117. The direction of the inequality is reversed, when both sides are multiplied by a negative real number.

119. A polynomial can change signs only at the x-values that make the polynomial zero. The zeros of the polynomial are called the ciritical numbers, and they are used ro determine the test intervals in solving polynomial inequalities.

121. $x^2 + 1 < 0$ is one example of a quadratic inequality that has no real solution. Any inequality of the form $x^2 + c < 0$, c any positive constant or $-x^2 - c > 0$, c any positive constant will not have a real solution.

Review Exercises for Chapter 6

1. $x^2 + 12x = 0$

$x(x + 12) = 0$

$x = 0 \qquad x + 12 = 0$

$x = 0 \qquad x = -12$

3. $4y^2 - 1 = 0$

$(2y - 1)(2y + 1) = 0$

$2y - 1 = 0 \qquad 2y + 1 = 0$

$y = \frac{1}{2} \qquad y = -\frac{1}{2}$

5. $4y^2 + 20y + 25 = 0$

$(2y + 5)(2y + 5) = 0$

$2y + 5 = 0 \qquad 2y + 5 = 0$

$2y = -5 \qquad 2y = -5$

$y = -\frac{5}{2} \qquad y = -\frac{5}{2}$

7. $2x^2 - 2x - 180 = 0$

$2(x^2 - x - 90) = 0$

$2(x - 10)(x + 9) = 0$

$x - 10 = 0 \qquad x + 9 = 0$

$x = 10 \qquad x = -9$

9. $6x^2 - 12x = 4x^2 - 3x + 18$

$2x^2 - 9x - 18 = 0$

$(2x + 3)(x - 6) = 0$

$x = -\frac{3}{2} \qquad x = 6$

11. $4x^2 = 10{,}000$

$x^2 = 2500$

$x = \pm\sqrt{2500}$

$x = \pm 50$

13. $y^2 - 12 = 0$

$y^2 = 12$

$y = \pm\sqrt{12}$

$y = \pm 2\sqrt{3}$

15. $(x - 16)^2 = 400$

$x - 16 = \pm\sqrt{400}$

$x = 16 \pm 20$

$x = 36, -4$

17. $z^2 = -121$

$z = \pm\sqrt{-121}$

$z = \pm 11i$

19. $y^2 + 50 = 0$

$y^2 = -50$

$y = \pm\sqrt{-50}$

$y = \pm 5\sqrt{2}i$

21. $(y + 4)^2 + 18 = 0$

$(y + 4)^2 = -18$

$y + 4 = \pm\sqrt{-18}$

$y = -4 \pm 3\sqrt{2}i$

23. $x^4 - 4x^2 - 5 = 0$

$(x^2 - 5)(x^2 + 1) = 0$

$x^2 + 1 = 0$

$x^2 - 5 = 0 \qquad x^2 = -1$

$x^2 = 5 \qquad x = \pm\sqrt{-1}$

$x = \pm\sqrt{5} \qquad x = \pm i$

25. $x - 4\sqrt{x} + 3 = 0$

$(\sqrt{x} - 3)(\sqrt{x} - 1) = 0$

$(\sqrt{x} - 3) = 0 \qquad (\sqrt{x} - 1) = 0$

$\sqrt{x} = 3 \qquad \sqrt{x} = 1$

$(\sqrt{x})^2 = 3^2 \qquad (\sqrt{x})^2 = 1^2$

$x = 9 \qquad x = 1$

Check:

$9 - 4\sqrt{9} + 3 \stackrel{?}{=} 0$

$9 - 12 + 3 \stackrel{?}{=} 0$

$0 = 0$

Check:

$1 - 4\sqrt{1} + 3 \stackrel{?}{=} 0$

$1 - 4 + 3 \stackrel{?}{=} 0$

$0 = 0$

27. $(x^2 - 2x)^2 - 4(x^2 - 2x) - 5 = 0$

$[(x^2 - 2x) - 5][(x^2 - 2x) + 1] = 0$

$(x^2 - 2x - 5)(x^2 - 2x + 1) = 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4 + 20}}{2}$

