed at the bottom of the window. More will said about the intersect feature in Section I.3.3.

I.3.2 Solving Equations by Graphing: Suppose you need to solve the equation $24x^3 - 36x + 17 = 0$. First graph $y = 24x^3 - 36x + 17$ in a window large enough to exhibit *all* its x-intercepts, corresponding to all the equation's real roots. Then use zoom and trace, or the TI-82's roots finder, to locate each one. In fact, this equation has just one real solution, $x \approx 1.414$.

Remember that when an equation has more than one x-intercept, it may be necessary to change the viewing window a few times to locate all of them.

Technology Tip: To solve an equation like $24x^3 + 17 = 36x$, you may first rewrite it in general form, $24x^3 - 36x + 17 = 0$, and proceed as above. However, you may also graph the *two* functions $y = 24x^3 + 17$ and $y = 36x$, then zoom and trace to locate their point of intersection.

I.3.3 Solving Systems by Graphing: The solutions to a system of equations correspond to the points of intersection of their graphs (Figure I.60). For example, to solve the system $y = 2x + 5$ and $y = -2x + 1$, first graph them together. Then use zoom and trace, or use the intersect option in the CALCULATE menu, to locate their point of intersection, which is $(-1, 3)$ (see Figure I.61).

The solution of the system of two equations $y = 2x + 5$ and $y = -2x + 1$ correspond to the solutions of the single equation $2x + 5 = -2x + 1$, which simplifies to $4x + 4 = 0$. So you may also graph $y = 4x + 4$ and find its x-intercept to solve the system.

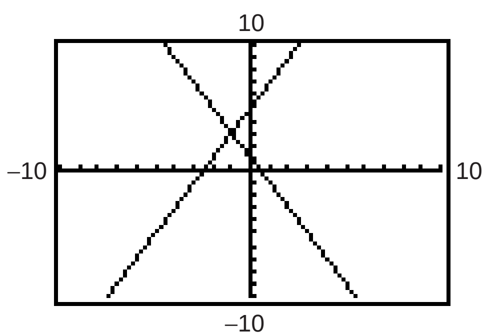


Figure I.60: Solving a system of equations

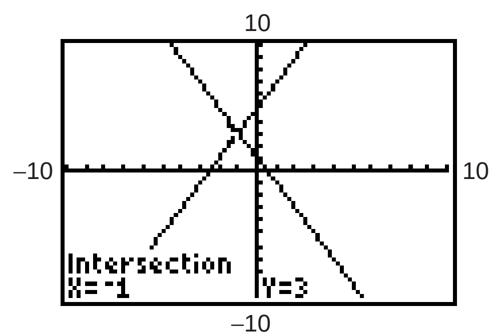


Figure I.61: The point of intersection is $(-1, 3)$.

I.3.4 Solving Inequalities by Graphing: Consider the inequality $1 - \frac{3x}{2} \geq x - 4$. To solve it with your TI-82, graph the two functions $y = 1 - \frac{3x}{2}$ and $y = x - 4$ (Figure I.62). First locate their point of intersection, at $x = 2$. The inequality is true when the graph of $y = 1 - \frac{3x}{2}$ lies *above* the graph of $y = x - 4$, and that occurs for $x < 2$. So the solution is $x \leq 2$, or $(-\infty, 2]$.

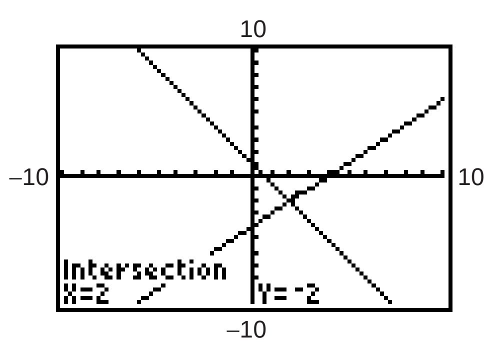


Figure I.62: Solving $1 - \frac{3x}{2} \geq x - 4$

The TI-82 is capable of shading the region above or below a graph or between two graphs. For example, to graph $y \geq x^2 - 1$, first graph the function $y = x^2 - 1$ as Y_1. Then press 2nd DRAW 7 *[Shade(]* 2nd Y-VARS 1 1 , 10 , 2) ENTER (see Figure I.63). These keystrokes instruct the TI-82 to shade the region *above* $y = x^2 - 1$ and *below* $y = 10$ (chosen because this is the greatest y-value in the graph window) with shading resolution value of 2. The result is shown in Figure I.64.

To clear the shading, press 2nd DRAW 1.

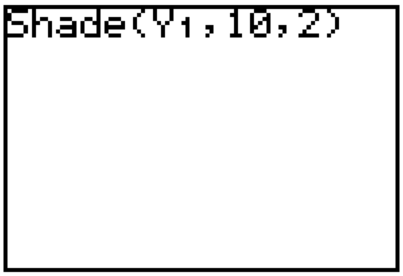


Figure I.63: DRAW Shade

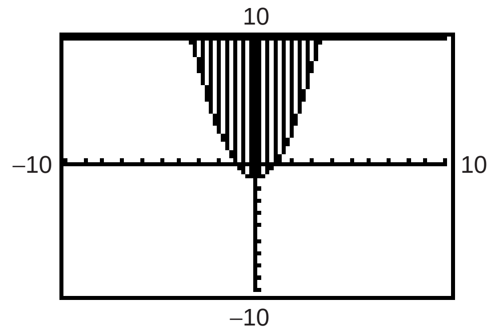


Figure I.64: Graph of $y \geq x^2 - 1$

Now use shading to solve the previous inequality, $1 - \frac{3x}{2} \geq x - 4$. The function whose graph forms the lower boundary is named *first* in the SHADE command (see Figure I.65). To enter this in your TI-82, press these keys: 2nd DRAW 7 X,T,θ − 4 , 1 − 3 X,T,θ ÷ 2 , 2) ENTER (Figure I.66). The shading extends left from $x = 2$, so the solution to $1 - \frac{3x}{2} \geq x - 4$ is $x \leq 2$, or $(-\infty, 2]$ (see Figure I.66).

