Section 6.4 Applications of Quadratic Equations

1. *Verbal model:* $\boxed{\text{Selling price per doz eggs}} = \boxed{\text{Cost per doz eggs}} + \boxed{\text{Profit per doz eggs}}$

Equation: $\dfrac{21.60}{x} = \dfrac{21.60}{x+6} + 0.30$

Labels: Number eggs sold $= x$

Number eggs purchased $= x + 6$

$$21.60(x+6) = 21.60x + 0.30x(x+6)$$
$$21.6x + 129.6 = 21.6x + 0.3x^2 + 1.8x$$
$$0 = 0.3x^2 + 1.8x - 129.6$$
$$0 = 3x^2 + 18x - 1296$$
$$0 = x^2 + 6x - 432$$
$$0 = (x+24)(x-18)$$
$$x = -24 \quad x = 18 \text{ dozen}$$

Selling price $= \dfrac{21.60}{18} = \$1.20$ per dozen

3. *Verbal model:* $\boxed{\text{Selling price per video}} = \boxed{\text{Cost per video}} + \boxed{\text{Profit per video}}$

Labels: Number videos sold $= x$

Number videos purchased $= x + 8$

Equation: $\dfrac{480}{x} = \dfrac{480}{x+8} + 10$

$$480(x+8) = 480x + 10x(x+8)$$
$$480x + 3840 = 480x + 10x^2 + 80x$$
$$0 = 10x^2 + 80x - 3840$$
$$0 = x^2 + 8x - 384$$
$$0 = (x+24)(x-16)$$
$$x = -24 \quad x = 16 \text{ videos}$$

Selling price $= \dfrac{480}{16} = \$30$

5. *Verbal model:* $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Labels: Length $= l$

Width $= 0.75\,l$

Equation: $2l + 2(0.75l) = 42$

$$2l + 1.5l = 42$$
$$3.5l = 42$$
$$l = 12 \text{ inches}$$
$$w = 0.75\,l = 9 \text{ inches}$$

Verbal model: $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $12 \cdot 9 = A$

108 square inches $= A$

7. *Verbal model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Length $= 2.5w$

Width $= w$

Equation: $250 = 2.5w \cdot w$

$$250 = 2.5w^2$$
$$100 = w^2$$
$$10 = w$$
$$25 = 2.5w$$

Verbal model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(25) + 2(10) = P$

70 feet $= P$

9. *Verbal model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels: Length $= l$

Width $= \frac{1}{3}l$

Equation: $l \cdot \frac{1}{3}l = 192$

$\frac{1}{3}l^2 = 192$

$l^2 = 576$

$l = 24$ inches

$w = \frac{1}{3}l = 8$ inches

Verbal model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(24) + 2(8) = P$

$48 + 16 = P$

$64 \text{ inches} = P$

11. *Verbal model:* $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Labels: Length $= w + 3$

Width $= w$

Equation: $2(w + 3) + 2w = 54$

$2w + 6 + 2w = 54$

$4w = 48$

$w = 12$ km

$l = w + 3 = 15$ km

Verbal model: $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $15 \cdot 12 = 180$ square kilometers $= A$

13. *Verbal model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels: Length $= l$

Width $= l - 20$

Equation: $l \cdot (l - 20) = 12{,}000$

$l^2 - 20l = 12{,}000$

$l^2 - 20l + 100 = 12{,}000 + 100$

$(l - 10)^2 = 12{,}100$

$l - 10 = \pm\sqrt{12{,}100}$

$l = 10 + 110 = 120$ meters

$w = l - 20 = 100$ meters

Verbal model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(120) + 2(100) = 440$ meters $= P$

15. *Verbal model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Length $= x + 4$

Width $= x$

Equation: $192 = (x + 4)x$

$192 = x^2 + 4x$

$0 = x^2 + 4x - 192$

$0 = (x + 16)(x - 12)$

$x = -16 \quad x = 12$ inches

$x + 4 = 16$ inches

17. *Verbal model:* $\boxed{\text{Area}} = \frac{1}{2} \cdot \boxed{\text{Height}} \cdot \boxed{\text{Base}}$

