97. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 10$

$y = -\frac{4}{9}(x^2 - 6x) + 10$

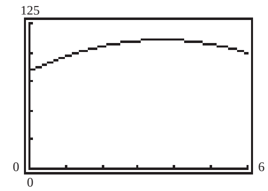
$y = -\frac{4}{9}(x^2 - 6x + 9 - 9) + 10$

$y = -\frac{4}{9}(x^2 - 6x + 9) + 4 + 10$

$y = -\frac{4}{9}(x - 3)^2 + 14$

The maximum height of the diver is 14 ft.

99. (a)



(b) vertex $= (3.65, 110, 810)$1993, 110,800 reserves

101. (a) $P = (100 + x)[90 - x(0.15)] - (100 + x)60$

$P = (100 + x)[90 - x(0.15) - 60]$

$P = (100 + x)(30 - 0.15x)$

$P = 3000 - 15x + 30x = 0.15x^2$

$P = 3000 + 15x - 0.15x^2$

$P = 3000 + 15x - \frac{3}{20}x^2$

(b) $P = -\frac{3}{20}x^2 + 15x + 3000$

$P = -\frac{3}{20}(x^2 - 100x + 2500) + 3000 + 375$

$P = -\frac{3}{20}(x - 50)^2 + 3375$

vertex $= (50, 3375)$

order size for maximum profit

$P = 100 + 50 = 150$ radios

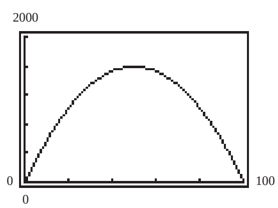
(c) Recommend pricing scheme if price reductions are restricted to orders between 100 and 150 orders.

103. $A = \dfrac{2}{\pi}(100x - x^2)$

Keystrokes:

[Y=] [(] 2 [÷] [π] [)] [(] 100 [X,T,θ] [−] [X,T,θ] [x²] [)] [GRAPH]

$x \approx 50$ when A is maximum



105. $100 = a(500 - 0)^2 + 0$

$100 = a(250{,}000)$

$\dfrac{100}{250{,}000} = a$

$\dfrac{1}{2500} = a$

$y = \dfrac{1}{2500}(x - 0)^2 + 0$

$y = \dfrac{1}{2500}x^2$

107. The graph of the quadratic function $f(x) = ax^2 + bx + c$ is a parabola.

109. To find any x-intercepts, set $y = 0$ and solve the resulting equation for x.

To find the y-intercept, set $x = 0$ and solve the resulting equation for y.

111. The discriminant of a quadratic function tells how many x-intercepts the parabola has. If positive, there are 2 x-intercepts; if zero, 1 x-intercept; and if negative, no x-intercepts.

113. Find the y-coordinate of the vertex. This is the maximum (or minimum) value of a quadratic function.

Mid-Chapter Quiz for Chapter 7

1. $A = kr^2$

2. $z = \dfrac{kx}{y^2}$

3. Distance: $d = rt$

Distance varies jointly proportional to rate and time.

4. Volume: $V = s^3$

The volume of a cube varies directly as the cube of the length of the sides.

5. $z = \dfrac{kx^2}{y}$ if $z = 6, x = 6, y = 4$ $z = \dfrac{2x^2}{3y}$

$$\text{then } 6 = \frac{k(6)^2}{4}$$
$$24 = k(6)^2$$
$$\frac{24}{36} = k$$
$$\frac{2}{3} = k$$

6. $2x - 3y \le 4$

(a) $2(5) - 3(2) \overset{?}{\le} 4$
$10 - 6 \le 4$
$4 \le 4$
$(5, 2)$ is a solution.

(b) $2(-2) - 3(4) \overset{?}{\le} 4$
$-4 - 12 \le 4$
$-16 \le 4$
$(-2, 4)$ is a solution.

(c) $2(2) - 3(-4) \overset{?}{\le} 4$
$4 + 12 \le 4$
$16 \le 4$
$(2, -4)$ is not a solution.

(d) $2(3) - 3(0) \overset{?}{\le} 4$
$6 - 0 \le 4$
$6 \le 4$
$(3, 0)$ is not a solution.

