18. $y = -0.005x^2 + x + 5$

$y = -0.005(x^2 - 200x \quad) + 5$

$y = -0.005(x^2 - 200x + 10{,}000) + 5 + 50$

$y = -0.005(x - 100)^2 + 55$

maximum height $= 55$ feet

Section 7.4 Conic Sections

1. $x^2 + y^2 = 9$ (c)

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (e)

5. $x^2 - y^2 = 4$ (a)

7. center: $(0, 0)$, radius: 5

$x^2 + y^2 = r^2$

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

9. center: $(0, 0)$, radius: $\frac{2}{3}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\frac{2}{3}\right)^2$

$x^2 + y^2 = \frac{4}{9}$ or $9x^2 + 9y^2 = 4$

11. center: $(0, 0)$, point: $(0, 8)$

$r = \sqrt{(0 - 0)^2 + (8 - 0)^2}$

$r = \sqrt{64}$

$r = 8$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$

13. center: $(0, 0)$, point: $(5, 2)$

$r = \sqrt{(5 - 0)^2 + (2 - 0)^2}$

$r = \sqrt{25 + 4}$

$r = \sqrt{29}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\sqrt{29}\right)^2$

$x^2 + y^2 = 29$

15. center: $(4, 3)$, radius: 10

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y - 3)^2 = 10^2$

$(x - 4)^2 + (y - 3)^2 = 100$

17. center: $(5, -3)$, radius: 9

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + [y - (-3)]^2 = 9^2$

$(x - 5)^2 + (y + 3)^2 = 81$

19. center: $(-2, 1)$, point: $(0, 1)$

$r = \sqrt{[0 - (-2)]^2 + (1 - 1)^2}$

$r = \sqrt{4 + 0}$

$r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 1)^2 = 2^2$

$(x + 2)^2 + (y - 1)^2 = 4$

21. center: $(3, 2)$, point: $(4, 6)$

$r = \sqrt{(4 - 3)^2 + (6 - 2)^2}$

$r = \sqrt{1 + 16}$

$= \sqrt{17}$

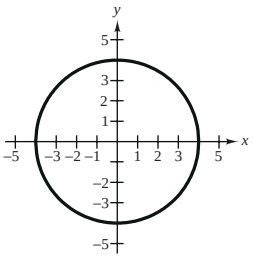
$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 2)^2 = \left(\sqrt{17}\right)^2$