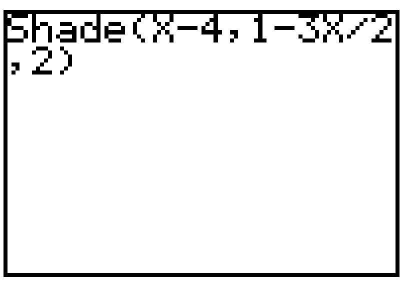


Figure I.65: DRAW Shade command

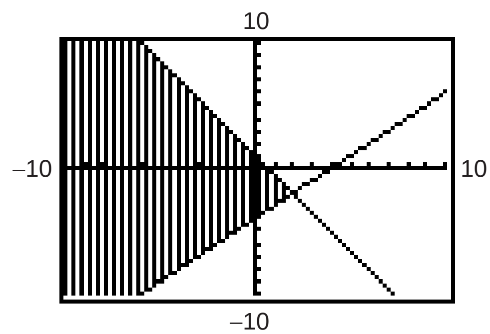


Figure I.66: Graph of $1 - \frac{3x}{2} \geq x - 4$

More information about the DRAW menu is in the TI-82 manual.

I.4 Trigonometry

I.4.1 Degrees and Radians: The trigonometric functions can be applied to angles measured either in radians or degrees, but you should take care that the TI-82 is configured for whichever measure you need. Press MODE to see the current settings. Press ▼ twice and move down to the third line of the mode menu where angle measure is selected. Then press ◀ or ▶ to move between the displayed options. When the blinking cursor is on the measure you want, press ENTER to select it. Then press CLEAR or 2nd QUIT to leave the mode menu.

It's a good idea to check the angle measure setting before executing a calculation that depends on a particular measure. You may change a mode setting at any time and not interfere with pending calculations. Try the following keystrokes to see this in action.

Expression	*Keystrokes*	*Display*
sin 45°	MODE ▼ ▼ ▶ ENTER CLEAR SIN 45 ENTER	.7071067812
sin $\pi°$	SIN 2nd π ENTER	.0548036651
sin π	MODE ▼ ▼ ENTER CLEAR SIN 2ND π ENTER	0
sin 45	SIN 45 ENTER	.8509035245
sin $\frac{\pi}{6}$	SIN (2nd π ÷ 6) ENTER	.5

The first line of keystrokes sets the TI-82 in degree mode and calculates the sine of 45 *degrees*. While the calculator is still in degree mode, the second line of keystrokes calculates the sine of π *degrees*, 3.1415°. The third line changes the radian mode just before calculating the sine of π *radians*. The fourth line calculates the sine of 45 *radians*. Finally, the fifth line calculates the sine of $\frac{\pi}{6}$ *radians* (the calculator remains in radian mode).

The TI-82 makes it possible to mix degrees and radians in a calculation. Execute these keystrokes to calculate $\tan 45° + \sin \frac{\pi}{6}$ as shown in Figure I.67. TAN 45 2nd ANGLE 1 *[°]* + SIN (2nd π ÷ 6) 2nd ANGLE 3 *[r]* ENTER. Do you get 1.5 whether your calculator is set *either* in degree mode *or* in radian mode?

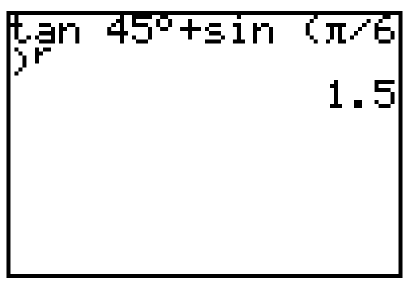


Figure I.67: Angle measure

I.4.2 Graphs of Trigonometric Functions: When you graph a trigonometric function, you need to pay careful attention to the viewing window and to your angle measure configuration. For example, graph $y = \frac{\sin 30x}{30}$ in the standard viewing window in radian mode. Trace along the curve to see where it is. Zoom in to a better window, or use the period and amplitude to establish better WINDOW values.

Technology Tip: Because $\pi \approx 3.1$, when in radian mode, set Xmin = 0 and Xmax = 6.3 to cover the interval from 0 to 2π.

Next graph $y = \tan x$ in the standard window first, then press ZOOM 7 *[ZTrig]* to change to a special window for trigonometric functions in which the Xscl is $\frac{\pi}{2} \approx 1.5708$ or 90° and the vertical range is from -4 to 4. The TI-82 plots consecutive points and then connects them with a segment, so the graph is not exactly what you should expect. You may wish to change from Connected line to Dot graph (see Section I.2.3) when you plot the tangent function.

I.5 Scatter Plots

I.5.1 Entering Data: This table shows total prize money (in millions of dollars) awarded at the Indianapolis 500 race from 1995 to 2003. (*Source:* Indy Racing League)

Year	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*	*2003*
Prize (in millions)	$8.06	$8.11	$8.61	$8.72	$9.05	$9.48	$9.61	$10.03	$10.15

We'll now use the TI-82 to construct a scatter plot that represents these points and to find a linear model that approximates the given data.

The TI-82 holds data in up to six *lists*. Before entering this new data, press STAT 4 *[ClrList]* 2nd L1, 2nd L2 , 2nd L3 , 2nd L4 , 2nd L5 , 2nd L6 ENTER to clear all data lists. This can also be done from within the list editor by highlighting each list title (L1, etc) and pressing CLEAR ENTER.

Now press STAT 1 *[Edit]* to reach the list editor. Instead of entering the full year, let $x = 5$ represent 1995, $x = 6$ represent 1996, and so on. Here are the keystrokes for the first three years: 5 ENTER 6 ENTER 7 ENTER and so on, then press ▶ to move to the first element of the next list and press 8.06 ENTER 8.11 ENTER 8.61 ENTER and so on (see Figure I.68). Press 2nd QUIT when you have finished.

