105. Equation of ellipse $= \frac{x^2}{50^2} + \frac{y^2}{40^2} = 1$

or

$$\frac{x^2}{2500} + \frac{y^2}{1600} = 1$$
$$\frac{45^2}{2500} + \frac{y^2}{1600} = 1$$
$$\frac{y^2}{1600} = 0.19$$
$$y^2 = 304$$
$$y = 17.435596 \approx 17 \text{ feet}$$

107.
$$A = \pi ab \qquad a + b = 20$$
$$301.59 = \pi ab \qquad b = 20 - a$$
$$\frac{301.59}{\pi} = ab$$
$$96 \approx ab$$
$$96 = a(20 - a)$$
$$0 = -a^2 + 20a - 96$$
$$0 = a^2 - 20a + 96$$
$$0 = (a - 12)(a - 8)$$
$a = 12$ $\quad$ $a = 8$

$b = 8$ $\quad$ $b = 12$

$$\frac{x^2}{144} + \frac{y^2}{64} = 1$$

109. The four types of conics are circles, parabolas, ellipses, and hyperbolas.

111. An ellipse is the set of all points (x, y) such that the sum of the distances between (x, y) and two distinct fixed points is a constant.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

113. An ellipse is a circle if the coefficients of the second degree terms are equal.

115. The central rectangle of a hyperbola can be used to sketch its asymptotes because the asymptotes are the extended diagonals of the central rectangle.

117. $y = \frac{3}{2}\sqrt{x^2 - 4}$ is the top half of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

Section 7.5 Graphs of Rational Functions

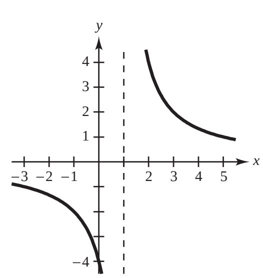
1. (a)

x	0	0.5	0.9	0.99	0.999
y	-4	-8	-40	-400	-4000

x	2	1.5	1.1	1.01	1.001
y	4	8	40	400	4000

x	2	5	10	100	1000
y	4	1	0.44444	0.0404	0.004

(b)



(c) Domain:

$$x - 1 \neq 0$$
$$x \neq 1$$
$$(-\infty, 1) \cup (1, \infty)$$

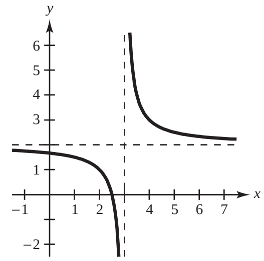
3. (a)

x	2	2.5	2.9	2.99	2.999
y	1	0	-8	-98	-998

x	4	3.5	3.1	3.01	3.001
y	3	4	12	102	1002

x	4	5	10	100	1000
y	3	2.5	2.143	2.010	2.001

(b)



(c) Domain:

$$x - 3 \neq 0$$
$$x \neq 3$$
$$(-\infty, 3) \cup (3, \infty)$$

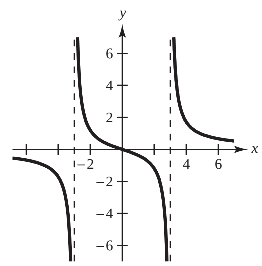
5. (a)

x	2	2.5	2.9	2.99	2.999
y	-1.2	-2.727	-14.75	-149.7	-1500

x	4	3.5	3.1	3.01	3.001
y	1.714	3.231	15.246	150.25	1500.2

x	4	5	10	100	1000
y	1.714	0.938	0.330	0.030	0.003

(b)



(c) Domain:

$$x^2 - 9 \neq 0$$
$$(x - 3)(x + 3) \neq 0$$
$$x \neq 3 \quad x \neq -3$$
$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

7. $f(x) = \dfrac{5}{x^2}$

Domain: $x^2 \neq 0$

$$x \neq 0$$
$$(-\infty, 0) \cup (0, \infty)$$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.

9. $f(x) = \dfrac{x}{x + 8}$

Domain: $x + 8 \neq 0$

$$x \neq -8$$
$$(-\infty, -8) \cup (-8, \infty)$$

Vertical asymptote: $x = -8$

Horizontal asymptote: $y = 1$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficients are 1.

