Graphical Approach to Compound Interest

A graphing utility can be used to investigate the rates of growth of different types of compound interest.

Example Comparing Balances

You deposit $1000 in a savings account. Which of the following will produce the largest balance?

a. 6% annual interest rate, compounded annually
b. 6% annual interest rate, compounded continuously
c. 6.25% annual interest rate, compounded quarterly

Solution

One way to compare all three options is to sketch their graphs in the same viewing window.

\[
\begin{align*}
\text{Option (a)} & \quad A = 1000(1 + 0.06)^t \\
\text{Option (b)} & \quad A = 1000e^{0.06t} \\
\text{Option (c)} & \quad A = 1000 \left(1 + \frac{0.0625}{4}\right)^{4t}
\end{align*}
\]

The graphs are shown at the left. From the graphs, you can conclude that option (c) is better than option (b), and option (b) is better than option (a). Note that for the first 50 years, there is little difference in the graphs. Between 50 and 100 years, however, the balances obtained begin to differ significantly. At the end of 100 years, the balances are (a) $339,302, (b) $403,429, and (c) $493,575.

Chapter Project Investigations

1. Which would produce a larger balance: an annual interest rate of 8.05% compounded monthly or an annual interest rate of 8% compounded continuously? Explain.

2. You deposit $1000 in each of two savings accounts. The interest for the accounts is paid according to the two options described in Question 1. How long would it take for the balance in one of the accounts to exceed the balance in the other account by $100? By $100,000?

3. No income tax is due on the interest earned in some types of investments. You deposit $25,000 in an account. Which of the following plans is better? Explain.

   (a) Tax-free The account pays 5% compounded annually. There is no income tax due on the earned interest.

   (b) Tax-deferred The account pays 7% compounded annually. At maturity, the earned interest is taxable at a rate of 40%.