

Chapter 8 Project ► Solving Systems of Equations

Matrices have always been a powerful mathematical tool—especially for dealing with problems that involve a lot of data. With technology available to perform the calculations, matrices have also become a practical mathematical tool.

Example ► Solving a Metallurgy Problem

	Alloy X	Alloy Y	Alloy Z
Carbon	1%	1%	4%
Chromium	0%	15%	3%
Iron	99%	84%	93%

Three iron alloys contain different percents of carbon, chromium, and iron as shown in the matrix at the left. Alloy X is a type of wrought iron, alloy Y is a type of stainless steel, and alloy Z is a type of cast iron. How much of each of the three alloys can you make with 15 tons of carbon, 39 tons of chromium, and 546 tons of iron?

Solution

Let x , y , and z represent the amounts of the three iron alloys. You can model the situation with the linear system

$$\begin{cases} 0.01x + 0.01y + 0.04z = 15 & \text{Carbon} \\ 0.15y + 0.03z = 39 & \text{Chromium} \\ 0.99x + 0.84y + 0.93z = 546 & \text{Iron} \end{cases}$$

The matrix equation $AX = B$ that represents this system is

$$\begin{bmatrix} 0.01 & 0.01 & 0.04 \\ 0 & 0.15 & 0.03 \\ 0.99 & 0.84 & 0.93 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 39 \\ 546 \end{bmatrix}.$$

With a graphing utility or computer, you can solve the equation as follows.

$$X = A^{-1}B = \begin{bmatrix} -25.4 & -5.4 & 1.267 \\ -6.6 & 6.733 & 0.067 \\ 33 & -0.333 & -0.333 \end{bmatrix} \begin{bmatrix} 15 \\ 39 \\ 546 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

So, you can make 100 tons of alloy X, 200 tons of alloy Y, and 300 tons of alloy Z.

Chapter Project Investigation

Three different gold alloys contain the percents of gold, copper, and silver shown in the matrix. You have 20,144 grams of gold, 766 grams of copper, and 1990 grams of silver. How much of each alloy can you make?

Percent by Weight

	Alloy X	Alloy Y	Alloy Z
Gold	94%	92%	80%
Copper	4%	2%	4%
Silver	2%	6%	16%