$(x - 3)^2 + (y - 2)^2 = 17$

23. $x^2 + y^2 = 16$

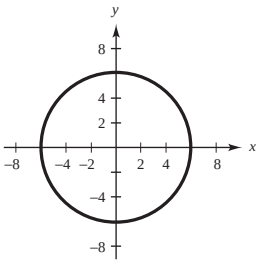
radius $= 4$ center $= (0, 0)$



25. $x^2 + y^2 = 36$

center $= (0, 0)$

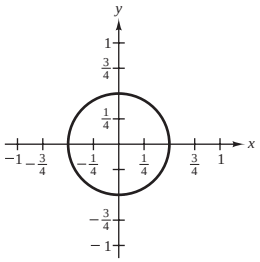
radius $= 6$



27. $4x^2 + 4y^2 = 1$ center $= (0, 0)$

$x^2 + y^2 = \frac{1}{4}$ radius $= \frac{1}{2}$

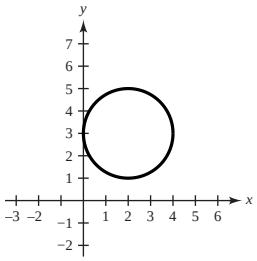
$r = \frac{1}{2}$



29. $(x - 2)^2 + (y - 3)^2 = 4$

center $= (2, 3)$

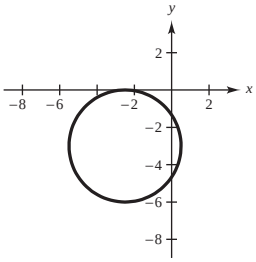
radius $= 2$



31. $\left(x + \frac{5}{2}\right)^2 + (y + 3)^2 = 9$

center $= \left(-\frac{5}{2}, -3\right)$

radius $= 3$



33. $x^2 + y^2 - 4x - 2y + 1 = 0$

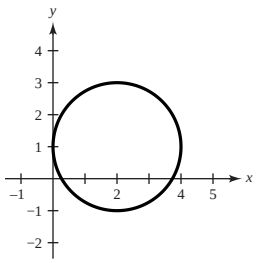
$x^2 - 4x + y^2 - 2y = -1$

$(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$

$(x - 2)^2 + (y - 1)^2 = 4$

center $= (2, 1)$

radius $= 2$



35. $x^2 + y^2 + 2x + 6y + 6 = 0$

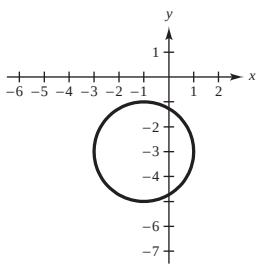
$x^2 + 2x + y^2 + 6y = -6$

$(x^2 + 2x + 1) + (y^2 + 6y + 9) = -6 + 1 + 9$

$(x + 1)^2 + (y + 3)^2 = 4$

center $= (-1, -3)$

radius $= 2$



37. $x^2 + y^2 = 30$

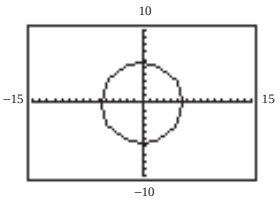
$y^2 = 30 - x^2$

$y = \pm\sqrt{30 - x^2}$

Keystrokes:

y_1 [Y=] [√] [(] 30 [−] [X,T,θ] [x²] [)] [ENTER]

y_2 [(−)] [√] [(] 30 [−] [X,T,θ] [x²] [)] [GRAPH]



39. $(x - 2)^2 + y^2 = 10$

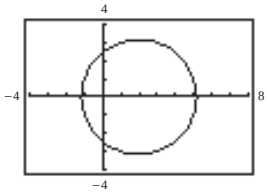
$y^2 = 10 - (x - 2)^2$

$y = \pm\sqrt{10 - (x - 2)^2}$

Keystrokes:

y_1 [Y=] [√] [(] 10 [−] [(] [X,T,θ] [−] 2 [)] [x²] [)] [ENTER]

y_2 [(−)] [√] [(] 10 [−] [(] [X,T,θ] [−] 2 [)] [x²] [)] [GRAPH]



41. center: $(0, 0)$

vertices: $(-4, 0), (4, 0)$

co-vertices: $(0, -3), (0, 3)$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

major axis is x-axis so $a = 4$

minor axis is y-axis so $b = 3$

$\dfrac{x^2}{4^2} + \dfrac{y^2}{3^2} = 1$

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

43. center: $(0, 0)$

vertices: $(-2, 0), (2, 0)$

co-vertices: $(0, -1), (0, 1)$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