Labels: Height $= x - 8$

Base $= x$

Equation: $192 = \frac{1}{2}(x - 8)x$

$384 = x^2 - 8x$

$0 = x^2 - 8x - 384$

$0 = (x - 24)(x + 16)$

$x = 24$ inches reject $x = -16$

$x - 8 = 16$ inches

19. *Verbal model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels: Length $= 350 - 2x$

Width $= x$

Equation: $(350 - 2x) \cdot x = 12{,}500$

$350x - 2x^2 = 12{,}500$

$2x^2 - 350x + 12{,}500 = 0$

$x^2 - 175x + 6{,}250 = 0$

$$x = \frac{175 \pm \sqrt{175^2 - 4(1)(6,250)}}{2(1)}$$

$$x = \frac{175 \pm \sqrt{5625}}{2} = \frac{175 \pm 75}{2}$$

$x = 125, 50$

$350 - 2x = 100, 250$

100 ft $\times$ 125 ft. or 50 ft $\times$ 250 ft.

21. *Verbal model:* $\boxed{\text{Side 1}} + \boxed{\text{Side 2}} + \boxed{\text{Side 3}} = 550$

Equation:
$$x + x + b = 550$$
$$2x + b = 550$$
$$b = 550 - 2x$$

Verbal model: $\boxed{\frac{1}{2}} \cdot \boxed{\text{Height}}\boxed{(}\boxed{\text{Base 1}} + \boxed{\text{Base 2}}\boxed{)} = \boxed{\text{Area}}$

Labels:
Height $= x$
Base 1 $= x$
Base 2 $= 6$

Equation:
$$\tfrac{1}{2}x(x + b) = 43{,}560$$
$$\tfrac{1}{2}x(x + 550 - 2x) = 43{,}560$$
$$\tfrac{1}{2}x(-x + 550) = 43{,}560$$
$$-\tfrac{1}{2}x^2 + 275x = 43{,}560$$
$$-x^2 + 550x = 87{,}120$$
$$0 = x^2 - 550x + 87{,}120$$

This has no real solution, so it would be impossible to have an area of 43,560 square feet.

23. *Verbal model:* $\boxed{\text{Height}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels:
Height $= x$
Width $= 48 - 2x$

Equation:
$$x \cdot (48 - 2x) = 288$$
$$2x^2 - 48x + 288 = 0$$
$$x^2 - 24x + 144 = 0$$
$$(x - 12)(x - 12) = 0$$
$$x = 12$$
$$\text{height} = 12 \text{ inches}$$
$$\text{width} = 48 - 2(12)$$
$$= 48 - 24 = 24 \text{ inches}$$

25.
$$A = P(1 + r)^2$$
$$3499.20 = 3000(1 + r)^2$$
$$1.1664 = (1 + r)^2$$
$$1.08 = 1 + r$$
$$0.08 = r \text{ or } 8\%$$

27.
$$A = P(1 + r)^2$$
$$280.90 = 250.00(1 + r)^2$$
$$\frac{280.90}{250.00} = (1 + r)^2$$
$$1.1236 = (1 + r)^2$$
$$1.06 = 1 + r$$
$$.06 = r$$
$$6\% = r$$

29.
$$A = P(1 + r)^2$$
$$8420.20 = 8000.00(1 + r)^2$$
$$1.052525 = (1 + r)^2$$
$$1.0259 \approx 1 + r$$
$$.0259 \approx r \text{ or } 2.59\%$$

31. *Verbal model:* $\boxed{\text{Cost per member}} \cdot \boxed{\text{Number of members}} = \boxed{\$240}$

Labels: Number of members $= x$

Number going to game $= x + 8$

Equation:
$$\left(\frac{240}{x} - 1\right) \cdot (x + 8) = 240$$
$$\left(\frac{240 - x}{x}\right)(x + 8) = 240$$
$$(240 - x)(x + 8) = 240x$$
$$240x + 1920 - x^2 - 8x = 240x$$
$$-x^2 - 8x + 1920 = 0$$
$$x^2 + 8x - 1920 = 0$$
$$(x + 48)(x - 40) = 0$$
$$x = -48 \quad x = 40$$
$$x + 8 = 48$$

33. *Verbal model:* $\boxed{\text{Investment per person; current group}} - \boxed{\text{Investment per person; new group}} = \boxed{6000}$

