

Chapter P Prerequisites

Section P.1 Real Numbers

Real numbers The set of numbers formed by joining the set of rational numbers and the set of irrational numbers.

Real number line A line used to graphically represent the set of real numbers.

Order If a and b are real numbers, a is less than b if $b - a$ is positive.

Inequality A statement that represents an order relationship.

Absolute value The magnitude or distance between the origin and the point representing a real number on the real number line.

Variables Letters that represent unknown quantities.

Evaluate To find the value of an algebraic expression by substituting numerical values for each of the variables in the algebraic expression.

Factors If a , b , and c are integers such that $ab = c$, then a and b are factors or divisors of c .

Section P.2 Exponents and Radicals

Exponential form A form of notation for writing repeated multiplication using exponents.

Scientific notation A real number written in the form $\pm c \times 10^n$, where $1 \leq c \leq 10$ and n is an integer.

Principal n th root Let a be a real number that has at least one n th root. The principal n th root of a is the n th root that has the same sign as a . It is denoted by the symbol: ${}^n\sqrt{a}$.

Rational exponent If a is a real number and n is a positive integer such that the principle n th root of a exists, then $a^{1/n} = {}^n\sqrt{a}$, where $1/n$ is the rational exponent of a .

Section P.3 Polynomials and Factoring

Polynomial Let $a_0, a_1, a_2, \dots, a_n$ be real numbers and let n be a nonnegative integer. A polynomial in x is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where $a_n \neq 0$.

Degree The sum of the exponents of the variables in a term. The degree of a polynomial is the highest degree of its

Section P.4 Rational Expressions

Domain The set of real numbers for which an algebraic expression is defined.

Rational expression The quotient of two polynomials.

Complex fractions A fractional expression with separate fractions in the numerator, denominator, or both.

Section P.5 Solving Equations

Equation A statement, usually involving x , that two algebraic expressions are equal.

Identity An equation that is true for every real number in the domain of the variable.

Conditional equation An equation that is true for just some (or even none) of the real numbers in the domain of the variable.

Equivalent equations Two equations that have exactly the same solution(s).

Extraneous solution A solution that does not satisfy the original equation.

Quadratic equation An equation in x that can be written in the general form $ax^2 + bx + c = 0$ where a, b , and c are real numbers with $a \neq 0$.

Section P.6 Solving Inequalities

Linear inequality An inequality in a single variable (usually x) that can be written in the form $ax + b < 0$ or $ax + b > 0$, where a and b are real numbers with $a \neq 0$.

Critical numbers The x -values that make the polynomial in a polynomial inequality equal to zero.

Section P.7 Errors and the Algebra of Calculus

Section P.8 Graphical Representation of Data

Rectangular coordinate system A plane, formed by using two real number lines intersecting at right angles, used to graphically represent ordered pairs of real numbers.

Ordered pair Two real numbers x and y , written (x, y) , which represent a point in the Cartesian plane.

Chapter 1 Functions and Their Graphs

Section 1.1 Graphs of Equations

Graph of an equation The set of all points that are solutions of the equation.

Intercepts The points at which a graph intersects the x - or y -axis.

Symmetry If a graph is folded along a dividing line and the portion of the graph on one side of the dividing line coincides with the portion of the graph on the other side of the dividing line, then the graph is said to have symmetry.

Circle The set of points that are equidistant from a fixed point, (h, k) , called the center.

Section 1.2 Linear Equations in Two Variables

Slope The number of units a nonvertical line rises (or falls) vertically for each unit of horizontal change from left to right.

Parallel Two distinct nonvertical lines are parallel if and only if their slopes are equal. That is, $m_1 = m_2$.

Perpendicular Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is, $m_1 = -1/m_2$.

Section 1.3 Functions

Function A function f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B .

Domain The set of inputs of the function f .

Range The set of all outputs for the given set of inputs of the function f .

Independent variable A variable in an equation that represents a function that can take on any value for which the function is defined.

Dependent variable A variable in an equation that represents a function whose value depends on the value of the independent variable

Section 1.4 Analyzing Graphs of Functions

Even function A function $y = f(x)$ is even if, for each x in the domain of f , $f(-x) = f(x)$.

Odd function A function $y = f(x)$ is odd if, for each x in the domain of f , $f(-x) = -f(x)$.

Section 1.5 Shifting, Reflecting, and Stretching Graphs

Section 1.6 Combinations of Functions

Arithmetic combination of functions Two or more functions combined by the operations of addition, subtraction, multiplication, or division.