$x = \dfrac{2 \pm \sqrt{24}}{2}$

$x = \dfrac{2 \pm 2\sqrt{6}}{2}$ $\quad (x - 1)^2 = 0$

$x = 1 \pm \sqrt{6}$ $\quad x = 1$

29. $x^{2/3} + 3x^{1/3} - 28 = 0$

$(x^{1/3} + 7)(x^{1/3} - 4) = 0$

$x^{1/3} + 7 = 0 \qquad x^{1/3} - 4 = 0$

$x^{1/3} = -7 \qquad x^{1/3} = 4$

$\sqrt[3]{x} = -7 \qquad \sqrt[3]{x} = 4$

$(\sqrt[3]{x})^3 = (-7)^3 \qquad (\sqrt[3]{x})^3 = 4^3$

$x = -343 \qquad x = 64$

31. $x^2 - 6x - 3 = 0$

$x^2 - 6x + 9 = 3 + 9$

$(x - 3)^2 = 12$

$x - 3 = \pm\sqrt{12}$

$x = 3 \pm 2\sqrt{3}$

33. $x^2 - 3x + 3 = 0$

$x^2 - 3x + \dfrac{9}{4} = -3 + \dfrac{9}{4}$

$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{-12 + 9}{4}$

$\left(x - \dfrac{3}{2}\right)^2 = -\dfrac{3}{4}$

$x - \dfrac{3}{2} = \pm\sqrt{-\dfrac{3}{4}}$

$x = \dfrac{3}{2} \pm \dfrac{i\sqrt{3}}{2}$

35. $y^2 - \dfrac{2}{3}y + 2 = 0$

$y^2 - \dfrac{2}{3}y = -2$

$y^2 - \dfrac{2}{3}y + \dfrac{1}{9} = -2 + \dfrac{1}{9}$

$\left(y - \dfrac{1}{3}\right)^2 = \dfrac{-17}{9}$

$y - \dfrac{1}{3} = \pm\sqrt{\dfrac{-17}{9}}$

$y = \dfrac{1}{3} \pm \dfrac{\sqrt{17}i}{3}$

37. $2y^2 + 10y + 3 = 0$

$y^2 + 5y + \dfrac{25}{4} = -\dfrac{3}{2} + \dfrac{25}{4}$

$\left(y + \dfrac{5}{2}\right)^2 = \dfrac{-6 + 25}{4}$

$\left(y + \dfrac{5}{2}\right)^2 = \dfrac{19}{4}$

$y + \dfrac{5}{2} = \pm\sqrt{\dfrac{19}{4}}$

$y = -\dfrac{5}{2} \pm \dfrac{\sqrt{19}}{2}$

39. $y^2 + y - 30 = 0$

$$y = \frac{-1 \pm \sqrt{1^2 - 4(1)(-30)}}{2(1)}$$

$$y = \frac{-1 \pm \sqrt{1 + 120}}{2}$$

$$y = \frac{-1 \pm \sqrt{121}}{2}$$

$$y = \frac{-1 \pm 11}{2}$$

$$y = 5, -6$$

41. $2y^2 + y - 21 = 0$

$$y = \frac{-1 \pm \sqrt{1^2 - 4(2)(-21)}}{2(2)}$$

$$y = \frac{-1 \pm \sqrt{1 + 168}}{4}$$

$$y = \frac{-1 \pm \sqrt{169}}{4}$$

$$y = \frac{-1 \pm 13}{4}$$

$$y = 3, -\frac{7}{2}$$

43. $5x^2 - 16x + 2 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(5)(2)}}{2(5)}$$

$$x = \frac{16 \pm \sqrt{256 - 40}}{10}$$

$$x = \frac{16 \pm \sqrt{216}}{10}$$

$$x = \frac{16 \pm 6\sqrt{6}}{10}$$

$$x = \frac{8 \pm 3\sqrt{6}}{5}$$

45. $0.3t^2 - 2t + 5 = 0$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(0.3)(5)}}{2(0.3)}$$

$$t = \frac{2 \pm \sqrt{4 - 6}}{0.6}$$

$$t = \frac{2 \pm \sqrt{-2}}{0.6}$$

$$t = \frac{2 \pm i\sqrt{2}}{0.6}$$

$$t = \frac{20 \pm 10\sqrt{2}i}{6} = \frac{10}{3} \pm \frac{5\sqrt{2}i}{3}$$

47. $x^2 + 4x + 4 = 0$

$$b^2 - 4ac = 4^2 - 4(1)(4)$$

$$= 16 - 16$$

$$= 0$$

One repeated rational solution.