major axis is x-axis so $a = 2$

minor axis is y-axis so $b = 1$

$\dfrac{x^2}{2^2} + \dfrac{y^2}{1^2} = 1$

$\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

45. center: $(0, 0)$

vertices: $(0, -4), (0, 4)$

co-vertices: $(-3, 0), (3, 0)$

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

major axis is y-axis so $a = 4$

minor axis is x-axis so $b = 3$

$\dfrac{x^2}{3^2} + \dfrac{y^2}{4^2} = 1$

$\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

47. center: $(0, 0)$

vertices: $(0, -2), (0, 2)$

co-vertices: $(-1, 0), (1, 0)$

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

major axis is y-axis so $a = 2$

minor axis is x-axis so $b = 1$

$\dfrac{x^2}{1^2} + \dfrac{y^2}{2^2} = 1$

$\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$

49. center: $(0, 0)$

major axis (vertical) 10 units

minor axis 6 units

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$b = 3 \quad a = 5$

$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

51. center: $(0, 0)$

major axis (horizontal) 20 units

minor axis 12 units

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

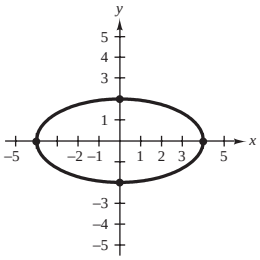
$a = 10 \quad b = 6$

$\frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$

$\frac{x^2}{100} + \frac{y^2}{36} = 1$

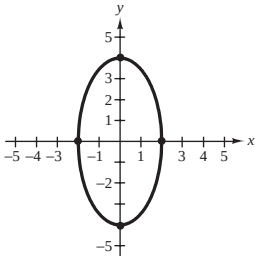
53. Vertices: $(-4, 0), (4, 0)$

Co-Vertices: $(0, 2), (0, -2)$



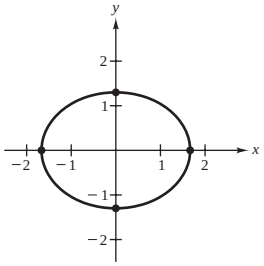
55. Vertices: $(0, 4), (0, -4)$

Co-Vertices: $(2, 0), (-2, 0)$



57. Vertices: $\left(-\frac{5}{3}, 0\right), \left(\frac{5}{3}, 0\right)$

Co-Vertices: $\left(0, \frac{4}{3}\right), \left(0, -\frac{4}{3}\right)$

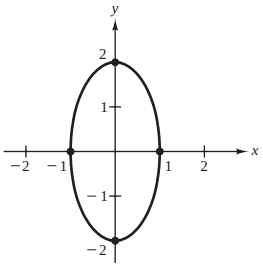


59. $4x^2 + y^2 - 4 = 0$

$\frac{x^2}{1} + \frac{y^2}{4} = 1$

Vertices: $(0, 2), (0, -2)$

Co-Vertices: $(1, 0), (-1, 0)$



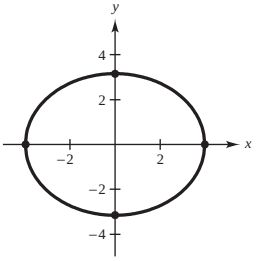
61. $10x^2 + 16y^2 - 160 = 0$

$\frac{10x^2}{160} + \frac{16y^2}{160} = \frac{160}{160}$

$\frac{x^2}{16} + \frac{y^2}{10} = 1$

Vertices: $(\pm 4, 0)$

Co-Vertices: $\left(0, \pm\sqrt{10}\right)$



63. $x^2 + 2y^2 = 4$

$$2y^2 = 4 - x^2$$

$$y^2 = \frac{4 - x^2}{2}$$

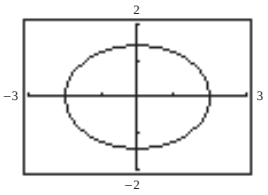
$$y = \pm\sqrt{\frac{4 - x^2}{2}}$$

Keystrokes:

y_1 [Y=] [√] [(] [(] 4 [−] [X,T,θ] [x²] [)] [÷] 2 [)] [ENTER]

y_2 [(−)] [√] [(] [(] 4 [−] [X,T,θ] [x²] [)] [÷] 2 [)] [GRAPH]

Vertices: $(\pm 2, 0)$



65. $3x^2 + y^2 - 12 = 0$

$$y^2 = 12 - 3x^2$$

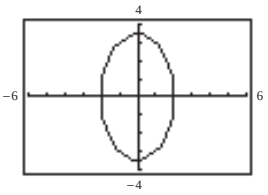
$$y = \pm\sqrt{12 - 3x^2}$$

Keystrokes:

y_1 [Y=] [√] [(] 12 [−] 3 [X,T,θ] [x²] [)] [ENTER]

y_2 [(−)] [√] [(] 12 [−] 3 [X,T,θ] [x²] [)] [GRAPH]

