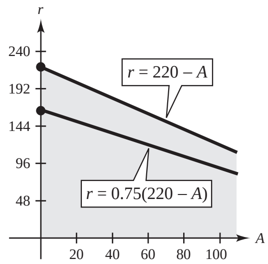
63. $r = 0.75(220 - A)$



65. (x_1, y_1) is a solution of a linear inequality in x and y means the inequality is true when x_1 and y_1 are substituted for x and y respectively.

67. The solution of $x - y > 1$ does not include the points on the line $x - y = 1$. The solution of $x - y \geq 1$ does include the points on the line $x - y = 1$.

69. On the real number line, the solution of $x \leq 3$ is an unbounded interval.

On a rectangular coordinate system, the solution of $x \leq 3$ is a half-plane.

Section 7.3 Graphs of Quadratic Functions

1. $y = 4 - 2x$ (e)

3. $y = x^2 - 3$ (b)

5. $y = (x - 2)^2$ (d)

7. $y = x^2 + 2 = (x - 0)^2 + 2$

vertex $(0, 2)$

9. $y = x^2 - 4x + 7$

$= (x^2 - 4x + 4) + 7 - 4$

$= (x - 2)^2 + 3$

vertex $= (2, 3)$

11. $y = x^2 + 6x + 5$

$y = (x^2 + 6x + 9) + 5 - 9$

$y = (x + 3)^2 - 4$

vertex $= (-3, -4)$

13. $y = -x^2 + 6x - 10$

$y = -1(x^2 - 6x) - 10$

$y = -1(x^2 - 6x + 9) - 10 + 9$

$y = -1(x - 3)^2 - 1$

vertex $(3, -1)$

15. $y = -x^2 + 2x - 7$

$= -1(x^2 - 2x + 1) - 7 + 1$

$= -1(x - 1)^2 - 6$

vertex $= (1, -6)$

17. $y = 2x^2 + 6x + 2$

$= 2\left(x^2 + 3x + \frac{9}{4}\right) + 2 - \frac{9}{2}$

$= 2\left(x + \frac{3}{2}\right)^2 - \frac{5}{2}$

vertex $= \left(-\frac{3}{2}, -\frac{5}{2}\right)$

19. $f(x) = x^2 - 8x + 15$

$a = 1 \quad b = -8$

$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)} = 4$

$f\left(-\frac{b}{2a}\right) = 4^2 - 8(4) + 15$

$= 16 - 32 + 15$

$= -1$

vertex $= (4, -1)$

21. $g(x) = -x^2 - 2x + 1$

$a = -1 \quad b = -2$

$x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = -1$

$g\left(\frac{-b}{2a}\right) = -(-1)^2 - 2(-1) + 1$

$= -1 + 2 + 1$

$= 2$

vertex $= (-1, 2)$

23. $y = 4x^2 + 4x + 4$

$a = 4 \quad b = 4$

$x = \frac{-b}{2a} = \frac{-4}{2(4)} = \frac{-1}{2}$

$y = 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 4$

$= 4\left(\frac{1}{4}\right) - 2 + 4$

$= 1 - 2 + 4$

$= 3$

vertex $= \left(-\frac{1}{2}, 3\right)$

25. $2 > 0$ opens upward

vertex $= (0, 2)$

27. $-1 < 0$ opens downward

vertex $= (10, 4)$

29. $1 > 0$ opens upward

vertex $= (0, -6)$

31. $-1 < 0$ opens downward

vertex $= (3, 0)$

33. $y = 25 - x^2$

$0 = 25 - x^2$

$x^2 = 25$

$x = \pm 5$

$(5, 0), (-5, 0)$