49. $s^2 - s - 20 = 10$

$$b^2 - 4ac = (-1)^2 - 4(1)(-20)$$

$$= 1 + 80$$

$$= 81$$

Two distinct rational solutions.

51. $3t^2 + 17t + 10 = 0$

$$b^2 - 4ac = 17^2 - 4(3)(10)$$

$$= 289 - 120$$

$$= 169$$

Two distinct rational solutions.

53. $v^2 - 6v + 21 = 0$

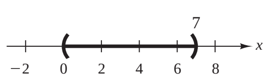
$$b^2 - 4ac = (-6)^2 - 4(1)(21)$$

$$= 36 - 84$$

$$= -48$$

Two distinct imaginary solutions.

55. $5x(7 - x) > 0$



Critical numbers: $x = 0, 7$

Test intervals:

Negative: $(-\infty, 0)$

Positive: $(0, 7)$

Negative: $(7, \infty)$

Solution: $(0, 7)$

57. $16 - (x - 2)^2 \le 0$

$(4 - x + 2)(4 + x - 2) \le 0$

$(6 - x)(2 + x) \le 0$

Critical numbers: $x = -2, 6$

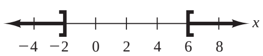
Test intervals:

Negative: $(-\infty, 2]$

Positive: $[-2, 6]$

Negative: $[6, \infty)$

Solution: $(-\infty, -2] \cup [6, \infty)$



59. $2x^2 + 3x - 20 < 0$

$(2x - 5)(x + 4) < 0$

Critical numbers: $x = -4, \frac{5}{2}$

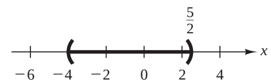
Test intervals:

Positive: $(-\infty, -4)$

Negative: $\left(-4, \frac{5}{2}\right)$

Positive: $\left(\frac{5}{2}, \infty\right)$

Solution: $\left(-4, \frac{5}{2}\right)$



61. $\dfrac{x + 3}{2x - 7} \ge 0$

Critical numbers: $x = -3, \frac{7}{2}$

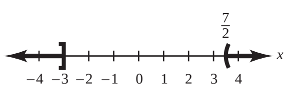
Test intervals:

Positive: $(-\infty, -3]$

Negative: $\left[-3, \frac{7}{2}\right]$

Positive: $\left(\frac{7}{2}, \infty\right)$

Solution: $[-\infty, -3] \cup \left(\frac{7}{2}, \infty\right)$



63. $\dfrac{2x - 2}{x + 6} + 2 < 0$

$\dfrac{2x - 2 + 2(x + 6)}{x + 6} < 0$

$\dfrac{2x - 2 + 2x + 12}{x + 6} < 0$

$\dfrac{4x + 10}{x + 6} < 0$

Critical numbers: $x = -6, -\frac{5}{2}$

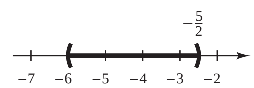
Test intervals:

Positive: $(-\infty, -6)$

Negative: $\left(-6, -\frac{5}{2}\right)$

Positive: $\left(-\frac{5}{2}, \infty\right)$

Solution: $\left(-6, -\frac{5}{2}\right)$



65. *Verbal model:* [Selling price per car] = [Cost per car] + [Profit per car]