Vertices: $\left(0, \pm 2\sqrt{3}\right)$



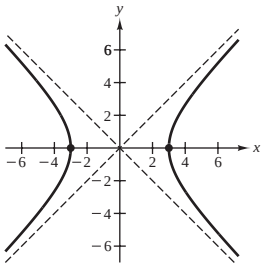
67. Vertices: $(3, 0), (-3, 0)$

Asymptotes: $y = \frac{3}{3}x$ $\quad y = -\frac{3}{3}x$

$y = x$ $\quad y = -x$

Equation: $x^2 - y^2 = 9$

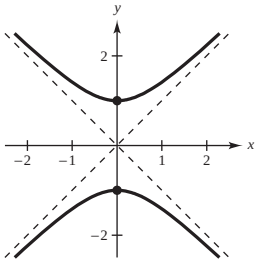
$$\frac{x^2}{9} - \frac{y^2}{9} = 1$$



69. Vertices: $(0, \pm 1)$

Asymptotes: $y = \pm x$

Equation: $y^2 - x^2 = 1$

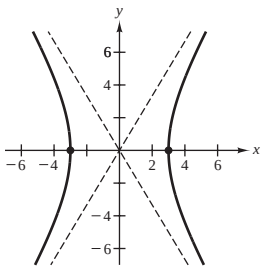


71. Vertices: $(3, 0), (-3, 0)$

Asymptotes: $y = \frac{5}{3}x$

$y = -\frac{5}{3}x$

Equation: $\frac{x^2}{9} - \frac{y^2}{25} = 1$

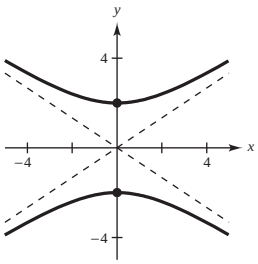


73. Vertices: $(0, \pm 2)$

Asymptotes: $y = \frac{2}{3}x$

$y = -\frac{2}{3}x$

Equation: $\frac{y^2}{4} - \frac{x^2}{9} = 1$

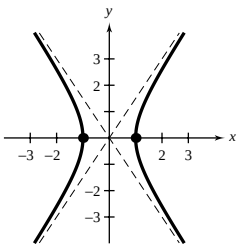


75. Vertices: $(\pm 1, 0)$

Asymptotes: $y = \pm\frac{\frac{3}{2}}{1}x$

$y = \pm\frac{3}{2}x$

Equation: $\frac{x^2}{1} - \frac{y^2}{\frac{9}{4}} = 1$



77. Vertices: $(4, 0), (-4, 0)$

Asymptotes: $y = \frac{2}{4}x = \frac{1}{2}x$

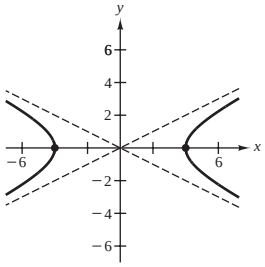
$y = -\frac{2}{4}x = -\frac{1}{2}x$

Equation: $4y^2 - x^2 + 16 = 0$

$\frac{4y^2}{-16} - \frac{x^2}{-16} = \frac{-16}{-16}$

$\frac{-y^2}{4} + \frac{x^2}{16} = 1$

$\frac{x^2}{16} - \frac{y^2}{4} = 1$



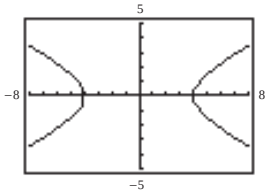
79. $\dfrac{x^2}{16} - \dfrac{y^2}{64} = 1$

81. $\dfrac{y^2}{16} - \dfrac{x^2}{64} = 1$

83. $\dfrac{x^2}{81} - \dfrac{y^2}{36} = 1$

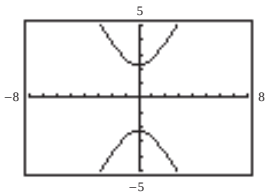
85. $\dfrac{y^2}{1} - \dfrac{x^2}{\frac{1}{4}} = 1$

87.
$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$
$$x^2 - 4y^2 = 16$$
$$x^2 - 16 = 4y^2$$
$$\frac{x^2 - 16}{4} = y^2$$
$$\pm\sqrt{\frac{x^2 - 16}{4}} = y$$