$y = 25 - x^2$

$y = 25 - 0^2$

$y = 25$

$(0, 25)$

35. $y = x^2 - 9x$

$0 = x^2 - 9x$

$0 = x(x - 9)$

$(0, 0), (9, 0)$

$y = x^2 - 9x$

$y = 0^2 - 9(0)$

$y = 0$

$(0, 0)$

37. $y = 4x^2 - 12x + 9$

$0 = 4x^2 - 12x + 9$

$0 = (2x - 3)^2$

$0 = 2x - 3$

$\frac{3}{2} = x$

$\left(\frac{3}{2}, 0\right)$

$y = 4x^2 - 12x + 9$

$y = 4(0)^2 - 12(0) + 9$

$y = 9$

$(0, 9)$

39. $y = x^2 - 3x + 3$

$0 = x^2 - 3x + 3$

$x = \dfrac{3 \pm \sqrt{9 - 12}}{2}$

$= \dfrac{3 \pm \sqrt{-3}}{2}$

no x-intercepts

$y = x^2 - 3x + 3$

$y = 0^2 - 3(0) + 3$

$y = 3$

$(0, 3)$

41. $g(x) = x^2 - 4$

x-intercepts

$0 = x^2 - 4$

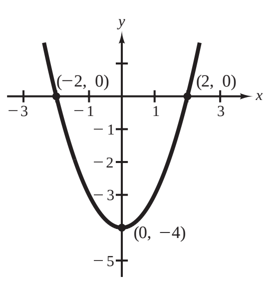
$0 = (x - 2)(x + 2)$

$x = 2 \quad x = -2$

vertex

$g(x) = (x - 0)^2 - 4$

$(0, -4)$



43. $f(x) = -x^2 + 4$

x-intercepts

$0 = -x^2 + 4$

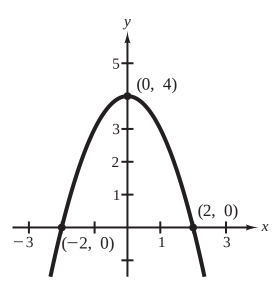
$x^2 = 4$

$x = \pm 2$

vertex

$f(x) = -(x - 0)^2 + 4$

$(0, 4)$



45. $f(x) = x^2 - 3x$

x-intercepts

$0 = x^2 - 3x$

$0 = x(x - 3)$

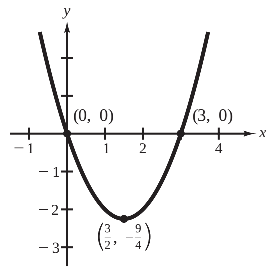
$0 = x \qquad x = 3$

vertex

$f(x) = \left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4}$

$f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$

$\left(\frac{3}{2}, -\frac{9}{4}\right)$



47. $f(x) = -x^2 + 3x$

x-intercepts

$0 = -x^2 + 3x$

$0 = -x(x - 3)$

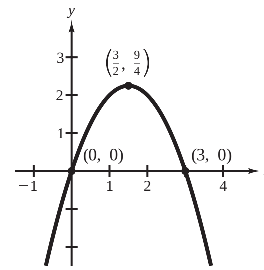
$0 = x \qquad x = 3$

vertex

$y = -1\left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4}$

$\quad = -1\left(x - \frac{3}{2}\right)^2 + \frac{9}{4}$

$\left(\frac{3}{2}, \frac{9}{4}\right)$



49. $f(x) = (x - 4)^2$

x-intercepts

$0 = (x - 4)^2$

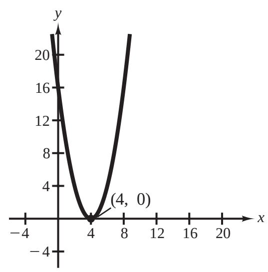
$0 = x - 4$

$4 = x$

vertex

$y = (x - 4)^2 + 0$

$(4, 0)$



51. $f(x) = x^2 - 8x + 15$

x-intercepts

$0 = x^2 - 8x + 15$

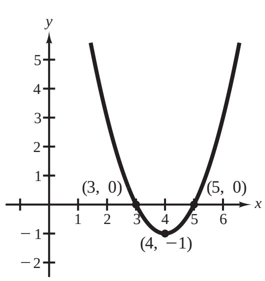
$0 = (x - 5)(x - 3)$

$5 = x \qquad x = 3$

vertex

$y = (x^2 - 8x + 16) + 15 - 16$

$\quad = (x - 4)^2 - 1$



53. $f(x) = -(x^2 + 6x + 5)$

x-intercepts

$0 = x^2 + 6x + 5$

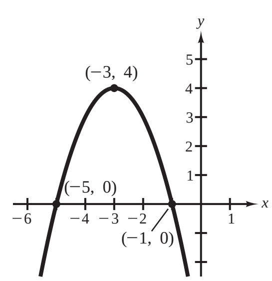
$0 = (x + 5)(x + 1)$

$-5 = x \qquad x = -1$

vertex

$y = -(x^2 + 6x + 9) - 5 + 9$

$y = -(x + 3)^2 + 4$



55. $g(x) = -x^2 + 6x - 7$

x-intercepts

$0 = -x^2 + 6x - 7$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(-1)(-7)}}{2(-1)}$