Labels: Number cars sold $= x$

Number cars purchased $= x + 4$

Equation:

$$\frac{80{,}000}{x} = \frac{80{,}000}{x + 4} + 1{,}000$$

$$x(x + 4)\left(\frac{80{,}000}{x}\right) = \left(\frac{80{,}000}{x + 4} + 1{,}000\right)x(x + 4)$$

$$80{,}000(x + 4) = 80{,}000x + 1{,}000x(x + 4)$$

$$80{,}000x + 320{,}000 = 80{,}000x + 1{,}000x^2 + 4{,}000x$$

$$0 = 1{,}000x^2 + 4{,}000x - 320{,}000$$

$$0 = x^2 + 4x - 320$$

$$0 = (x + 20)(x - 16)$$

reject $x = -20$ $\quad x = 16$ cars

Average price per car $= \dfrac{80{,}000}{16} = \$5{,}000$

67. *Verbal model:* [Area] = [Length] · [Width]

Labels: Width $= x$

Length $= x + 12$

Equation:

$$108 = (x + 12)x$$
$$0 = x^2 + 12x - 108$$
$$0 = (x + 18)(x - 6)$$

reject $x = -18$ $\quad$ $x = 6$ inches

$x + 12 = 18$ inches

69. Formula: $A = P(1 + r)^2$

$$21{,}424.50 = 20{,}000(1 + r)^2$$
$$1.071225 = (1 + r)^2$$
$$1.035 = 1 + r$$
$$.035 = r \text{ or } 3.5\%$$

71. *Verbal model:* [Cost per person Current Group] − [Cost per person New Group] = [$1.50]

Labels: Number in Current Group $= x$

Number in New Group $= x + 8$

Equation:

$$\frac{360}{x} - \frac{360}{x + 8} = 1.50$$
$$[x(x + 8)]\left(\frac{360}{x} - \frac{360}{x + 8}\right) = (1.50)[x(x + 8)]$$
$$360(x + 8) - 360x = 1.50(x^2 + 8x)$$
$$360x + 2880 - 360x = 1.50x^2 + 12x$$
$$0 = 1.5x^2 + 12x - 2880$$
$$0 = x^2 + 8x - 1920$$
$$0 = (x + 48)(x - 40)$$

$x + 48 = 0$ $\quad$ $x - 40 = 0$

~~$x = -48$~~ $\quad$ $x = 40$

$x + 8 = 48$

73. *Verbal model:* [Cost per ticket] · [Number of tickets] = [$96]

Labels: Number in team $= x$

Number going to game $= x + 3$

Equation:

$$\left(\frac{96}{x} - 1.60\right)(x + 3) = 96$$
$$\left(\frac{96 - 1.60x}{x}\right)(x + 3) = 96$$
$$(96 - 1.6x)(x + 3) = 96x$$
$$96x - 1.6x^2 - 4.8x + 288 = 96x$$
$$1.6x^2 + 4.8x - 288 = 0$$
$$x^2 + 3x - 180 = 0$$
$$(x - 12)(x + 15) = 0$$

$x - 12 = 0$ $\quad$ $x + 15 = 0$

$x = 12$ $\quad$ $x = -15$ reject

$x + 3 = 15$

75. *Formula:* $c^2 = a^2 + b^2$

Labels: $c = 100$

$a = x$

$b = 140 - x$

$a + b = 140$

$x + b = 140$

$b = 140 - x$

Equation:

$$100^2 = x^2 + (140 - x)^2$$
$$10{,}000 = x^2 + 19{,}600 - 280x + x^2$$
$$0 = 2x^2 - 280x + 9{,}600$$
$$0 = x^2 - 140x + 4800$$
$$0 = (x - 60)(x - 80)$$

$x = 60$ $\quad$ $x = 80$

$140 - x = 80$ $\quad$ $140 - x = 60$

60 feet and 80 feet

77. *Verbal model:* $\boxed{\begin{array}{c}\text{Work done}\\ \text{by Person 1}\end{array}} + \boxed{\begin{array}{c}\text{Work done}\\ \text{by Person 2}\end{array}} = \boxed{\text{One complete job}}$