Keystrokes:
y_1: Y= √ ((X,T,θ x² − 16) ÷ 4) ENTER
y_2: (−) √ ((X,T,θ x² − 16) ÷ 4) GRAPH

89.
$$5x^2 - 2y^2 + 10 = 0$$
$$5x^2 + 10 = 2y^2$$
$$\frac{5x^2 + 10}{2} = y^2$$
$$\pm\sqrt{\frac{5x^2 + 10}{2}} = y$$



Keystrokes:
y_1: Y= √ ((5 X,T,θ x² + 10) ÷ 2) ENTER
y_2: (−) √ ((5 X,T,θ x² + 10) ÷ 2) GRAPH

91. Parabola

93. Ellipse

95. Hyperbola

97. Circle

99. Line

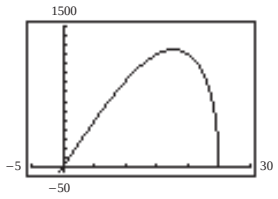
101. $x^2 + y^2 = 4500^2$
$x^2 + y^2 = 20{,}250{,}000$

103. (a) $x^2 + y^2 = 625$ (equation of circle)

(x, y) of the rectangle is also the point on the circle, so y-coordinate equals:

$$x^2 + y^2 = 625$$
$$y^2 = 625 - x^2$$
$$y = \sqrt{625 - x^2}$$
$$\text{width} = 2\left(\sqrt{625 - x^2}\right)$$
$$\text{area} = 2x \cdot 2\left(\sqrt{625 - x^2}\right)$$
$$\text{area} = 4x\sqrt{625 - x^2}$$

(b)



$x \approx 17.68$

105. Equation of ellipse $= \dfrac{x^2}{50^2} + \dfrac{y^2}{40^2} = 1$

or

$$\frac{x^2}{2500} + \frac{y^2}{1600} = 1$$

$$\frac{45^2}{2500} + \frac{y^2}{1600} = 1$$

$$\frac{y^2}{1600} = 0.19$$

$$y^2 = 304$$

$$y = 17.435596 \approx 17 \text{ feet}$$

107.

$$A = \pi ab \qquad a + b = 20$$

$$301.59 = \pi ab \qquad b = 20 - a$$

$$\frac{301.59}{\pi} = ab$$

$$96 \approx ab$$

$$96 = a(20 - a)$$

$$0 = -a^2 + 20a - 96$$

$$0 = a^2 - 20a + 96$$

$$0 = (a - 12)(a - 8)$$

$a = 12$ $\qquad$ $a = 8$

$b = 8$ $\qquad$ $b = 12$

$$\frac{x^2}{144} + \frac{y^2}{64} = 1$$

109. The four types of conics are circles, parabolas, ellipses, and hyperbolas.

111. An ellipse is the set of all points (x, y) such that the sum of the distances between (x, y) and two distinct fixed points is a constant.

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

113. An ellipse is a circle if the coefficients of the second degree terms are equal.

115. The central rectangle of a hyperbola can be used to sketch its asymptotes because the asymptotes are the extended diagonals of the central rectangle.

117. $y = \frac{3}{2}\sqrt{x^2 - 4}$ is the top half of the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$.

Section 7.5 Graphs of Rational Functions

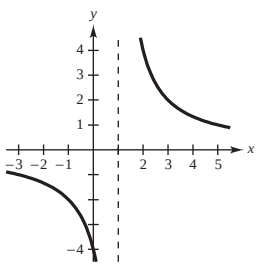
1. (a)

x	0	0.5	0.9	0.99	0.999
y	-4	-8	-40	-400	-4000

x	2	1.5	1.1	1.01	1.001
y	4	8	40	400	4000

x	2	5	10	100	1000
y	4	1	0.44444	0.0404	0.004

(b)



(c) Domain:

$x - 1 \neq 0$

$x \neq 1$

$(-\infty, 1) \cup (1, \infty)$