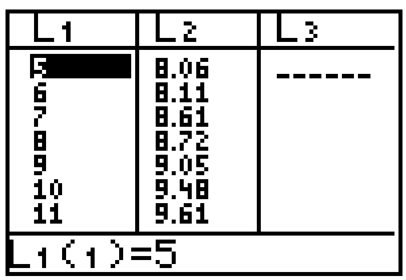
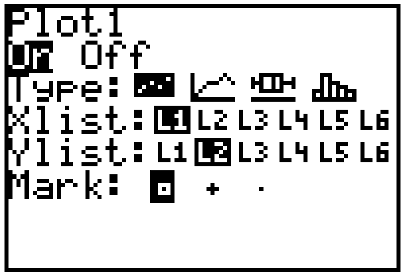


Figure I.68: Entering data points

You may edit statistical data in the same way you edit expressions in the home screen. Move the cursor to any value you wish to change, then type the correction. To insert or delete data, move the cursor over the data point you wish to add or delete. Press 2nd INS and a new data point is created; press DEL and the data point is deleted.

I.5.2 Plotting Data: Once all the data points have been entered, press 2nd STAT PLOT 1 to display the Plot1 screen. Press ENTER to turn Plot1 on, select the other options shown in figure I.69, and press GRAPH. (Make sure that you have cleared or turned off any functions in the Y= screen, or those functions will be graphed simultaneously.) Figure I.70 shows this plot in a window from 0 to 15 in both directions. You may now press TRACE to move from data point to data point.



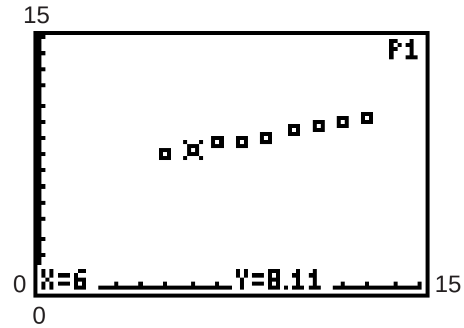


Figure I.69: Plot1 menu

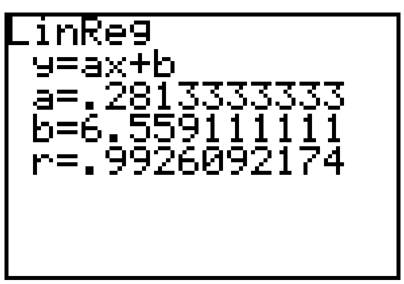
Figure I.70: Scatter plot

To draw the scatter plot in a window adjusted automatically to include all the data you entered, press ZOOM 9 *[ZoomStat]*.

When you no longer want to see the scatter plot, press 2nd STAT PLOT 1, move the cursor to OFF, and press ENTER. The TI-82 still retains all the data you entered.

I.5.3 Regression Line: The TI-82 calculates the slope and *y*-intercept for the line that best fits all the data. The TI-82 can calculate regression lines in two equivalent forms. After the data points have been entered, press STAT ▶ 5 *[LinReg(ax + b)]* ENTER to calculate a linear regression model with the slope named a and the *y*-intercept named b (Figure I.71). By pressing STAT ▶ 9 *[LinReg(a + bx)]* ENTER, the TI-82 produces a linear regression model with the roles of a and b reversed. The number r (between -1 and 1) is called the *correlation coefficient* and measures how well the linear regression equation fits the data. The closer $|r|$ is to 1, the better the fit; the closer $|r|$ is to 0, the worse the fit.

Turn Plot1 on again, if it is not currently displayed. Graph the regression line $y = ax + b$ by pressing Y=, inactivating any existing functions, moving to a free line or clearing one, then pressing VARS 5 *[Statistics...]* ▶ ▶ 7 *[RegEQ]* GRAPH. See how well this line fits with your data points (Figure I.72).



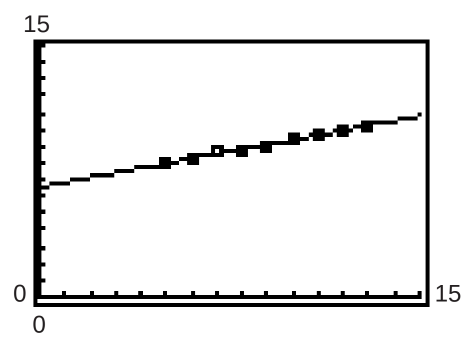


Figure I.71: Linear regression model

Figure I.72: Linear regression line

I.5.4 Other Regression Models: After data points have been entered, you can choose from seven different regression models. They are all located in the CALC sub-menu of the STAT menu.

I.6 Matrices

I.6.1 Making a Matrix: The TI-82 can display and use five different matrices (A through E). Here's how to store this 3×4 matrix $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & 4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$ in your calculator.

Press MATRX to see the matrix menu (Figure I.73); then press ▶ ▶ or just ◀ to switch to the matrix EDIT menu. Whenever you enter the matrix EDIT menu, the cursor starts at the top matrix. Move to another matrix by repeatedly pressing ▼. For now, press ENTER to edit matrix [A].

The display will show the dimension of matrix [A] if the matrix exists; otherwise, it will display 1×1. Change the dimensions of matrix [A] to 3×4 by pressing 3 ENTER 4 ENTER. Simply press ENTER or an arrow key to accept an existing dimension. The matrix shown in the window changes in size to reflect a changed dimension.