Composition of functions The composition of the function f with the function g is $(f \circ g)(x) = f(g(x))$.

Section 1.7 Inverse Functions

Inverse Let f and g be two functions. If $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f , then g is the inverse of the function f . The function g is denoted by f^{-1} .

Horizontal Line Test A function f has an inverse if and only if no horizontal line intersects the graph of f at more than one point.

Section 1.8 Mathematical Modeling

Directly proportional If $y = kx$ for some nonzero constant k , then y is said to be directly proportional to x . This can also be described as y varies directly as x .

Least squares regression line The best fitting linear model with the least sum of square differences.

Chapter 2 Polynomial and Rational Functions

Section 2.1 Quadratic Functions

Constant function A polynomial function with degree 0. That is, $f(x) = a$, $a \neq 0$.

Linear function A polynomial function with degree 1. That is, $f(x) = ax + b$, $a \neq 0$.

Quadratic function Let a , b , and c be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ is called a quadratic function.

Axis A line about which a parabola is symmetric. Also called the axis of symmetry.

Vertex The point where the axis intersects the parabola.

Section 2.2 Polynomial Functions of Higher Degree

Continuous The graph of a polynomial function has no breaks, holes, or gaps.

Repeated zero If $(x - a)^k$, $k > 1$ is a factor of a polynomial, then $x = a$ is a repeated zero with multiplicity k .

Intermediate Value Theorem Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

Section 2.3 Polynomial and Synthetic Division

Division Algorithm If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = d(x)q(x) + r(x)$ where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$.

Improper rational expression A rational expression $f(x)/d(x)$ where the degree of $f(x)$ is greater than or equal to the degree of $d(x)$.

Proper rational expression A rational expression $r(x)/d(x)$ where the degree of $r(x)$ is less than the degree of $d(x)$.

Synthetic division A shortcut for long division of polynomials when dividing by divisors of the form $x - k$.

Remainder Theorem If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Factor Theorem A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

Section 2.4 Complex Numbers

Complex number If a and b are real numbers, the number $a + bi$, where the number a is called the real part and the number bi is called the imaginary part, is a complex number written in standard form.

Imaginary number If $b \neq 0$, the number $a + bi$ is called an imaginary number.

Complex conjugates A pair of complex numbers of the form $a + bi$ and $a - bi$.

Section 2.5 Zeros of Polynomial Functions

Fundamental Theorem of Algebra If $f(x)$ is a polynomial of degree n , where $n > 0$, then f has at least one zero in the complex number system.

Linear Factorization Theorem If $f(x)$ is a polynomial of degree n , where $n > 0$, then f has precisely n linear factors $f(x) = a_n(x - c_1)(x - c_2) \dots (x - c_n)$ where c_1, c_2, \dots, c_n are complex numbers.

Irreducible over the reals A quadratic factor with no real zeros; also known as prime.

Variation in sign Two consecutive coefficients have opposite signs.

Upper bound A real number b is an upper bound for the real zeros of f if no real zeros of f are greater than b .

Lower bound A real number b is a lower bound for the real zeros of f if no real zeros of f are less than b .

Section 2.6 Rational Functions

Rational function A function that can be written in the form: $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial.

Vertical asymptote The line $x = a$ is a vertical asymptote of the graph of f if $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ as $x \rightarrow a$, either from the right or from the left.

Horizontal asymptote The line $y = b$ is a horizontal asymptote of the graph of f if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or $x \rightarrow -\infty$.

Slant (or oblique) asymptote If the degree of the numerator of a rational function is exactly one more than the degree of the denominator, then the line determined by the quotient of the denominator into the numerator is a slant asymptote of the graph of the rational function.

Section 2.7 Partial Fractions

Partial fraction A rational expression can be written as the sum of two or more simpler fractions. Each simpler fraction is called a partial fraction. The rational expression written as the sum of two or more partial fractions is called the partial fraction decomposition of the rational expression.

Chapter 3 Exponential and Logarithmic Functions

Section 3.1 Exponential Functions and Their Graphs

Algebraic functions Functions of x that can be expressed as a finite number of sums, differences, multiples, quotients, powers and roots.

Transcendental functions Functions that are not algebraic.

Natural base e The irrational number $e \approx 2.718281828 \dots$.

Continuous compounding Increasing the number of compoundings in the compound interest formula without bound leads to continuous compounding, which is given by the formula $A = Pe^{rt}$.

Section 3.2 Logarithmic Functions and Their Graphs

Common logarithmic function The logarithmic function with base 10.