$x = \dfrac{-6 \pm \sqrt{36 - 28}}{-2}$

$x = \dfrac{-6 \pm \sqrt{8}}{-2}$

$x = \dfrac{-6 \pm 2\sqrt{2}}{2}$

$x = -3 \pm \sqrt{2}$

vertex

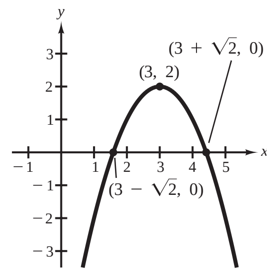
$q(x) = -x^2 + 6x - 7$

$q(x) = -(x^2 - 6x) - 7$

$q(x) = -(x^2 - 6x + 9 - 9) - 7$

$q(x) = -(x^2 - 6x + 9) + 9 - 7$

$q(x) = -(x - 3)^2 + 2$



57. $f(x) = 2(x^2 + 6x + 8)$

vertex

$y = 2(x^2 + 6x + 9) + 16 - 18$

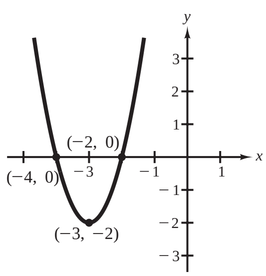
$y = 2(x + 3)^2 - 2$

x-intercepts

$0 = x^2 + 6x + 8$

$0 = (x + 4)(x + 2)$

$-4 = x \qquad x = -2$



59. $f(x) = \frac{1}{2}(x^2 - 2x - 3)$

vertex

$y = \frac{1}{2}(x^2 - 2x + 1) - \frac{3}{2} - \frac{1}{2}$

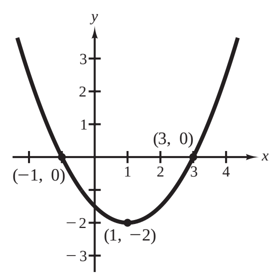
$y = \frac{1}{2}(x - 1)^2 - 2$

x-intercepts

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$3 = x \qquad x = -1$



61. $y = \frac{1}{5}(3x^2 - 24x + 38)$

$y = \frac{3}{5}(x^2 - 8x + 16) + \frac{38}{5} - \frac{48}{5}$

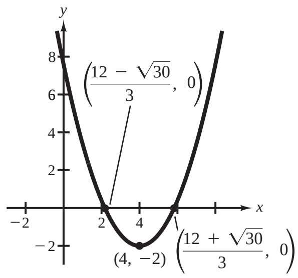
$y = \frac{3}{5}(x - 4)^2 - 2$

$0 = 3x^2 - 24x + 38$

$x = \frac{24 \pm \sqrt{576 - 456}}{6}$

$x = \frac{24 \pm \sqrt{120}}{6} = \frac{12 \pm \sqrt{30}}{3}$

$\approx 5.83, 2.17$



63. $f(x) = 5 - \frac{1}{3}x^2$

$f(x) = -\frac{1}{3}x^2 + 5$

$f(x) = -\frac{1}{3}(x - 0)^2 + 5$

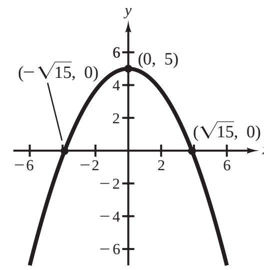
$0 = -\frac{1}{3}x^2 + 5$

$\frac{1}{3}x^2 = 5$

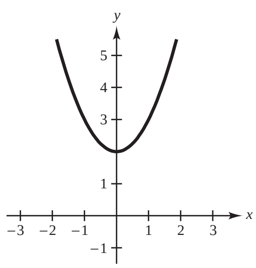
$x^2 = 15$

$x = \pm\sqrt{15}$

$x \approx 3.87, -3.87$

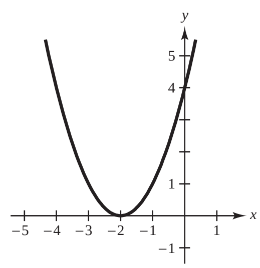


65. $h(x) = x^2 + 2$



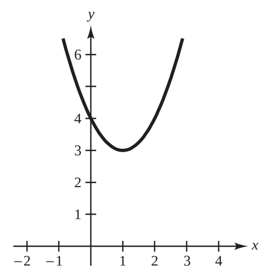
Vertical shift 2 units up.

67. $h(x) = (x + 2)^2$



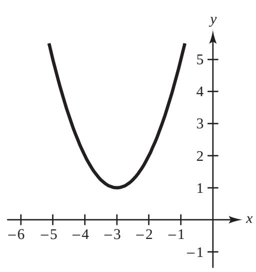
