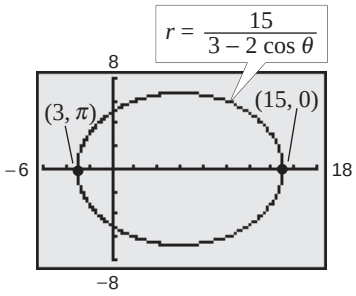
Chapter 10 Project ▶ Polar, Rectangular, and Parametric Forms



In this project, you will compare the polar, rectangular, and parametric forms of equations for conics.

(a) Consider the polar equation

$$r = \frac{15}{3 - 2\cos\theta}.$$

In Section 10.9, you learned that the graph of this polar equation is an ellipse and that one of the ellipse's foci is at the pole, as shown in the graph at the left. To find the rectangular equation of this ellipse, begin by rewriting the equation as

$$3r - 2r\cos\theta = 15.$$

Then use the substitutions $r = \sqrt{x^2 + y^2}$ and $r\cos\theta = x$ to find the rectangular equation. After you find the rectangular equation, write it in the standard form

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1.$$

(b) Use the standard form of the ellipse in part (a) to find the center, foci, and eccentricity of the ellipse. Compare your results with those obtained in Example 1 in Section 10.9. Use the *function* mode of a graphing utility to graph the ellipse.

(c) Use the standard form of the ellipse and the identity $\sin^2\theta + \cos^2\theta = 1$ to write parametric equations for the ellipse. Then use the *parametric* mode of a graphing utility to graph the ellipse.

Chapter Project Investigations

1. Sketch the graph of the polar equation

$$r = \frac{32}{3 - 5\sin\theta}.$$

Which type of conic is this?

2. Write the standard form of the rectangular equation of the conic in Question 1. Then sketch the graph using the *function* mode of a graphing utility. Does your graph agree with the graph obtained in Question 1? Which of the two graphs is easier to obtain? Explain your reasoning.

3. Write parametric equations for the conic given in Question 1. Then use the *parametric* mode of a graphing utility to graph the conic. Does your graph agree with the graph obtained in Question 1?

4. Consider the parametric equations

$$x = 4 + 5\cos t \quad \text{and} \quad y = 3\sin t.$$

With appropriate scaling of the coordinate system, could these equations represent the motion of a comet about the sun? Explain your reasoning.