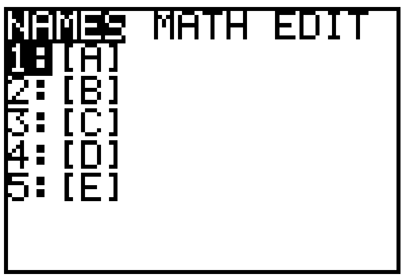


Figure I.73: MATRX menu

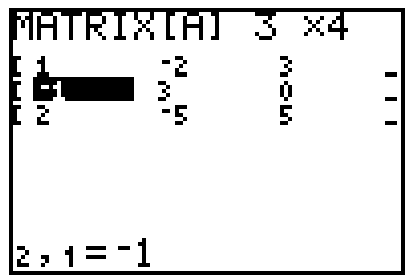


Figure I.74: Editing a matrix

Use the arrow keys or press ENTER repeatedly to move the cursor to a matrix element you want to change. If you press ENTER, you will move right across a row and then back to the first column of the next row. At the right edge of the screen in Figure I.74, there are dashes to indicate more columns than are shown. Go to them by pressing ▶ as many times as necessary. The ordered pair at the bottom left of the screen show the cursor's current location withing the matrix. The element in the second row and first column in Figure I.74 is highlighted, so the ordered pair at the bottom of the window is 2 , 1, and the screen shows that element's current value. Continue to enter all the elements of matrix [A]; press ENTER after inputting each value.

When you are finished, leave the matrix editing screen by pressing 2nd QUIT to return to the home screen.

I.6.2 Matrix Math: From the home screen you can perform many calculations with matrices. To see matrix [A], press MATRX 1 ENTER (Figure I.75).

Perform the scalar multiplication 2[A] by pressing 2 MATRX 1 ENTER. The resulting matrix is displayed on the screen. To replace matrix [B] by 2[A], press 2 MATRX 1 STO ▶ MATRX 2 ENTER (see Figure I.76), or if you do this immediately after calculating 2[A], press only STO ▶ MATRX 2 ENTER. Press MATRX ▶ ▶ 2 to verify that the dimensions and entries of matrix [B] have been changed automatically to reflect these new values.

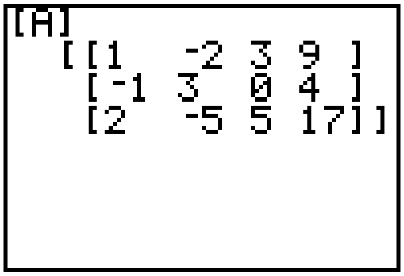


Figure I.75: Matrix [A]

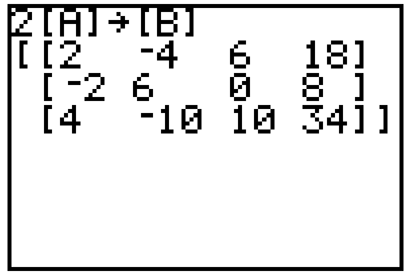


Figure I.76: Matrix [B]

Add the two matrices say [A] and [B], create [B] (with the same dimensions as [A]) and then press MATRX 1 + MATRX 2 ENTER. Subtraction is performed in a similar manner. Now set the dimensions of matrix [C] to 2×3 and enter the matrix: $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -5 & -1 \end{bmatrix}$ as [C]. For matrix multiplication of [C] by [A], press MATRX 3 × MATRX 1 ENTER. If you tried to multiply [A] by [C], your TI-82 would signal an error because the dimensions of the two matrices do not permit multiplication in this way.

I.6.3 Row Operations: Here are the keystrokes necessary to perform elementary row operations on a matrix. Your textbook provides more careful explanation of the elementary row operations and their uses.

To interchange the second and third rows of the matrix [A] that was defined in Figure I.75, press MATRX ▶ 8 *[rowSwap(]* MATRX 1 , 2 , 3) ENTER (see Figure I.77). The format of this command is rowSwap*(matrix, row1, row 2)*.

To add row 2 and row 3 and store the results in row 3, press MATRX ▶ 9 *[row + (]* MATRX 1 , 2 , 3) ENTER. The format of this command is row+*(matrix, row1, row2)*.

To multiply row 2 by −4 and *store* the results in row 2, thereby replacing row 2 with new values, press MATRX ▶ 0 *[*row(]* (-) 4 , MATRX 1 , 2) ENTER. The format of this command *row(*scalar, matrix, row*).

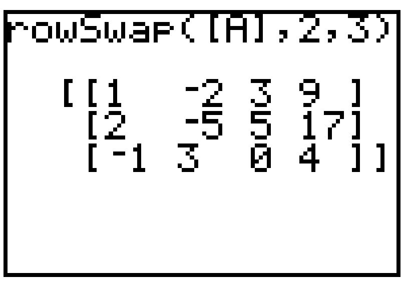


Figure I.77: Interchange rows 2 and 3

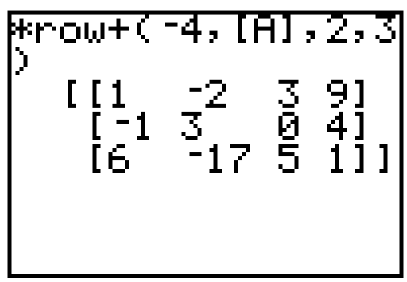


Figure I.78: Add −4 times row 2 to row 3

To multiply row 2 by −4 and *add* the results to row 3, thereby replacing row 3 with new values, press MATRX ▶ ALPHA A *[*row + (]* (-) 4 , MATRX 1 , 2 , 3) ENTER (see Figure I.78). The format of this command is *row+(*scalar, matrix, row1, row2*).

Technology Tip: Note that your TI-82 does *not* store a matrix obtained as the result of any row operations. So when you need to perform several row operations in succession, it is a good idea to store the result of each one in a temporary place. You may wish to use matrix [E] to hold such intermediate results.

For example, use elementary row operations to solve this system of linear equations: $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4. \\ 2x - 5y + 5z = 17 \end{cases}$

First enter this *augmented matrix* as [A] in your TI-82 : $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$. Next store this matrix in [E] (press MATRX 1 STO ▶ MATRX 5 ENTER) so you may keep the original in case you need to recall it.

Here are the row operations and their associated keystrokes. At each step, the result is stored in [E] and replaces the previous matrix [E]. The matrix in row-echelon form is shown in Figure I.79.