11. $g(t) = \dfrac{2t - 5}{3t - 9}$

Domain: $3t - 9 \neq 0$

$$t \neq 3$$
$$(-\infty, 3) \cup (3, \infty)$$

Vertical asymptote: $t = 3$

Horizontal asymptote: $y = \frac{2}{3}$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 2 and the leading coefficient of the denominator is 3.

13. $y = \dfrac{3 - 5x}{1 - 3x}$

Domain: $1 - 3x \neq 0$

$$1 \neq 3x$$
$$\tfrac{1}{3} \neq x$$
$$\left(-\infty, \tfrac{1}{3}\right) \cup \left(\tfrac{1}{3}, \infty\right)$$

Vertical asymptote: $x = \frac{1}{3}$

Horizontal asymptote: $y = \frac{5}{3}$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is -5 and the leading coefficient of the denominator is -3.

15. $g(t) = \dfrac{3}{t(t-1)}$

$t(t-1) \neq 0$

$t \neq 0 \quad t - 1 \neq 0$

$t \neq 1$

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

Vertical asymptotes: $t = 0, t = 1$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.

17. $y = \dfrac{2x^2}{x^2 + 1}$

Domain: $x^2 + 1 \neq 0$

$(-\infty, \infty)$

no real solution

Vertical asymptote: none

Horizontal asymptote: $y = 2$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 2 and the leading coefficient of the denominator is 1.

19. $y = \dfrac{x^2 - 4}{x^2 - 1}$

Domain: $x^2 - 1 \neq 0$

$(x-1)(x+1) \neq 0$

$x \neq 1 \quad x \neq -1$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Vertical asymptotes: $x = 1, x = -1$

Horizontal asymptote: $y = 1$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 1 and the leading coefficient of the denominator is 1.

21. $g(z) = 1 - \dfrac{2}{z}$

$g(z) = \dfrac{z}{z} \cdot \dfrac{1}{1} - \dfrac{2}{z} = \dfrac{z-2}{z}$

Domain: $z \neq 0$

$(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $z = 0$

Horizontal asymptote: $y = 1$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficients are 1.

23. $g(x) = 2x + \dfrac{4}{x} = \dfrac{x}{x} \cdot \dfrac{2x}{1} + \dfrac{4}{x} = \dfrac{2x^2 + 4}{x}$

Domain: $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

Horizontal asymptote: none since the degree of the numerator is greater than the degree of the denominator.

25. $f(x) = \dfrac{2}{x+1}$ matches with graph (d).

Vertical asymptote: $x + 1 = 0$

$x = -1$

Horizontal asymptote: $y = 0$

27. $f(x) = \dfrac{x-2}{x-1}$ matches with graph (b).

Vertical asymptote: $x - 1 = 0$

$x = 1$

Horizontal asymptote: $y = 1$

29. (d)

31. (a)

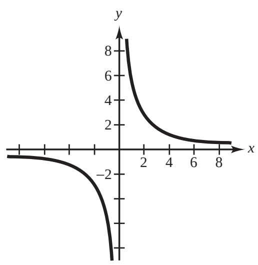
33. $g(x) = \dfrac{5}{x}$

y-intercept: $g(0) = \dfrac{5}{0} =$ undefined, none

x-intercept: none, numerator is never zero.

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



35. $g(x) = \dfrac{5}{x - 4}$

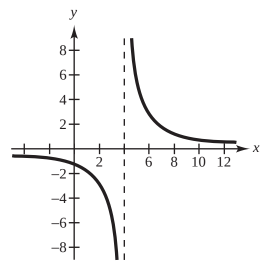
y-intercept: $g(0) = \dfrac{5}{0 - 4} = -\dfrac{5}{4}$

x-intercept: none, numerator is never zero.

Vertical asymptote: $x - 4 = 0$

$x = 4$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



37. $f(x) = \dfrac{1}{x - 2}$

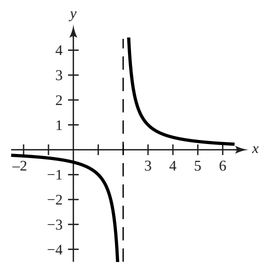
y-intercept: $f(0) = \dfrac{1}{0 - 2} = -\dfrac{1}{2}$

x-intercept: none, numerator is never zero.