Labels: Number in current group $= x$

Number in new group $= x + 3$

Equation:
$$\frac{80{,}000}{x} - \frac{80{,}000}{x + 3} = 6000$$
$$x(x + 3)\left(\frac{80{,}000}{x} - \frac{80{,}000}{x + 3}\right) = (6000)x(x + 3)$$
$$80{,}000(x + 3) - 80{,}000x = 6000(x^2 + 3x)$$
$$80{,}000x + 240{,}000 - 80{,}000x = 6000x^2 + 18{,}000x$$
$$0 = 6000x^2 + 18{,}000x - 240{,}000$$
$$0 = x^2 + 3x - 40$$
$$0 = (x + 8)(x - 5)$$
$$x + 8 = 0 \qquad x - 5 = 0$$
$$\cancel{x = -8} \qquad x = 5 \text{ investors}$$

35. *Common Formula:* $a^2 + b^2 = c^2$

Equation:
$$x^2 + (18 - x)^2 = 16^2$$
$$x^2 + 324 - 36x + x^2 = 256$$
$$2x^2 - 36x + 68 = 0$$
$$x^2 - 18x + 34 = 0$$
$$x = \frac{18 \pm \sqrt{18^2 - 4(1)(34)}}{2(1)}$$
$$x = \frac{18 \pm \sqrt{324 - 136}}{2} = \frac{18 \pm \sqrt{188}}{2}$$
$$x = 15.855655, \quad \text{reject } 2.1443454$$
$$\approx 15.86 \text{ miles}$$

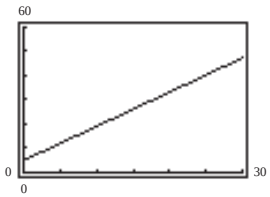
37. (a) $d = \sqrt{(3 + x)^2 + (4 + x)^2}$

Keystrokes:

Y= $\sqrt{\ }$ ((3 + X,T,θ) x^2 + (4 + X,T,θ) x^2) GRAPH

Approximate value of $x \approx 3.55$ when $d = 10$.

(b) $10 = \sqrt{(3 + x)^2 + (4 + x)^2}$



$$100 = (3 + x)^2 + (4 + x)^2$$

$$= 9 + 6x + x^2 + 16 + 8x + x^2$$

$$0 = 2x^2 + 14x - 75$$

$$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(-75)}}{2(2)}$$

$$x = \frac{-14 \pm \sqrt{196 + 600}}{4}$$

$$x = \frac{-14 \pm \sqrt{796}}{4}$$

$$x = \frac{14 \pm 2\sqrt{199}}{4}$$

$$x = \frac{-7 \pm \sqrt{199}}{2} \approx 3.55 \text{ meters}$$

39. *Verbal model:* [Work done by Person 1] + [Work done by Person 2] = [One complete job]

Labels: Time to do job by Person 1 = x

Time to do job by Person 2 = $x + 2$

Equation:

$$\frac{1}{x}(5) + \frac{1}{x + 2}(5) = 1$$

$$x(x + 2)\left[(5)\left(\frac{1}{x} + \frac{1}{x + 2}\right) = 1\right]x(x + 2)$$

$$5(x + 2) + 5x = x(x + 2)$$

$$5x + 10 + 5x = x^2 + 2x$$

$$-x^2 + 8x + 10 = 0$$

$$x^2 - 8x - 10 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 + 40}}{2}$$

$$x = \frac{8 \pm \sqrt{104}}{2}$$

$x \approx 9.1$ hours, reject -1.1

$x + 2 \approx 11.1$ hours

41. *Verbal model:* [Rate Company] + [Rate Company] = [Rate together]