Horizontal shift 2 units left.

69. $h(x) = (x - 1)^2 + 3$



Horizontal shift 1 unit right.
Vertical shift 3 units up.

71. $h(x) = (x + 3)^2 + 1$



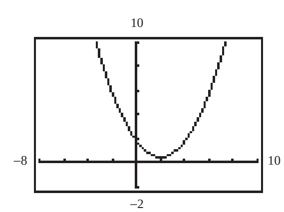
Horizontal shift 3 units left.
Vertical shift 1 unit up.

73. $y = \frac{1}{6}(2x^2 - 8x + 11)$

Keystrokes:

[Y=] [(] 1 [÷] 6 [)] [(] 2 [X,T,θ] [x²] [−] 8 [X,T,θ] [+] 11 [)] [GRAPH]

vertex = (2, 0.5)

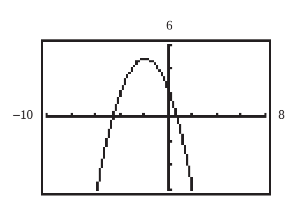


75. $y = -0.7x^2 - 2.7x + 2.3$

Keystrokes:

[Y=] [(−)] .7 [X,T,θ] [x²] [(−)] 2.7 [X,T,θ] [+] 2.3 [GRAPH]

vertex = (−1.9, 4.9)



77. vertex = $(0, 4)$ point = $(-2, 0)$

$y = a(x - 0)^2 + 4$

$0 = a(-2 - 0)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$-1 = a$

$y = -1(x - 0)^2 + 4$

$y = -x^2 + 4$

79. vertex = $(-2, 2)$ point = $(0, 2)$

$y = a(x - (-2))^2 + (-2)$

$y = a(x + 2)^2 - 2$

$2 = a(0 + 2)^2 - 2$

$2 = 4a - 2$

$4 = 4a$

$1 = a$

$y = 1(x + 2)^2 - 2$

$y = (x + 2)^2 - 2$

$y = x^2 + 4x + 4 - 2$

$y = x^2 + 4x + 2$

81. vertex = $(2, 6)$ point = $(0, 4)$

$y = a(x - 2)^2 + 6$

$4 = a(0 - 2)^2 + 6$

$4 = a(4) + 6$

$-2 = a(4)$

$-\frac{2}{4} = a$

$y = -\frac{1}{2}(x - 2)^2 + 6$

$y = -\frac{1}{2}(x^2 - 4x + 4) + 6$

$y = -\frac{1}{2}x^2 + 2x - 2 + 6$

$y = -\frac{1}{2}x^2 + 2x + 4$

83. vertex = $(2, 1)$ $a = 1$

$y = 1(x - 2)^2 + 1 = x^2 - 4x + 5$

85. vertex = $(2, -4)$ point = $(0, 0)$

$0 = a(0 - 2)^2 - 4$

$4 = a(4)$

$1 = a$

$y = 1(x - 2)^2 - 4 = x^2 - 4x$

87. vertex = $(3, 2)$ point = $(1, 4)$

$4 = a(1 - 3)^2 + 2$

$2 = a(4)$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x - 3)^2 + 2 = \frac{1}{2}x^2 - 3x + \frac{13}{2}$

