# CHAPTER 6 Quadratic Equations and Inequalities

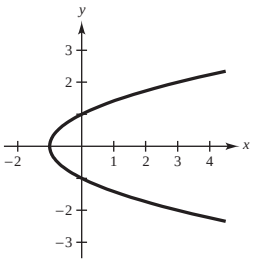
## SECTION 6.1 Factoring and Extracting Square Roots

**1.** The leading coefficient is $-3$ because $-3t^3$ is the term of highest degree.

**2.** $(y^2 - 2)(y^3 + 7) = y^5 + 7y^2 - 2y^3 - 14$

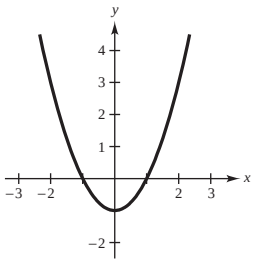
Degree: 5 (the highest power)

**3.**




For some values of $x$ there correspond two values of $y$.

**4.**




For each value of $x$ there corresponds exactly one value of $y$.

**5.** $(x^3 \cdot x^{-2})^{-3} = (x^{3+(-2)})^{-3} = (x^1)^{-3} = x^{-3} = \dfrac{1}{x^3}$

**6.** $(5x^{-4}y^5)(-3x^2y^{-1}) = -15x^{-4+2}y^{5+(-1)}$

$= -15x^{-2}y^4 = \dfrac{-15y^4}{x^2}$

**7.** $\left(\dfrac{2x}{3y}\right)^{-2} = \left(\dfrac{3y}{2x}\right)^2 = \dfrac{9y^2}{4x^2}$

**8.** $\left(\dfrac{7u^{-4}}{3v^{-2}}\right)\left(\dfrac{14u}{6v^2}\right)^{-1} = \left(\dfrac{7u^{-4}}{3v^{-2}}\right)\left(\dfrac{6v^2}{14u}\right)$

$= \dfrac{42u^{-4-1}v^{2-(-2)}}{42} = u^{-5}v^4 = \dfrac{v^4}{u^5}$

**9.** $\dfrac{6u^2v^{-3}}{27uv^3} = \dfrac{2u^{2-1}v^{-3-3}}{9} = \dfrac{2u^1v^{-6}}{9} = \dfrac{2u}{9v^6}$

**10.** $\dfrac{-14r^4s^2}{-98rs^2} = \dfrac{1r^{4-1}s^{2-2}}{7} = \dfrac{r^3s^0}{7} = \dfrac{r^3}{7}$

**11.** $N = \dfrac{k}{\sqrt{t+1}}$

$300 = \dfrac{k}{\sqrt{0+1}}$

$300 = k$

$N = \dfrac{300}{\sqrt{8+1}}$

$N = 100$ prey

**12.** $t = \dfrac{k}{r}$

$2 = \dfrac{k}{58}$

$116 = k$

$t = \dfrac{116}{72}$

$t = \dfrac{29}{18} \approx 1.6$ hours

$k$ measures the distance traveled in $t$ hours at $r$ miles per hour.

## SECTION 6.2 Completing the Square

**1.** $(ab)^4 = a^4b^4$

**2.** $(a^r)^8 = a^{r\cdot 8} = a^{8r}$

**3.** $\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r = \frac{b^r}{a^r},\ a \neq 0, b \neq 0$

**4.** $a^{-r} = \frac{1}{a^r},\quad a \neq 0$

**5.**
$$\frac{4}{x} - \frac{2}{3} = 0$$
$$(3x)\left(\frac{4}{x} - \frac{2}{3}\right) = (0)(3x)$$
$$12 - 2x = 0$$
$$-2x = -12$$
$$x = 6$$

**6.**
$$2x - 3[1 + (4 - x)] = 0$$
$$2x - 3[1 + 4 - x] = 0$$
$$2x - 3[5 - x] = 0$$
$$2x - 15 + 3x = 0$$
$$5x = 15$$
$$x = 3$$

**7.** $3x^2 - 13x - 10 = 0$

$(3x + 2)(x - 5) = 0$

$3x + 2 = 0 \qquad x - 5 = 0$

$x = -\frac{2}{3} \qquad x = 5$

**8.** $x(x - 3) = 40$

$x^2 - 3x - 40 = 0$

$(x - 8)(x + 5) = 0$

$x - 8 = 0 \qquad x + 5 = 0$

$x = 8 \qquad x = -5$

**9.** $g(x) = \frac{2}{3}x - 5$

*y*-intercept:

$g(0) = \frac{2}{3}(0) - 5 = -5$

*x*-intercept:

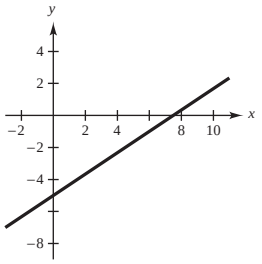
$0 = \frac{2}{3}x - 5$

$0 = 2x - 15$

$15 = 2x$

$\frac{15}{2} = x$

$7.5 = x$




**10.** $h(x) = 5 - \sqrt{x}$

*y*-intercept:

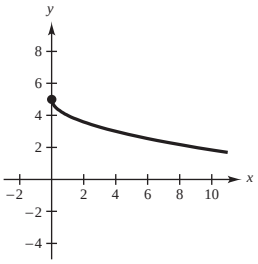
$h(0) = 5 - \sqrt{0} = 5$

*x*-intercept:

$0 = 5 - \sqrt{x}$

$\sqrt{x} = 5$

$x = 5^2 = 25$




**11.** $f(x) = \frac{4}{x + 2}$

*y*-intercept:

$f(0) = \frac{4}{0 + 2} = 2$

*x*-intercept:

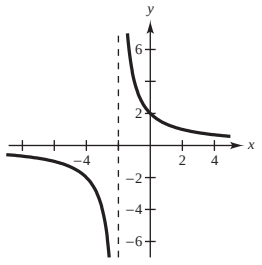
$0 = \frac{4}{x + 2}$

$0 = 4$, none

Vertical asymptote: $x + 2 = 0$

$x = -2$

Horizontal asymptote: $y = 0$ since the degree of the numerator is less than the degree of the denominator.




**12.** $f(x) = 2x + |x - 1|$

*y*-intercept:

$f(0) = 2(0) + |0 - 1| = 1$

*x*-intercept:

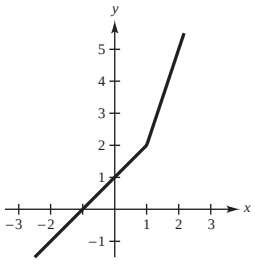
$0 = 2x + |x - 1|$

$-2x = |x - 1|$

$-2x = x - 1 \qquad -2x = -x + 1$

$-3x = -1 \qquad -x = 1$

$x = \frac{1}{3} \qquad x = -1$




**Check:**

$-2\left(\frac{1}{3}\right) \stackrel{?}{=} \left|\frac{1}{3} - 1\right|$

$-\frac{2}{3} \neq \frac{2}{3}$

**Check:**

$-2(-1) = |-1 - 1|$

$2 = |-2|$

$2 = 2$

## SECTION 6.3 The Quadratic Formula

**1.** Multiplication Property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

**2.** Division Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad b \neq 0$

**3.** $\sqrt{72}$ is not in simplest form. A factor (36) of 72 is a perfect square.

$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$

**4.** $10/\sqrt{5}$ is not in simplest form. There is a radical in the denominator which needs to be rationalized.

$\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

**5.** $\sqrt{128} + 3\sqrt{50} = \sqrt{64 \cdot 2} + 3\sqrt{25 \cdot 2}$

$= 8\sqrt{2} + 15\sqrt{2}$

$= 23\sqrt{2}$

**6.** $3\sqrt{5}\sqrt{500} = 3\sqrt{5 \cdot 500} = 3\sqrt{2500} = 3 \cdot 50 = 150$

**7.** $\left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right) = 3^2 - \left(\sqrt{2}\right)^2 = 9 - 2 = 7$

**8.** $\left(3 + \sqrt{2}\right)^2 = 3^2 + 2(3)\sqrt{2} + \left(\sqrt{2}\right)^2$

$= 9 + 6\sqrt{2} + 2$

$= 11 + 6\sqrt{2}$

**9.** $\frac{8}{\sqrt{10}} = \frac{8}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{8\sqrt{10}}{10} = \frac{4\sqrt{10}}{5}$