Natural logarithmic function The logarithmic function with base e given by $f(x) = \ln x, x > 0$.

Section 3.3 Properties of Logarithms

Section 3.4 Exponential and Logarithmic Equations

Section 3.5 Exponential and Logarithmic Models

Bell-shaped curve The graph of a Gaussian model.

Logistic curve A model for describing populations initially having rapid growth followed by a declining rate of growth.

Sigmoidal curve Another name for a logistic growth curve.

Chapter 4 Trigonometry

Section 4.1 Radian and Degree Measure

Trigonometry The Greek word for “measurement of triangles.”

Central angle of a circle An angle whose vertex is the center of the circle.

Complementary angles Two positive angles whose sum is $\pi/2$ radians or 90° .

Supplementary angles Two positive angles whose sum is π radians or 180° .

Degree The most common unit of angle measure, denoted by the symbol $^\circ$. A measure of one degree (1°) is equivalent to a rotation of $1/360$ of a complete revolution about the vertex of an angle.

Section 4.2 Trigonometric Functions: The Unit Circle

Unit circle A circle of radius 1 centered at the origin and given by the equation $x^2 + y^2 = 1$. The basis for one of the perspectives of trigonometry.

Period A function f is periodic if there exists a positive real number c such that $f(t + c) = f(t)$ for all t in the domain of f . The smallest number c for which f is periodic is called the period of f .

Section 4.3 Right Triangle Trigonometry

Section 4.4 Trigonometric Functions of Any Angle

Reference angles Let θ be an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

Section 4.5 Graphs of Sine and Cosine Functions

Amplitude Represents half the distance between the maximum and minimum values of $y = a \sin x$ or $y = a \cos x$ and is given by amplitude = $|a|$.

Phase shift The amount c/b by which the graph of $y = a \sin bx$ is shifted to obtain the graph of $y = a \sin(bx - c)$.

Section 4.6 Graphs of Other Trigonometric Functions

Damping factor When a trigonometric function is multiplied by another function, this other function is called the damping factor.

Section 4.7 Inverse Trigonometric Functions

Section 4.8 Applications and Models

Angle of elevation The angle from the horizontal upward to an object.

Angle of depression The angle from the horizontal downward to an object.

Bearings A measure giving the acute angle a path or line of sight makes with a fixed north-south line. Used to give directions in surveying and navigation.

Simple harmonic motion The vibration, oscillation, or rotation of an object under ideal conditions such that the object's uniform and regular motion can be described by a sine or cosine function.

Chapter 5 Analytic Trigonometry

Section 5.1 Using Fundamental Identities

Section 5.2 Verifying Trigonometric Identities

Section 5.3 Solving Trigonometric Equations

Section 5.4 Sum and Difference Formulas

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

Chapter 6 Additional Topics in Trigonometry

Section 6.1 Law of Sines

Oblique triangle A triangle that has no right angles.

Section 6.2 Law of Cosines

Section 6.3 Vectors in the Plane

Vector \mathbf{v} in the plane The set of all directed line segments that are equivalent to given directed line segment PQ , written $\mathbf{v} = PQ$.

Standard position The representative of a set of equivalent directed line segments whose initial point is the origin.

Zero vector A vector whose initial point and terminal point both lie at the origin; denoted by $\mathbf{0} = \langle 0, 0 \rangle$.

Unit vector A vector \mathbf{v} such that $\|\mathbf{v}\| = 1$.

Standard unit vectors The unit vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$, denoted by $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$, which can be used to represent any vector $\mathbf{v} = \langle v_1, v_2 \rangle$.

Direction angle The angle (measured counterclockwise) that a unit vector makes with the positive x -axis.

Section 6.4 Vectors and Dot Products

Angle between two nonzero vectors The angle θ , $0 \leq \theta \leq \pi$, between the two nonzero vectors' respective standard position vectors.

Orthogonal Meeting at right angles; essentially the same meaning as “perpendicular.”

Section 6.5 Trigonometric Form of a Complex Number

Real axis The horizontal axis in the complex plane.

Imaginary axis The vertical axis in the complex plane.

Absolute value of a complex number $a + bi$ The distance between the origin $(0, 0)$ and the point (a, b) .

n th roots of unity The n distinct n th roots of 1.

Chapter 7 Systems of Equations and Inequalities

Section 7.1 Solving Systems of Equations

System of equations A model used for solving a problem that involves two or more equations in two or more variables.

Solution of a system of equations (in two variables) An ordered pair that satisfies each equation in the system.