Vertical asymptote: $x - 2 = 0$

$x = 2$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



39. $g(x) = \dfrac{1}{2 - x}$

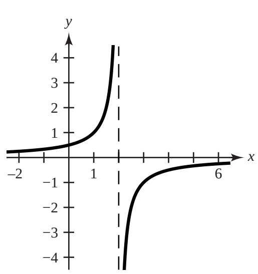
y-intercept: $g(0) = \dfrac{1}{2 - 0} = \dfrac{1}{2}$

x-intercept: none, numerator is never zero

Vertical asymptote: $2 - x = 0$

$x = 2$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



41. $y = \dfrac{3x}{x^2 + 4x}$

y-intercept: $y = \dfrac{3(0)}{0^2 + 4(0)} =$ undefined, none

x-intercept: $0 = \dfrac{3x}{x^2 + 4x} = \dfrac{3x}{x(x + 4)}$

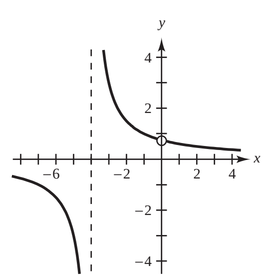
$0 = \dfrac{3}{x + 4}$; none

Vertical asymptote: $x^2 + 4x = 0$

$x(x + 4) = 0$

$x = -4$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



43. $h(u) = \dfrac{3u^2}{u^2 - 3u}$

y-intercept: $h(0) = \dfrac{3(0)^2}{0^2 - 3(0)} =$ undefined, none

x-intercept: $0 = \dfrac{3u^2}{u^2 - 3u} = \dfrac{3u^2}{u(u-3)}$

$0 = \dfrac{3u}{u - 3}$

$0 = 3u$

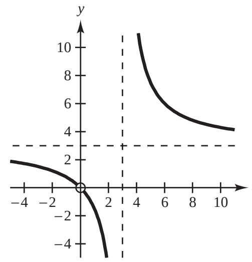
$0 = u$, none, since $h(0)$ is undefined.

Vertical asymptote: $u^2 - 3u = 0$

$u(u - 3) = 0$

$u = 3$

Horizontal asymptote: $y = 3$ since the degrees are equal and the leading coefficient of the numerator is 3 and the leading coefficient of the denominator is 1.



45. $y = \dfrac{2x + 4}{x}$

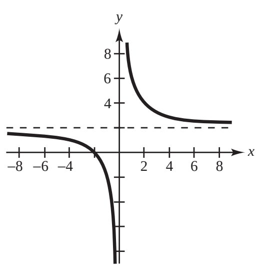
y-intercept: $y = \dfrac{2(0) + 4}{0} =$ undefined, none.

x-intercept: $2x + 4 = 0$

$x = -2$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 2$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 2 and the leading coefficient of the denominator is 1.



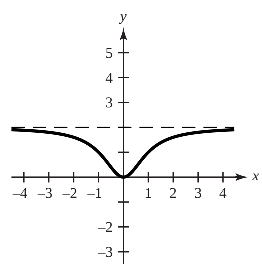
47. $y = \dfrac{2x^2}{x^2 + 1}$

y-intercept: $y = \dfrac{2(0)^2}{0^2 + 1} = 0$

x-intercept: $x = 0$

Vertical asymptote: none, $x^2 + 1 = 0$ has no real solutions.

Horizontal asymptote: $y = 2$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 2 and the leading coefficient of the denominator is 1.



49. $y = \dfrac{4}{x^2 + 1}$

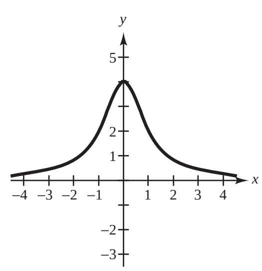
y-intercept: $y = \dfrac{4}{0^2 + 1} = 4$

x-intercept: none, numerator is never zero.

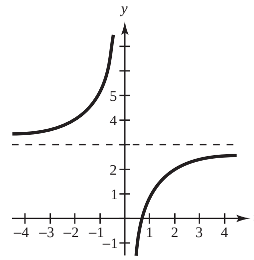
Vertical asymptote: none, $x^2 + 1 \neq 0$

no real solution

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.