Row Operation	*Keystrokes*
Add row 1 to row 2.	MATRX ▶ 9 MATRX 5 , 1 , 2) STO ▶ MATRX 5 ENTER
Add -2 times row 1 to row 3.	MATRX ▶ ALPHA A (-) 2 , MATRX 5 , 1 , 3) STO ▶ MATRX 5 ENTER
Add row 2 to row 3.	MATRX ▶ 9 MATRX 5 , 2 , 3) STO ▶ MATRX 5 ENTER
Multiply row 3 by $\frac{1}{2}$.	MATRX ▶ 0 1 ÷ 2 , MATRX 5 , 3) STO ▶ MATRX 5 ENTER

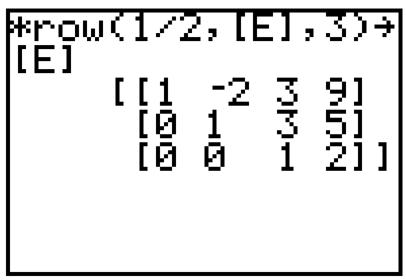


Figure I.79: Row-echelon form of matrix after row operations

So, $z = 2$, $y = -1$, and $x = 1$.

I.6.4 Determinants and Inverses: Enter this 3×3 square matrix as [A]: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. To calculate its determinant $\begin{vmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{vmatrix}$, go to the home screen and press MATRX ▶ 1 *[det]* MATRX 1 ENTER.

You should find that the determinant is 2, as shown in Figure I.80.

Because the determinant of the matrix is not zero, it has an inverse, $[A]^{-1}$. Press MATRX 1 x^{-1} ENTER to calculate the inverse of matrix [A], also shown in Figure I.80.

Now let's solve a system of linear equations by matrix inversion. Once more, consider $\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$.

The coefficient matrix for this system is the matrix $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$, that was entered as matrix [A] in the previous example.

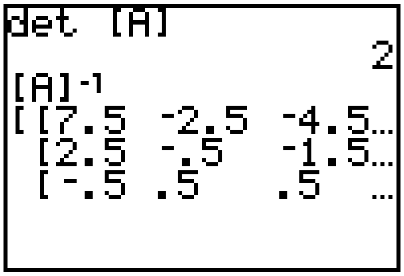


Figure I.80: $|[A]|$ and $[A]^{-1}$

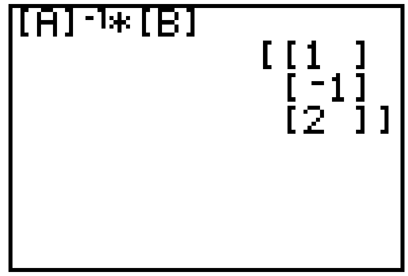


Figure I.81: Solution matrix

Now enter the 3×1 matrix $\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$ as [B]. Then press MATRX 1 x^{-1} $\times$ MATRX 2 ENTER to calculate the solution matrix (Figure I.81). The solution is still $x = 1$, $y = -1$, and $z = 2$.

I.7 Sequences

I.7.1 Iteration with ANS Key: The ANS feature permits you to perform *iteration*, the process of evaluating a function repeatedly. As an example, calculate $\frac{n-1}{3}$ for $n = 27$. Then calculate $\frac{n-1}{3}$ for $n =$ the answer to the previous calculation. Continue to use each answer as n in the *next* calculation. Here are keystrokes to accomplish this iteration on TI-82 calculator (see the results in Figure I.82). Notice that when you use ANS in place of n in a formula, it is sufficient to press ENTER to continue an iteration.

Iteration	*Keystrokes*	*Display*
1	27 ENTER	27
2	(2nd ANS $-$ 1) $\div$ 3 ENTER	8.666666667
3	ENTER	2.555555556
4	ENTER	.5185185185
5	ENTER	-.1604938272

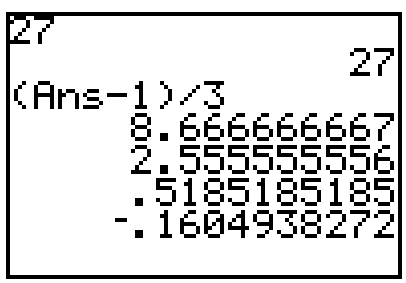


Figure I.82: Iteration

Press ENTER several more times and see what happens with this iteration. You may wish to try it again with a different starting value.

I.7.2 Terms of Sequences: Another way to display the terms of a sequence is to enter the sequence and the number of terms you want listed. For example, to find the first five terms of the sequence $u_n = -n + 4$, press 2nd LIST 5 *[seq(]* (-) ALPHA N + 4, ALPHA N , 1 , 5 , 1) ENTER (see Figure I.83). The format of this command is seq(*expression*, *variable*, *begin*, *end*, *increment*).

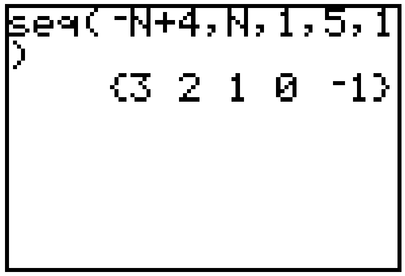


Figure I.83: Terms of sequence $u_n = -n + 4$

I.7.3 Arithmetic and Geometric Sequences: Use iteration with the ANS variable to determine the nth term of a sequence. For example, find the 18th term of an arithmetic sequence whose first term is 7 and whose common difference is 4. Enter the first term 7, then start the progression with the recursion formula, 2nd ANS4 ENTER. This yields the 2nd term, so press ENTER sixteen more times to find the 18th term, 75. For a *geometric* sequence whose common ration is 4, start the progression with 2nd ANS × 4 ENTER.