Solving a system of equations The process of finding the set of all solutions of a system.

Method of substitution A method for solving systems of equations which involves substituting for one of the variables in one equation in the system an expression that represents that variable in terms of the other variable.

Graphical method A method for finding the solution(s) to a system of equations in which the solutions are determined by finding points of intersection of the graphs of the equations.

Points of intersection The points, which correspond to the solutions of the system, where the graphs of the equations in a system of equations intersect.

Break-even point The point reached by sales when enough units of a product have been sold so that the total revenue R equals the total cost C .

Section 7.2 Two-Variable Linear Systems

Method of elimination A method for solving a system of equations which involves obtaining coefficients that differ only in sign for one of the variables, so that adding the equations eliminates that variable.

Equivalent systems Two systems that have precisely the same solution set.

Consistent system A system that has at least one solution.

Inconsistent system A system that has no solution.

Section 7.3 Multivariable Linear Systems

Row-echelon form A system that has a “stair-step” pattern with leading coefficients of 1.

Ordered triple Real numbers (x, y, z) which correspond to the solution of a system of three equations in three unknowns.

Row operations A set of operations that can be performed on a system of equations in order to produce an equivalent system of equations.

Gaussian elimination The process of rewriting a system of equations in row-echelon form through a chain of equivalent systems each of which is obtained by using one of the three row operations.

Nonsquare system of equations A system of equations in which the number of equations differs from the number of variables.

Section 7.4 Systems of Inequalities

Solution of an inequality An ordered pair (a, b) is a solution of an inequality in x and y if the inequality is true when a and b are substituted for x and y respectively.

Graph of an inequality The collection of all solutions of the inequality represented on the Cartesian plane.

Linear inequalities An inequality written in the form $ax + by < c$.

Solution of a system of inequalities A point (x, y) that satisfies each inequality in the system.

Consumer surplus The area of the region that lies below the demand curve, above the horizontal line passing through the equilibrium point, and to the right of the p -axis.

Producer surplus The area of the region that lies above the supply curve, below the horizontal line passing through the equilibrium point, and to the right of the p -axis.

Section 7.5 Linear Programming

Optimization A process in which the minimum or maximum of a quantity is sought.

Linear programming An optimization strategy.

Objective function Gives the quantity that is to be maximized or minimized.

Constraints A system of linear inequalities that determines the region of feasible solutions.

Feasible solutions All points in a region satisfying each of the constraints.

Chapter 8 Matrices and Determinants

Section 8.1 Matrices and Systems of Equations

Entry of a matrix One of the real numbers that makes up a matrix.

Order of a matrix Indicates the number of rows and columns of a matrix. A matrix having m rows and n columns is said to be of order $m \times n$.

Square matrix A matrix in which the number of rows and the number of columns is equal.

Main diagonal For a square matrix, all entries, a_{ij} , in which $i = j$.

Row matrix A matrix that has only one row.

Column matrix A matrix that has only one column.

Elementary row operations A set of operations that can be performed on an augmented matrix of a given system of linear equations that produce a new augmented matrix corresponding to a new (but equivalent) system of linear equations.

Gauss-Jordan elimination The process of reducing a matrix to reduced row-echelon form.

Section 8.2 Operations with Matrices

Scalars Real numbers used in operations with matrices.

Scalar multiple If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, the scalar multiple of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$.

Zero matrix A matrix consisting entirely of zeros.

Matrix multiplication If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix $AB = [c_{ij}]$ where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$.

Identity matrix of order n The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere.

Section 8.3 The Inverse of a Square Matrix

Inverse of a matrix Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$ then A^{-1} is called the inverse of A .

Section 8.4 The Determinant of a Square Matrix

Determinant If A is a square matrix (of order 2×2 or greater), the determinant of A is the sum of the entries in any row (or column) of A multiplied by their respective cofactors. The determinant is a real number.

Minors If A is a square matrix, the minor M_{ij} of the entry a_{ij} is the determinant of the matrix obtained by deleting the i th row and j th column of A .

Cofactors The cofactor C_{ij} of the entry a_{ij} is $C_{ij} = (-1)^{i+j}M_{ij}$.

Section 8.5 Applications of Matrices and Determinants

Chapter 9 Sequences, Series, and Probability

Section 9.1 Sequences and Series

Terms of a sequence The function values $a_1, a_2, a_3, a_4, \dots, a_n, \dots$ of an infinite sequence.

Recursive A sequence is recursive if one or more of the first few terms are given and all other terms are defined using previous terms.