51. $g(t) = 3 - \dfrac{2}{t}$



y-intercept: $g(0) = 3 - \dfrac{2}{0} = \text{undefined, none}$

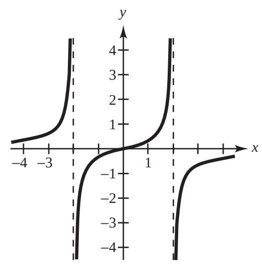
x-intercept: $0 = 3 - \dfrac{2}{t}$

$$0 = 3t - 2$$
$$2 = 3t$$
$$\frac{2}{3} = t$$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = 3$

53. $y = -\dfrac{x}{x^2 - 4}$



y-intercept: $y = \dfrac{-0}{0^2 - 4} = 0$

x-intercept: $0 = -\dfrac{x}{x^2 - 4}$

$$0 = -x$$
$$0 = x$$

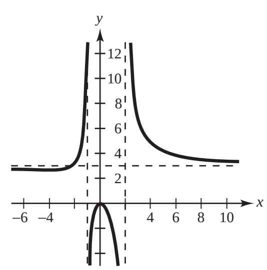
Vertical asymptote: $x = 2, x = -2$

$$x^2 - 4 = 0$$
$$(x - 2)(x + 2) = 0$$
$$x = 2 \quad x = -2$$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.

55. $f(x) = \dfrac{3x^2}{x^2 - x - 2}$

y-intercept: $y = \dfrac{3(0)^2}{0^2 - 0 - 2} = \dfrac{0}{-2} = 0$



x-intercept: $3x^2 = 0$

$$x^2 = 0$$
$$x = 0$$

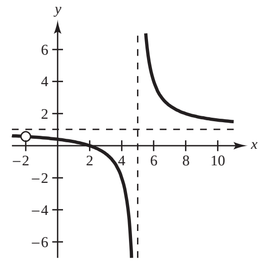
Vertical asymptotes: $x^2 - x - 2 = 0$

$$(x - 2)(x + 1) = 0$$
$$x - 2 = 0 \qquad x - 2 = 0$$
$$x = 2 \qquad x = 2$$

Vertical asymptote: none

Horizontal asymptote: $y = 3$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 3 and the leading coefficient of the denominator is 1.

57. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x - 10}$



y-intercept: $f(0) = \dfrac{0^2 - 4}{0^2 - 3(0) - 10} = \dfrac{4}{10} = \dfrac{2}{5}$

x-intercept: $0 = \dfrac{x^2 - 4}{x^2 - 3x - 10}$

$0 = x^2 - 4$

$0 = (x - 2)(x + 2)$

$x = 2$ undefined at $x = -2$

Vertical asymptotes: $x^2 - 3x - 10 = 0$

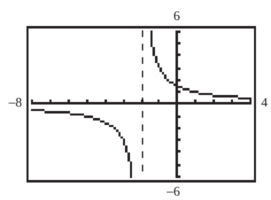
$(x - 5)(x + 2) = 0$

$x = 5 \quad x = -2 \rightarrow$ hole in graph

Horizontal asymptote: $y = 1$ since the degrees are equal and the leading coefficients are 1.

$f(x) = \dfrac{(x - 2)\cancel{(x + 2)}}{(x - 5)\cancel{(x + 2)}}\Bigg\}$ gives a hole in graph at $x = -2$

59. $f(x) = \dfrac{3}{x + 2}$



Domain: $x + 2 \neq 0$

$x \neq -2$

$(-\infty, -2) \cup (-2, \infty)$

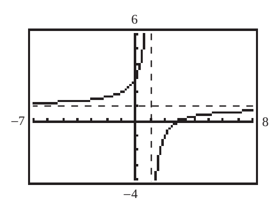
Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

Keystrokes:

y_1 [Y=] 3 [÷] [(] [X,T,θ] [+] 2 [)] [GRAPH]

61. $h(x) = \dfrac{x - 3}{x - 1}$



Domain: $x - 1 \neq 0$

$x \neq 1$

$(-\infty, 1) \cup (1, \infty)$

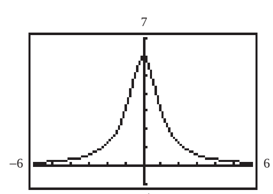
Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 1$