You can also define the sequence recursively with the TI-82 by selecting Seq in the MODE menu (see Figure I.1). Once again, let's find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Press Mode ▼ ▼ ▼ ▶ ▶ ▶ ENTER 2nd QUIT. Then press Y= to edit either the TI-82's two sequences, U_n and V_n. Make $u_n = u_{n-1} + 4$ by pressing 2nd $U_{n\text{-}1}$ + 4. Now make $u_1 = 7$ by pressing WINDOW and setting U*n*Start = 7 and *n*Start = 1 (because the first term is u_1 where $n = 1$). Press 2nd QUIT to leave this menu and return to the home screen. To find the 18th term of this sequence, calculate u_{18} by pressing 2nd Y-VARS 4 1 (18) ENTER (see Figure I.84).

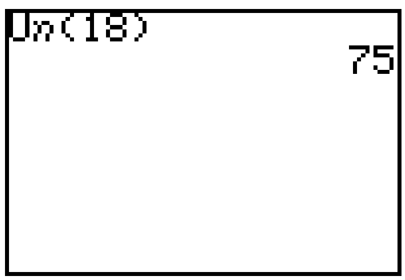


Figure I.84: Sequence mode

Of course, you could use the *explicit* formula for the nth term of an arithmetic sequence, $t_n = a + (n - 1)d$. First enter values for the variables a, d, and n, then evaluate the formula by pressing ALPHA A + (ALPHA N − 1) ALPHA D ENTER. For a geometric sequence whose nth term is given by $t_n = a \cdot r^{n-1}$, enter values for the variables a, r, and n, then evaluate the formula by pressing ALPHA A ALPHA R ^ (ALPHA N − 1) ENTER.

To use the explicit formula in Seq MODE, make $u_n = 7 + (n - 1) \cdot 4$ by pressing Y= and then 7 + (2nd *n* − 1) × 4 ENTER 2nd QUIT. Once more, calculate u_{18} by pressing 2nd Y-VARS 4 1 (18) ENTER.

There are more instructions for using sequence mode in the TI-82 manual.

I.7.4 Sums of Sequences: You can find the sum of a sequence by combining the sum feature on the MATH sub-menu of the LIST menu with the seq(feature on the OPS sub-menu of the LIST menu. The format of the sum command is sum *list*, *start*, *end*, where the optional arguments *start* and *end* determine which elements of list are summed. The format of the seq(command is seq(*expression*, *variable*, *begin*, *end*, *increment*), where the argument *increment* indicates the difference between successive points at which *expression* is evaluated.

For example, suppose you want to find the sum $\sum_{n=1}^{12} 4(0.3)^n$. Press 2nd LIST ▶ 5 *[sum]* 2nd LIST 5 *[seq(]* 4 (.3) ^ ALPHA N , ALPHA N , 1 , 12 , 1) ENTER (Figure I.85). Note that the sum command does not need a starting or ending point, because every term in the sequence is being summed. Also, any letter can be used for the variable in the sum, i.e., the N could just have easily been an A or a K.

Now calculate the sum starting at $n = 0$ by using 2nd ENTRY to edit the starting value. You should obtain a sum of approximately 5.714284803.

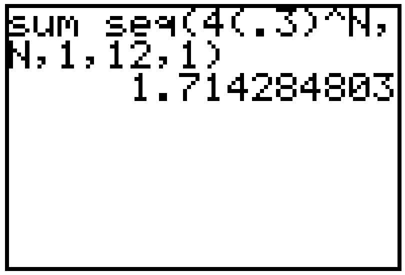


Figure I.85: $\sum_{n=1}^{12} 4(0.3)^n$

I.8 Parametric and Polar Graphs

I.8.1 Graphing Parametric Equations: The TI-82 plots up to six pairs of parametric equations as easily as it plots functions. In the MODE menu (Figure I.1), go to the fourth line from the top, and change the setting to Par. Be sure, if the independent parameter is an angle measure, that the angle measure in the MODE menu is set to whichever you need, Radian or Degree.

For example, here are the keystrokes needed to graph the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$. First check that angles are currently being measured in radians and change to parametric mode. Then press Y = (COS X,T,θ) ^ 3 ENTER (SIN X,T,θ) ^ 3 ENTER (Figure I.86). Note that when you press the variable key X,T,θ you get a T because the calculator is in parametric mode.

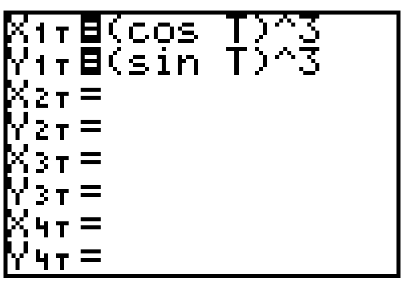


Figure I.86: Parametric Y= menu

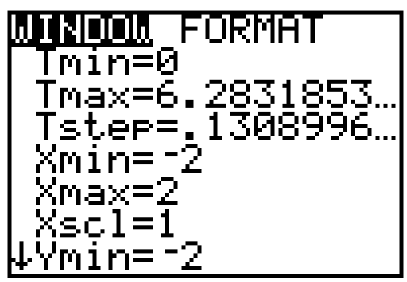


Figure I.87: Parametric WINDOW menu

Press WINDOW to set the graphing window and to initialize the values of T. In the standard window, the values of T go from 0 to 2π in steps of $\frac{\pi}{24} \approx 0.1309$, with the view from -10 to 10 in both directions. In order to provide a better viewing window, press ENTER three times to move the cursor down, then set the window to extend from -2 to 2 in both directions (Figure I.87). Press GRAPH to see the parametric graph (Figure I.88).

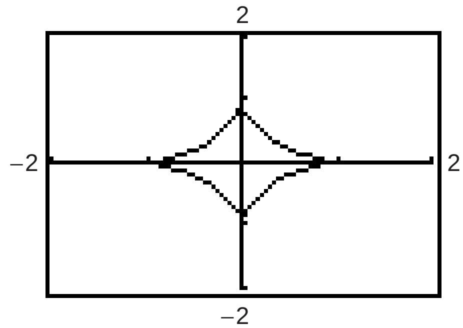


Figure I.88: Parametric graph of $x = \cos^3 t$ and $y = \sin^3 t$

You may ZOOM and TRACE along parametric graphs just as you did with function graphs. However, unlike with function graphs, the cursor will not move to values outside of the T range, so the left arrow ◀ will not work when T $= 0$, and the right arrow ▶ will not work when T $= 2\pi$. As you trace along this graph, notice that the cursor moves in the *counterclockwise* direction as T increases.