**10.** $\frac{5}{\sqrt{12} - 2} = \frac{5}{\sqrt{12} - 2} \cdot \frac{\sqrt{12} + 2}{\sqrt{12} + 2}$

$= \frac{5\left(\sqrt{12} + 2\right)}{\left(\sqrt{12}\right)^2 - 2^2}$

$= \frac{5\left(\sqrt{12} + 2\right)}{12 - 4}$

$= \frac{5\left(2\sqrt{3} + 2\right)}{8}$

$= \frac{10\left(\sqrt{3} + 1\right)}{8}$

$= \frac{5\left(\sqrt{3} + 1\right)}{4}$

**11.** *Verbal Model:* $\boxed{\text{Perimeter}} = 2 \cdot \boxed{\text{Length}} + 2 \cdot \boxed{\text{Width}}$

$50 = 2l + 2w$

$25 = l + w$

$25 - w = l$

*Common Formula:* $a^2 + b^2 = c^2$

*Equation:* $(25 - w)^2 + w^2 = \left(5\sqrt{13}\right)^2$

$625 - 50w + w^2 + w^2 = (25)(13)$

$2w^2 - 50w + 300 = 0$

$w^2 - 25w + 150 = 0$

$(w - 10)(w - 15) = 0$

$w = 10 \qquad w = 15$

$25 - w = 15 \qquad 25 - w = 10$

10 inches $\times$ 15 inches

**12.** $p = 75 - \sqrt{1.2(x - 10)}$

$59.90 = 75 - \sqrt{1.2(x - 10)}$

$-15.10 = -\sqrt{1.2(x - 10)}$

$(-15.10)^2 = \left(-\sqrt{1.2(x - 10)}\right)^2$

$228.01 = 1.2(x - 10)$

$190.00 \approx x - 10$

$200 \text{ units} \approx x$

## SECTION 6.4 Applications of Quadratic Equations

**1.** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

**2.** (a) Slope-intercept form: $y = mx + b$

(b) Point-slope form: $y - y_1 = m(x - x_1)$

(c) General form: $Ax + By + C = 0$

(d) Horizontal line: $y - b = 0$

**3.** $(0, 0), (4, -2)$

$$m = \frac{-2 - 0}{4 - 0} = \frac{-2}{4} = \frac{-1}{2}$$

$$y = -\frac{1}{2}x$$

$$2y = -x$$

$$x + 2y = 0$$

**4.** $(0, 0), (100, 75)$

$$m = \frac{75 - 0}{100 - 0} = \frac{75}{100} = \frac{3}{4}$$

$$y - 0 = \frac{3}{4}(x - 0)$$

$$y = \frac{3}{4}x$$

$$4y = 3x$$

$$0 = 3x - 4y$$

**5.** $(-1, -2), (3, 6)$

$$m = \frac{6 - (-2)}{3 - (-1)} = \frac{6 + 2}{3 + 1} = \frac{8}{4} = 2$$

$$y - 6 = 2(x - 3)$$

$$y - 6 = 2x - 6$$

$$0 = 2x - y$$

**6.** $(1, 5), (6, 0)$

$$m = \frac{0 - 5}{6 - 1} = \frac{-5}{5} = -1$$

$$y - 0 = -1(x - 6)$$

$$y = -x + 6$$

$$x + y - 6 = 0$$

**7.** $\left(\frac{3}{2}, 8\right), \left(\frac{11}{2}, \frac{5}{2}\right)$

$$m = \frac{\frac{5}{2} - 8}{\frac{11}{2} - \frac{3}{2}} \cdot \frac{2}{2} = \frac{5 - 16}{11 - 3} = \frac{-11}{8}$$