Labels: Time Person 1 $= x$

Labels: Time Person 2 $= x + 2$

Equation:

$$\frac{1}{x}(10) + \frac{1}{x+2}(10) = 1$$

$$x(x+2)\left[10\left(\frac{1}{x} + \frac{1}{x+2}\right)\right] = [1]x(x+2)$$

$$10(x+2) + 10x = x(x+2)$$

$$10x + 20 + 10x = x^2 + 2x$$

$$0 = x^2 - 18x - 20$$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(-20)}}{2(1)}$$

$$x = \frac{18 \pm \sqrt{324 + 80}}{2}$$

$$x = \frac{18 \pm \sqrt{404}}{2}$$

$$x = \frac{18 \pm 2\sqrt{101}}{2}$$

$$x = 9 \pm \sqrt{101}$$

$x \approx 19$ ~~$x \approx -1$~~

$x + 2 \approx 21$

19 hours, 21 hours

79. (a) $256 = -16t^2 + 64t + 192$

$0 = -16t^2 + 64t - 64$

$0 = t^2 - 4t + 4$

$0 = (t - 2)^2$

$t = 2$ seconds

(b) $0 = -16t^2 + 64t + 192$

$0 = -16(t^2 - 4t - 12)$

$0 = -16(t + 2)(t - 6)$

$t + 2 = 0 \qquad t - 6 = 0$

discard $t = -2 \qquad t = 6$ seconds

81. $\overline{C} = \dfrac{C}{x} = \dfrac{50{,}000 + 1.2x}{x} = \dfrac{50{,}000}{x} + 1.2$

$\overline{C} < 5$

$$\frac{50{,}000}{x} + 1.2 < 5$$

$$\frac{50{,}000}{x} - 3.8 < 0$$

$$\frac{50{,}000 - 3.8x}{x} < 0$$

Critical numbers: $x = 0, 13158$

Test intervals:

x must be positive

Positive: $(0, 13{,}158)$

Negative: $(13{,}158, \infty)$

Solution: $(13{,}158, \infty)$

83. $h = -16t^2 + 312t$

$-16t^2 + 312t > 1200$

$-16t^2 + 312t - 1200 > 0$ (Divide by -16)

$t^2 - 19.5t + 75 < 0$

$t = \dfrac{-(-19.5) \pm \sqrt{(-19.5)^2 - 4(1)(75)}}{2(1)}$

$t = \dfrac{19.5 \pm \sqrt{80.25}}{2}$

$t \approx 14.2, 5.3$

Critical numbers: $t = 14.2, 5.3$

Test intervals:

Positive: $(-\infty, 5.3)$

Negative: $(5.3, 14.2)$

Positive: $(14.2, \infty)$

Solution: $(5.3, 14.2)$

$5.3 < t < 14.2$

Chapter Test for Chapter 6

1. $x(x + 5) - 10(x + 5) = 0$

$(x + 5)(x - 10) = 0$

$x + 5 = 0 \qquad x - 10 = 0$

$x = -5 \qquad x = 10$

2. $8x^2 - 21x - 9 = 0$

$(8x + 3)(x - 3) = 0$

$8x + 3 = 0 \qquad x - 3 = 0$

$x = -\frac{3}{8} \qquad x = 3$

3. $(x - 2)^2 = 0.09$

$x - 2 = \pm 0.3$

$x = 2 \pm 0.3$

$x = 2.3, 1.7$

4. $(x + 3)^2 + 81 = 0$

$(x + 3)^2 = -81$

$x + 3 = \pm\sqrt{-81}$

$x = -3 \pm 9i$

5. $2x^2 - 6x + 3 = 0$

$x^2 - 3x + \dfrac{9}{4} = -\dfrac{3}{2} + \dfrac{9}{4}$

$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{-6 + 9}{4}$

$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{3}{4}$

$x - \dfrac{3}{2} = \pm\sqrt{\dfrac{3}{4}}$

$x = \dfrac{3}{2} \pm \dfrac{\sqrt{3}}{2}$

6. $2y(y - 2) = 7$

$2y^2 - 4y - 7 = 0$

$y = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-7)}}{2(2)}$

$y = \dfrac{4 \pm \sqrt{16 + 56}}{4}$

$y = \dfrac{4 \pm \sqrt{72}}{4}$

$y = \dfrac{4 \pm 6\sqrt{2}}{4}$

$y = \dfrac{2 \pm 3\sqrt{2}}{2} \approx 7.41$ and -0.41