7. $m = \dfrac{3 - 5}{5 - 1} = -\dfrac{2}{4} = -\dfrac{1}{2}$

$y - 3 = -\dfrac{1}{2}(x - 5)$

$y - 3 = -\dfrac{1}{2}x + \dfrac{5}{2}$

$2y - 6 = -x + 5$

$x + 2y = 11$ line

Shaded region: $x + 2y \le 11$

8. $m = \dfrac{3 - 1}{4 - (-2)} = \dfrac{2}{6} = \dfrac{1}{3}$

$y - 3 = \dfrac{1}{3}(x - 4)$

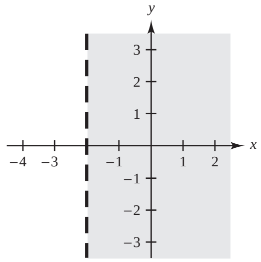
$y - 3 = \dfrac{1}{3}x - \dfrac{4}{3}$

$3y - 9 = x - 4$

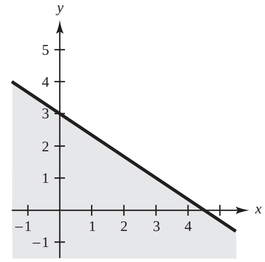
$x - 3y = -5$ line

Shaded region: $x - 3y > -5$

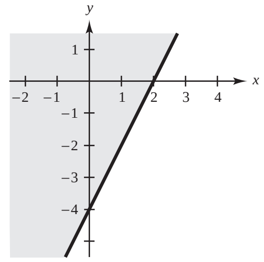
9. $x > -2$



10. $2x + 3y \le 9$



11. $2x - y \le 4$



12. $3 = a(5-3)^2 - 1 \qquad y = 1(x-3)^2 - 1$

$3 = a(4) - 1$

$4 = a(4)$

$1 = a$

13. $3 = a(3-5)^2 + 4 \qquad y = -\frac{1}{4}(x-5)^2 + 4$

$3 = a(4) + 4$

$-1 = a(4)$

$-\frac{1}{4} = a$

14. vertex $= (-3, 2)$

$y = -\frac{1}{4}(x^2 + 6x \qquad) - \frac{1}{4}$

$y = -\frac{1}{4}(x^2 + 6x + 9) - \frac{1}{4} + \frac{9}{4}$

$y = -\frac{1}{4}(x+3)^2 + 2$

x-intercepts

$0 = -\frac{1}{4}(x^2 + 6x + 1)$

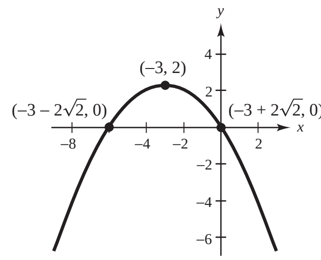
$0 = x^2 + 6x + 1$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36 - 4}}{2}$

$x = \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = \frac{-3 \pm 2\sqrt{2}}{2}$

$x \approx -0.17$ and -5.83



15. vertex $= (1, -9)$

$y = 2(x^2 - 2x \qquad) - 7$

$y = 2(x^2 - 2x + 1) - 7 - 2$

$y = 2(x-1)^2 - 9$

x-intercepts

$0 = 2x^2 - 4x - 7$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-7)}}{2(2)}$

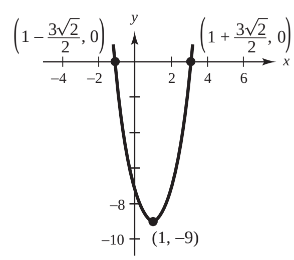
$x = \frac{4 \pm \sqrt{16 + 56}}{4}$

$x = \frac{4 \pm \sqrt{72}}{4}$

$x = \frac{4 \pm 6\sqrt{2}}{4}$

$x = \frac{2 \pm 3\sqrt{2}}{2} = 1 \pm \frac{3\sqrt{2}}{2}$

$x \approx -1.12$ and 3.12



16. $g = kt$

$.02 = k(12)$

$\frac{.02}{12} = k$

$\frac{2}{1200} = k$

$\frac{1}{600} = k$

$.05 = \frac{1}{600}t$

$(.05)(600) = t$

$30 = t$ minutes

17. $900x + 1400y \leq 20{,}000$

$9x + 14y \leq 200$

18. $y = -0.005x^2 + x + 5$

$y = -0.005(x^2 - 200x \quad) + 5$

$y = -0.005(x^2 - 200x + 10{,}000) + 5 + 50$

$y = -0.005(x - 100)^2 + 55$

maximum height $= 55$ feet

Section 7.4 Conic Sections

1. $x^2 + y^2 = 9$ (c)

3. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ (e)

5. $x^2 - y^2 = 4$ (a)

7. center: $(0, 0)$, radius: 5

$x^2 + y^2 = r^2$

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

9. center: $(0, 0)$, radius: $\frac{2}{3}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\frac{2}{3}\right)^2$

$x^2 + y^2 = \frac{4}{9}$ or $9x^2 + 9y^2 = 4$

11. center: $(0, 0)$, point: $(0, 8)$

$r = \sqrt{(0 - 0)^2 + (8 - 0)^2}$

$r = \sqrt{64}$

$r = 8$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$

13. center: $(0, 0)$, point: $(5, 2)$

$r = \sqrt{(5 - 0)^2 + (2 - 0)^2}$

$r = \sqrt{25 + 4}$

$r = \sqrt{29}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\sqrt{29}\right)^2$

$x^2 + y^2 = 29$

15. center: $(4, 3)$, radius: 10

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y - 3)^2 = 10^2$

$(x - 4)^2 + (y - 3)^2 = 100$

17. center: $(5, -3)$, radius: 9

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + [y - (-3)]^2 = 9^2$

$(x - 5)^2 + (y + 3)^2 = 81$

19. center: $(-2, 1)$, point: $(0, 1)$

$r = \sqrt{[0 - (-2)]^2 + (1 - 1)^2}$

$r = \sqrt{4 + 0}$

$r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 1)^2 = 2^2$

$(x + 2)^2 + (y - 1)^2 = 4$

21. center: $(3, 2)$, point: $(4, 6)$

$r = \sqrt{(4 - 3)^2 + (6 - 2)^2}$

$r = \sqrt{1 + 16}$

$= \sqrt{17}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 2)^2 = \left(\sqrt{17}\right)^2$

$(x - 3)^2 + (y - 2)^2 = 17$