Keystrokes:

[Y=] [(] [X,T,θ] [−] 3 [)] [÷] [(] [X,T,θ] [−] 1 [GRAPH]

63. $f(t) = \dfrac{6}{t^2 + 1}$



Domain: $t^2 + 1 \neq 0$

$(-\infty, \infty)$

Vertical asymptote: none

Horizontal asymptote: $y = 0$

Keystrokes:

[Y=] 6 [÷] [(] [X,T,θ] [x^2] [+] 1 [)] [GRAPH]

65. $y = \dfrac{2(x^2 + 1)}{x^2}$

Domain: $x^2 \neq 0$

$x \neq 0$

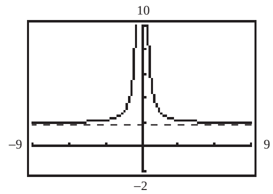
$(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 2$

Keystrokes:

[Y=] [(] 2 [(] [X,T,θ] [x^2] [+] 1 [)] [)] [÷] [X,T,θ] [x^2] [GRAPH]



67. $y = \dfrac{3}{x} + \dfrac{1}{x - 2}$

$y = \dfrac{3}{x} + \dfrac{1}{x - 2} = \dfrac{4x - 6}{x^2 - 2x}$

Domain: $x \neq 0 \quad x - 2 \neq 0$

$x \neq 2$

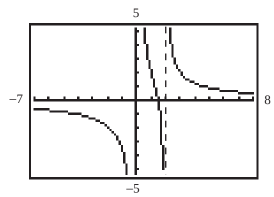
$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

Vertical asymptotes: $x^2 - 2x = 0, x(x - 2) = 0, x = 0, x = 2$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.

Keystrokes: [Y=] 3 [÷] [X,T,θ] [+] 1 [÷] [(] [X,T,θ] [−] 2 [)] [GRAPH] or

[Y=] (4 [X,T,θ] [−] 6) ÷ ([X,T,θ] [x^2] [−] 2 [X,T,θ]) [GRAPH]

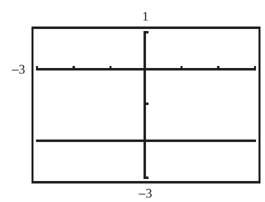


69. Reduce $g(x)$ to lowest terms.

$g(x) = \dfrac{4 - 2x}{x - 2} = \dfrac{2(2 - x)}{x - 2} = -2$

Keystrokes: [Y=] [(] 4 [−] 2 [X,T,θ] [)] [÷] [(] [X,T,θ] [−] 2 [)] [GRAPH]

There is no vertical asymptote because the fraction is not reduced to lowest terms.



71. (a) Average cost $= \dfrac{\text{Cost}}{\text{Number of units}}$

$\overline{C} = \dfrac{2500 + 0.50x}{x}, \quad 0 < x$

(b) $\overline{C} = \dfrac{2500 + 0.50(1000)}{1000} = \$3$

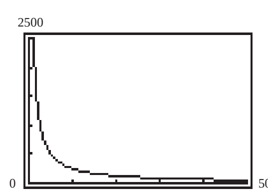
$\overline{C} = \dfrac{2500 + 0.50(10{,}000)}{10{,}000} = \$0.75$

(c) *Keystrokes:*

[Y=] [(] 2500 [+] .5 [X,T,θ] [)] [÷] [X,T,θ] [GRAPH]

Horizontal asymptote

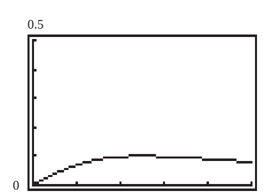
$\overline{C} = \$0.50$ since the degree of the numerator is equal to the degree of the denominator and the leading coefficient of the numerator is 0.50 and the leading coefficient of the denominator is 1. As the number of units produced increases, the average cost is approximately \$0.50.



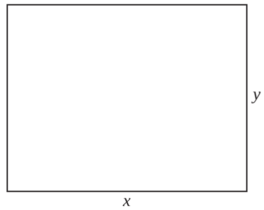
73. (a) $C = 0$ is the horizontal asymptote, since the degree of the numerator is less than the degree of the denominator. The meaning in the context of the problem is that the chemical is eliminated from the body.