$$y - 8 = \frac{-11}{8}\left(x - \frac{3}{2}\right)$$

$$y - 8 = \frac{-11}{8}x + \frac{33}{16}$$

$$16y - 128 = -22x + 33$$

$$22x + 16y - 161 = 0$$

**8.** $(0, 2), (7.3, 15.4)$

$$m = \frac{15.4 - 2}{7.3 - 0} = \frac{13.4}{7.3} = \frac{134}{73}$$

$$y - 2 = \frac{134}{73}(x - 0)$$

$$y - 2 = \frac{134}{73}x$$

$$73y - 146 = 134x$$

$$0 = 134x - 73y + 146$$

**9.** $(0, 8), (5, 8)$

$$m = \frac{8 - 8}{5 - 0} = \frac{0}{5} = 0$$

$$y - 8 = 0(x - 0)$$

$$y - 8 = 0$$

**10.** $(-3, 2), (-3, 5)$

$$m = \frac{5 - 2}{(-3) - (-3)} = \frac{3}{0} = \text{undefined}$$

$$x = -3$$

$$x + 3 = 0$$

**11.** *Verbal Model:* $\boxed{\text{Cost per person current group}} - \boxed{\text{Cost per person new group}} = 6250$

*Labels:* Number current group $= x$

Number new group $= x + 2$

*Equation:*
$$\frac{250{,}000}{x} - \frac{250{,}000}{x+2} = 6250$$
$$x(x+2)\left(\frac{250{,}000}{x} - \frac{250{,}000}{x+2}\right) = (6250)x(x+2)$$
$$250{,}000(x+2) - 250{,}000x = 6250(x^2 + 2x)$$
$$250{,}000x + 500{,}000 - 250{,}000x = 6250x^2 + 12{,}500x$$
$$0 = 6250x^2 + 12{,}500x - 500{,}000$$
$$0 = x^2 + 2x - 80$$
$$0 = (x+10)(x-8)$$
$x = -10$ $\quad x = 8$ people

Reject

**12.** *Verbal Model:* $\boxed{\text{Time upstream}} + \boxed{\text{Time downstream}} = \boxed{\text{Total time}}$

*Labels:* Speed of the current $= x$

*Equation:*
$$\frac{35}{18-x} + \frac{35}{18+x} = 4$$
$$(18-x)(18+x)\left(\frac{35}{18-x} + \frac{35}{18+x}\right) = (4)(18-x)(18+x)$$
$$35(18+x) + 35(18-x) = 4(324 - x^2)$$
$$630 + 35x + 630 - 35x = 1296 - 4x^2$$
$$4x^2 - 36 = 0$$
$$4(x^2 - 9) = 0$$
$$x^2 = 9$$
$$x = \pm 3$$

Reject $-3$

$x = 3$ miles per hour

## SECTION 6.5 Quadratic and Rational Inequalities

**1.** $36.82 \times 10^8$ is not written in scientific notation. The number must be between 1 and 10 such as $3.682 \times 10^9$.

**2.** $(n_1 \times 10^2)(n_2 \times 10^4) = n_1 \cdot n_2 \cdot 10^{2+4}$

$= n_1 \cdot n_2 \cdot 10^6$

$1 \le n_1 < 10 \quad \text{and} \quad 1 \le n_2 < 10$

$[1 \cdot 1 \le n_1 \cdot n_2 < 10 \cdot 10]10^6$

$10^6 \le (n_1 \times 10^2)(n_2 \times 10^4) < 10^8$

**3.** $6u^2v - 192v^2 = 6v(u^2 - 32v)$

**4.** $5x^{2/3} - 10x^{1/3} = 5x^{1/3}(x^{1/3} - 2)$

**5.** $x(x - 10) - 4(x - 10) = (x - 4)(x - 10)$

**6.** $x^3 + 3x^2 - 4x - 12 = (x^3 + 3x^2) + (-4x - 12)$

$= x^2(x + 3) - 4(x + 3)$

$= (x + 3)(x^2 - 4)$

$= (x + 3)(x - 2)(x + 2)$

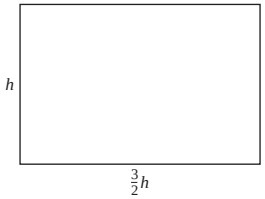
**7.** $16x^2 - 121 = (4x - 11)(4x + 11)$

**8.** $4x^3 - 12x^2 + 16x = 4x(x^2 - 3x + 4)$

**9.** Area = Length · Width

$A = \frac{3}{2}h \cdot h$

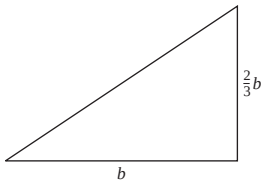
$= \frac{3}{2}h^2$




**10.** Area $= \frac{1}{2} \cdot$ Base · Height

$= \frac{1}{2} \cdot b \cdot \frac{2}{3}b$

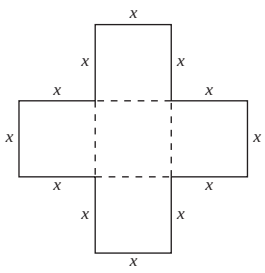
$= \frac{1}{3}b^2$




**11.** Divide figure into 5 congruent squares, each with side length $x$.

Area = 5 · Area of square

$= 5 \cdot x^2$




**12.** Area = Area of rectangle + Area of triangle

$= x \cdot (x + 6) + \frac{1}{2} \cdot x \cdot 4$

$= x^2 + 6x + 2x$

$= x^2 + 8x$