I.8.2 Rectangular-Polar Coordinate Conversion: The 2nd ANGLE menu provides functions for converting between rectangular and polar coordinate systems. These functions use the current angle measure setting, so it is a good idea to check the default angle measure before any conversion. Of course, you may override the current angle measure setting, as explained in Section I.4.1. For the following examples, the TI-82 is set to radian measure.

Given the rectangular coordinates $(x, y) = (4, -3)$, convert *from* these rectangular coordinates *to* polar coordinates (r, θ) by pressing 2nd ANGLE 5 *[R►Pr(]* 4 , (-) 3) ENTER to display the value of r. Now press 2nd ANGLE 6 *[R► Pθ(]* 4 , (-) 3) ENTER to display the value of θ (see Figure I.89).

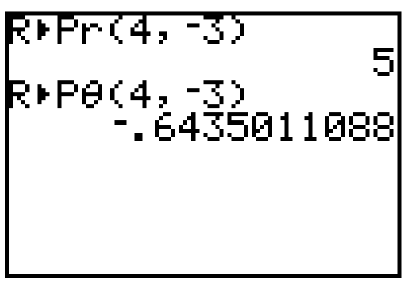


Figure I.89: Rectangular to polar coordinates

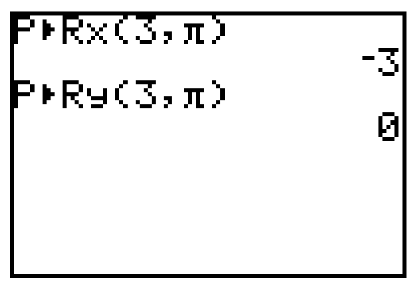


Figure I.90: Polar to rectangular coordinates

Suppose $(r, \theta) = (3, \pi)$. To convert *from* these polar coordinates *to* rectangular coordinates (x, y), press 2nd ANGLE 7 *[P► Rx(]* 3 , 2nd π) ENTER to display the x-coordinate. Now press 2nd ANGLE 8 *[P► Ry(]* 3 , 2nd π) ENTER to display the y-coordinate (see Figure I.90).

I.8.3 Graphing Polar Equations: The TI-82 graphs polar functions in the form $r = f(\theta)$. In the fourth line of the MODE menu, select Pol for polar graphs. You may now graph up to six different polar functions at a time. Be sure that the angle measure has been set to whichever you need, Radian or Degree. Here we will use radian measure.

For example, to graph $r = 4 \sin \theta$, press Y= for the polar graph editing screen. Then enter the expression $4 \sin \theta$ for r1 by pressing 4 SIN X,T,θ. Note that when you press the variable key X,T,θ, you get a θ because the calculator is in polar mode (see Figure I.91). Choose a good viewing window and an appropriate interval and increment for θ. In Figure I.92, the viewing window is roughly “square” and extends from -6 and 6 horizontally and from -4 to 4 vertically.

Figure I.91: Polar Y= menu

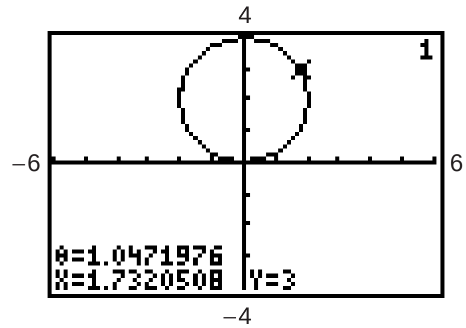


Figure I.92: Polar graph of $r = 4 \sin \theta$

Figure I.92 shows *rectangular* coordinates of the cursor's location on the graph. You may sometimes wish to trace along the curve and see *polar* coordinates of the cursor's location. The first line of the WINDOW FORMAT menu (Figure I.18) has options for displaying the cursor's position in rectangular (RectGC) or polar (PolarGC) form.

I.9 Probability and Statistics

I.9.1 Random Numbers: The command rand generates a number between 0 and 1. You will find this command in the PRB (probability) sub-menu of the MATH menu. Press MATH ◀ 1 *[rand]* ENTER to generate a random number. Press ENTER to generate another number; keep pressing ENTER to generate more of them.

If you need a random number between, say, 0 and 10, then press 10 MATH ◀ 1 ENTER. To get a random number between 5 and 15, press 5 + 10 MATH ◀ 1 ENTER.

I.9.2 Permutations and Combinations: To calculate the number of *permutations* of 12 objects taken 7 at a time, ${}_{12}P_7$, press 12 MATH ◀ 2 *[nPr]* 7 ENTER. So, ${}_{12}P_7 = 3{,}991{,}680$, as shown in Figure I.93.

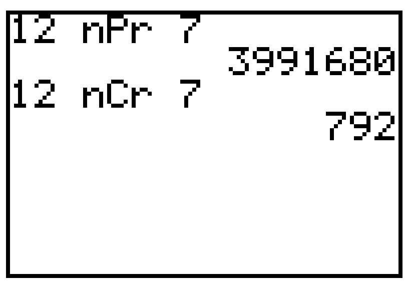


Figure I.93: ${}_{12}P_7$ and ${}_{12}C_7$

For the number of *combinations* of 12 objects taken 7 at a time, ${}_{12}C_7$, press 12 MATH ◀ 3 *[nCr]* 7 ENTER. So, ${}_{12}C_7 = 792$, as shown in Figure I.93.