(b) *Keystrokes:* [Y=] 2 [X,T,θ] [÷] [(] 4 [X,T,θ] [x^2] [+] 25 [)] [GRAPH]

Maximum occurs when $t \approx 2.5$.



75. (a) answers will vary.



(b)

$$A = x \cdot y \qquad P = 2l + 2w$$

$$400 = x \cdot y \qquad P = 2(l + w)$$

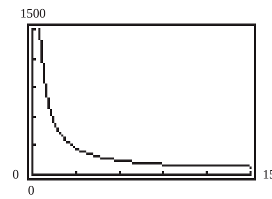
$$\frac{400}{x} = y \qquad P = 2\left(x + \frac{400}{x}\right)$$

(c) Domain: $x > 0$ or $(0, \infty)$

(d) Minimum perimeter: 20 units × 20 units

Keystrokes:

[Y=] 2 [(] [X,T,θ] [+] 400 [÷] [X,T,θ] [)] [GRAPH]



77. $y = \dfrac{2(x + 1)}{x - 3}$

79. $y = \dfrac{x - 6}{(x - 4)(x + 2)}$

81. (c) $y = \dfrac{48.4 - 4.79x}{1 - 0.13x}$

Domain:

$$1 - 0.13x \neq 0$$

$$-0.13x \neq -1$$

$$x \neq \frac{-1}{-0.13}$$

$$x \neq 7.69$$

Horizontal asymptote: $y = \dfrac{-4.79}{-0.13}$

$$y \approx 36.85$$

since the degrees are equal.

x-intercept:

$$0 = \frac{48.4 - 4.79x}{1 - 0.13x}$$

$$0 = 48.4 - 4.79x$$

$$4.79x = 48.4$$

$$x = \frac{48.4}{4.79} = 10.10$$

Vertical asymptote: $x \approx 7.69$

(the excluded value of the domain)

(d) *Keystrokes:*

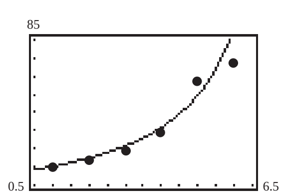
[Y=] [(] 48.4 [−] 4.79 [X,T,θ] [)] [÷] [(] 1 [−] .13 [X,T,θ] [)] [GRAPH]

Plot (1, 50.1), (2, 51.9), (3, 54.8), (4, 59.3), (5, 73.6), (6, 78.7)

in

[STAT] 1 then enter 1, 2, 3, 4, 5, 6, in L_1 and enter 50.1, 51.9, 54.8, 59.3, 73.6, 78.7, in L_2.

[STAT PLOT] 1 [ON] [GRAPH]



The model appears to be accurate for the restricted domain.

—CONTINUED—

81. —CONTINUED—

(e) The models are not accurate for the years before 1991 and after 1996. Use the quadratic model to estimate the value of the shipment in 1998, because the rational function evaluated at $x = 8$ is negative.

83. An asymptote of a graph is a line to which the graph becomes arbitrarily close as $|x|$ or $|y|$ increases without bound.

85. No, not when the domain is all reals. For example,

$f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

Review Exercises for Chapter 7

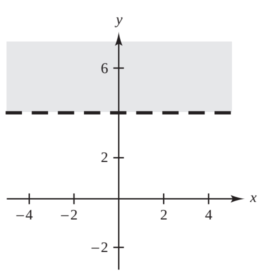
1. P varies directly as the cube of t. $P = kt^3$

3. z varies inversely as the square of s. $z = \dfrac{k}{s^2}$

5.
$$y = k\sqrt[3]{x}$$
$$12 = k\sqrt[3]{8}$$
$$6 = \frac{12}{\sqrt[3]{8}} = k$$
$$y = 6\sqrt[3]{x}$$

7. $T = krs^2$
$$5000 = k(0.09)(1000)^2$$
$$\frac{5000}{90{,}000} = k$$
$$\frac{1}{18} = k$$
$$T = \frac{1}{18}rs^2$$

9. $y > 4$



11. $x - 2 \geq 0$

$x \geq 2$