Section 9.2 Arithmetic Sequences and Partial Sums

Arithmetic sequence A sequence in which the differences between consecutive terms are the same.

Common difference The difference d between consecutive terms of an arithmetic sequence. That is, $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots$

Section 9.3 Geometric Sequences and Series

Geometric sequence A sequence in which the ratios of consecutive terms are the same.

Common ratio The ratio r between consecutive terms of a geometric sequence. That is, $r = a_2/a_1 = a_3/a_2 = a_4/a_3 = \dots, r \neq 0$.

Infinite geometric series or geometric series The summation of the terms of an infinite geometric sequence.

Section 9.4 Mathematical Induction

Mathematical induction A form of mathematical proof in which it must be shown for a statement, P_n , involving the positive integer n that P_1 is true and that the truth of P_k implies the truth of P_{k+1} for every positive k .

First differences The differences found by subtracting consecutive terms of a sequence.

Second differences The differences found by subtracting consecutive first differences.

Section 9.5 The Binomial Theorem

Binomial coefficients The coefficients of a binomial expansion.

Pascal's Triangle A triangular pattern, named for the French mathematician Blaise Pascal, in which the first and last numbers in each row are 1 and every other number in each row is formed by adding the two numbers immediately above the number. These numbers are precisely the same as the coefficients of binomial expansions.

Section 9.6 Counting Principles

Fundamental Counting Principle Let E_1 and E_2 be two events. The first event E_1 can occur in m_1 different ways. After E_1 has occurred, E_2 can occur in m_2 different ways. The number of ways that the two events can occur is $m_1 \cdot m_2$.

Permutation An ordering of n different elements such that one element is first, one is second, one is third, and so on.

Distinguishable permutations Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + \dots + n_k$. Then the number of distinguishable permutations of the n objects is $\frac{n!}{(n_1! \cdot n_2! \cdot n_3! \cdot \dots \cdot n_k!)}$.

Combination A subset of a set of n elements in which the order is not important.

Section 9.7 Probability

Experiment Any happening for which the result is uncertain.

Outcomes The possible results of an experiment.

Sample space The set of all possible outcomes of an experiment.

Event Any sub-collection of a sample space.

Mutually exclusive Two events A and B (from the same sample space) are mutually exclusive if A and B have no outcomes in common.

Independent events Two events are independent if the occurrence of one has no effect on the occurrence of the other.

Complement of an event The collection of all outcomes in the sample space that are not in the event.

Chapter 10 Topics in Analytic Geometry

Section 10.1 Lines

Inclination (of a nonhorizontal line) The positive angle θ (less than π) measured counterclockwise from the x -axis to the line.

Angle between two lines The smaller angle of the two pairs of opposite angles (one acute and one obtuse) formed by the intersection of two distinct lines in a plane.

Section 10.2 Introduction to Conics: Parabolas

Directrix A fixed line in the plane from which each point on a parabola is the same distance as the distance from the point to a fixed point in the plane.

Focus A fixed point in the plane from which each point on a parabola is the same distance as the distance from the point to a fixed line in the plane.

Focal chord A line segment that passes through the focus of a parabola and has endpoints on the parabola.

Latus rectum The specific focal chord perpendicular to the axis of a parabola.

Tangent A line is tangent to a parabola at a point on the parabola if the line intersects, but does not cross, the parabola at the point.

Section 10.3 Ellipses

Foci Distinct fixed points in the plane such that the sum of the distances from each point on an ellipse is constant.

Vertices Points of intersection of an ellipse and the line through its foci.

Major axis The chord connecting the vertices of an ellipse.

Center The midpoint of the major axis of an ellipse.

Minor axis The chord perpendicular to the major axis at the center of an ellipse.

Section 10.4 Hyperbolas

Branches The two disconnected parts of the graph of a hyperbola.

Transverse axis The line segment connecting the vertices of a hyperbola.

Conjugate axis The line segment in a hyperbola of length $2b$ joining $(0, b)$ and $(0, -b)$ [or $(-b, 0)$ and $(b, 0)$].

Section 10.5 Rotation of Conics

Invariant under rotation A term or quantity in the equation of a conic that remains the same during a rotation of the coordinate axes through an angle θ .

Discriminant The quantity $B^2 - 4AC$, of the general conic equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, which can be used to classify the type of conic.

Section 10.6 Parametric Equations

Parameter A third variable introduced to represent a curve in the plane.

Section 10.7 Polar Coordinates

Section 10.8 Graphs of Polar Equations

Section 10.9 Polar Equations of Conics