I.9.3 Probability of Winning: A state lottery is configured so that each player chooses six different numbers from 1 to 40. If these six numbers match the six numbers drawn by the State Lottery Commission, the player wins the top prize. There are ${}_{40}C_6$ ways for the six numbers to be drawn. If you purchase a single lottery ticket, your probability of winning is 1 in ${}_{40}C_6$. Press 1 ÷ 40 MATH ◀ 3 6 ENTER to calculate your chances, but don't be disappointed.

I.9.4 Sum of Data: The following data are a student's scores on 8 quizzes and 2 tests throughout an algebra course.

25, 20, 18, 89, 17, 24, 23, 22, 25, 93

To find the total points earned by the student, first enter the data using the TI-82's list editor, as shown in Figure I.94. Then press 2nd LIST ▶ 5 2ND L_1 ENTER. From Figure I.95, the student earned 356 points throughout the algebra course.

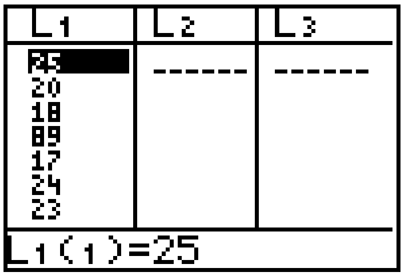


Figure I.94: List editor

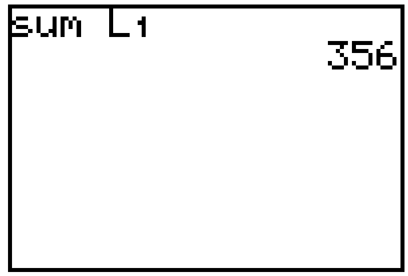


Figure I.95: Sum

I.9.5 Statistics: The following data are the high temperatures (in degrees Fahrenheit) recorded in Lincoln, Nebraska from October 1, 2003 to October 12, 2003 (*Source:* University of Nebraska-Lincoln).

65, 68, 74, 79, 83, 81, 80, 80, 79, 72, 67, 71

To find the mean and median of these temperatures, first enter the data using the TI-82's list editor, as shown in Figure I.96. Then to find the mean, press 2nd LIST ▶ 3 *[mean(]* 2nd L_1) ENTER and to find the median, press 2nd LIST ▶ 4 *[median(]* 2nd L_1) ENTER (see Figure I.97). So, the mean of the temperatures is approximately 75°F and the median is 76.5°F.

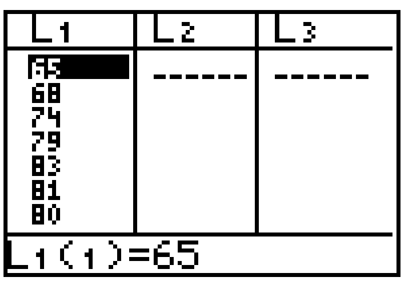


Figure I.96: List editor

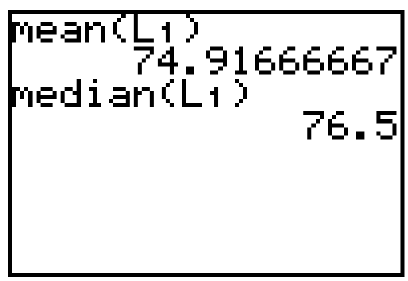


Figure I.97: Mean and median

You can also find the mean and median of the above data by finding the 1-Var Stats (one-variable statistics) of the data. Using the data you entered in Figure I.96, press STAT ▶ 1 2nd L1 ENTER. You should obtain a list of several different statistical values. The first line represents the mean of the data which is approximately 75°F (see Figure I.98). The second line is the sum of the data, the third line is the sum of the squares of the data, the fourth line is the sample standard deviation of the data, the fifth line is the population standard deviation of the data, the sixth line is the number of data values, the seventh line is the minimum value of the data, the eighth value is the first quartile of the data, and the ninth line is the median of the data which is 76.5°F (see Figure I.99). The tenth line is the third quartile of the data and the eleventh line is the maximum value of the data.

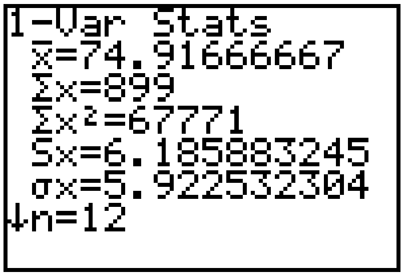


Figure I.98: 1-Var Stats

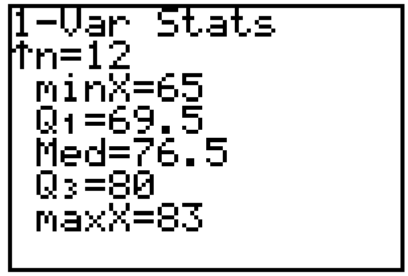


Figure I.99: 1-Var Stats

You can scroll through the list of statistical values by pressing ▲ or ▼.

I.10 Programming

I.10.1 Entering a Program: The TI-82 is a programmable calculator that can store sequences of commands for later replay. Press PRGM to access the programming menu. The TI-82 has space for many programs, each called by a title you give it. The title should be descriptive and can be eight characters, letters, or numerals long (but the first character must be a letter).

In the program, each line begins with a colon : supplied automatically by the calculator. Any command you could enter directly in the TI-82's home screen can be entered as a line in a program. There are also special programming commands.

You may interrupt programming input at any stage by pressing 2nd QUIT. To return later for more editing, press PRGM ▶, move the cursor down to the program's name, and press ENTER.

You may remove a program from memory by pressing 2nd MEM 2 *[Delete…]* 6 *[Prgm…]*. Then move the cursor to the program's name and press ENTER to delete the entire program.

I.10.2 Executing a Program: To execute a program you entered, press PRGM and then the number or letter it was named. If you have forgotten its name, use the arrow keys to move through the program listing to find its description. Then press ENTER to select the program and enter again to execute it.

If you need to interrupt a program during execution, press ON.

The instruction manual for your TI-82 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.