89. vertex = $(-1, 5)$ point = $(0, 1)$

$1 = a(0 - (-1))^2 + 5$

$1 = a(1) + 5$

$-4 = a$

$y = -4(x + 1)^2 + 5$

$y = -4(x^2 + 2x + 1) + 5$

$y = -4x^2 - 8x - 4 + 5$

$y = -4x^2 - 8x + 1$

91. Horizontal shift 3 units right

93. Horizontal shift 2 units right

Vertical shift 3 units down

95. $y = -\frac{1}{12}x^2 + 2x + 4$

(a) $y = -\frac{1}{12}(0)^2 + 2(0) + 4$

$y = 4$ feet

(b) $y = -\frac{1}{12}x^2 + 2x + 4$

$y = -\frac{1}{12}(x^2 - 24x + 144) + 4 + 12$

$y = -\frac{1}{12}(x - 12)^2 + 16$

Maximum height = 16 feet

(c) $0 = -\frac{1}{12}x^2 + 2x + 4$

$0 = x^2 - 24x - 48$

$x = \dfrac{24 \pm \sqrt{576 + 192}}{2}$

≈ 25.86 feet

97. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 10$

$y = -\frac{4}{9}(x^2 - 6x) + 10$

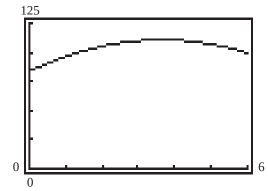
$y = -\frac{4}{9}(x^2 - 6x + 9 - 9) + 10$

$y = -\frac{4}{9}(x^2 - 6x + 9) + 4 + 10$

$y = -\frac{4}{9}(x - 3)^2 + 14$

The maximum height of the diver is 14 ft.

99. (a)



(b) vertex $= (3.65, 110, 810)$1993, 110,800 reserves

101. (a) $P = (100 + x)[90 - x(0.15)] - (100 + x)60$

$P = (100 + x)[90 - x(0.15) - 60]$

$P = (100 + x)(30 - 0.15x)$

$P = 3000 - 15x + 30x = 0.15x^2$

$P = 3000 + 15x - 0.15x^2$

$P = 3000 + 15x - \frac{3}{20}x^2$

(b) $P = -\frac{3}{20}x^2 + 15x + 3000$

$P = -\frac{3}{20}(x^2 - 100x + 2500) + 3000 + 375$

$P = -\frac{3}{20}(x - 50)^2 + 3375$

vertex $= (50, 3375)$

order size for maximum profit

$P = 100 + 50 = 150$ radios

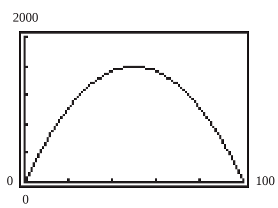
(c) Recommend pricing scheme if price reductions are restricted to orders between 100 and 150 orders.

103. $A = \dfrac{2}{\pi}(100x - x^2)$

Keystrokes:

[Y=] [(] 2 [÷] [π] [)] [(] 100 [X,T,θ] [−] [X,T,θ] [x²] [)] [GRAPH]

$x \approx 50$ when A is maximum



105. $100 = a(500 - 0)^2 + 0$

$100 = a(250{,}000)$

$\dfrac{100}{250{,}000} = a$

$\dfrac{1}{2500} = a$

$y = \dfrac{1}{2500}(x - 0)^2 + 0$

$y = \dfrac{1}{2500}x^2$

107. The graph of the quadratic function $f(x) = ax^2 + bx + c$ is a parabola.

109. To find any x-intercepts, set $y = 0$ and solve the resulting equation for x.

To find the y-intercept, set $x = 0$ and solve the resulting equation for y.

111. The discriminant of a quadratic function tells how many x-intercepts the parabola has. If positive, there are 2 x-intercepts; if zero, 1 x-intercept; and if negative, no x-intercepts.

113. Find the y-coordinate of the vertex. This is the maximum (or minimum) value of a quadratic function.

Mid-Chapter Quiz for Chapter 7

1. $A = kr^2$

2. $z = \dfrac{kx}{y^2}$

3. Distance: $d = rt$

Distance varies jointly proportional to rate and time.