Labels: Time Company A $= x + 3$

Time Company B $= x$

Equation: $\dfrac{1}{x+3} + \dfrac{1}{x} = \dfrac{1}{4}$

$$4x(x+3)\left(\frac{1}{x+3} + \frac{1}{x}\right) = \left(\frac{1}{4}\right)4x(x+3)$$

$$4x + 4(x+3) = x(x+3)$$

$$4x + 4x + 12 = x^2 + 3x$$

$$0 = x^2 - 5x - 12$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 48}}{2}$$

$$x = \frac{5 \pm \sqrt{73}}{2}$$

$x \approx 6.8$ days ~~$x = -1.8$~~

$x + 3 \approx 9.8$

43. $h = h_0 - 16t^2$

$0 = 144 - 16t^2$

$16t^2 = 144$

$t^2 = 9$

$t = 3$ seconds

45. $h = h_0 - 16t^2$

$0 = 1454 - 16t^2$

$16t^2 = 1454$

$t^2 = 90.875$

$t = 9.532838$ seconds ≈ 9.5 seconds

47. $h = 3 + 75t - 16t^2$

$0 = 3 + 75t - 16t^2$

$0 = 16t^2 - 75t - 3$

$$t = \frac{75 \pm \sqrt{(-75)^2 - 4(16)(-3)}}{2(16)}$$

$$t = \frac{75 \pm \sqrt{5625 + 192}}{32}$$

$$t = \frac{75 \pm \sqrt{5817}}{32}$$

$$t = \frac{75 \pm 76.26926}{32}$$

$t = 4.7271644$, reject -0.0396644

≈ 4.7 seconds

49. (a) $336 = -16t^2 + 160t$

$0 = -16t^2 + 160t - 336$

$0 = t^2 - 10t + 21$

$0 = (t-7)(t-3)$

at 3 seconds and at 7 seconds

(b) $0 = -16t^2 + 160t$

$0 = -16t(t - 10)$

$t = 0, 10$

after 10 seconds.

51. *Verbal model:* [Integer] · [Integer] = [Product]

Labels: First integer $= n$

Second integer $= n + 1$

Equation: $n \cdot (n + 1) = 240$

$$n^2 + n + \frac{1}{4} = 240 + \frac{1}{4}$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{960 + 1}{4}$$

$$n + \frac{1}{2} = \pm\sqrt{\frac{961}{4}}$$

$$n = -\frac{1}{2} \pm \frac{\sqrt{961}}{2}$$

$$n = \frac{-1 \pm 31}{2}$$

$n = 15$	$n = -16$
$n + 1 = 16$	$n + 1 = 2 - 15$

} reject

53. *Verbal model:* [Even integer] · [Even integer] = [Product]

Labels: First even integer $= 2n$

Second even integer $= 2n + 2$

Equation: $2n \cdot (2n + 2) = 224$

$$4n^2 + 4n = 224$$

$$n^2 + n = 56$$

$$n^2 + n - 56 = 0$$

$$(n + 8)(n - 7) = 0$$

$n + 8 = 0$	$n - 7 = 0$
$n = -8$	$n = 7$
$2n = -16$	$2n = 14$
$2n + 2 = -14$	$2n + 2 = 16$

reject

55. *Verbal model:* [Odd integer] · [Odd integer] = [Product]

Labels: First odd integer $= 2n + 1$

Second odd integer $= 2n + 3$

Equation: $(2n + 1) \cdot (2n + 3) = 483$

$$4n^2 + 8n + 3 = 483$$

$$4n^2 + 8n - 480 = 0$$

$$n^2 + 2n - 120 = 0$$

$$(n + 12)(n - 10) = 0$$

$n + 12 = 0$	$n - 10 = 0$
~~$n = -12$~~	$n = 10$
	$2n + 1 = 21$
	$2n + 3 = 23$

57. *Verbal model:* [Original time] = [New time] + $\left[\frac{1}{5}\right]$

Labels: Speed $= x$

Increased speed $= x + 40$

Equation: $$\frac{720}{x} = \frac{720}{x + 40} + \frac{1}{5}$$

$$720(5)(x + 40) = 720(5x) + x(x + 40)$$

$$3600x + 144{,}000 = 3600x + x^2 + 40x$$

$$0 = x^2 + 40x - 144{,}000$$

$$x = \frac{-40 \pm \sqrt{40^2 - 4(1)(-144{,}000)}}{2(1)}$$

$$x = \frac{40 \pm \sqrt{1600 + 576{,}000}}{2}$$

$$x = \frac{-40 \pm 760}{2}$$

$$x = 360, \text{ ~~400~~}$$

$$x + 40 = 400 \text{ miles per hour}$$

59. *Verbal model:* Total Cost = Wage Cost + Fuel Cost

Label: Time $= x$

Equation: $20.39 = 5x + x\left[\dfrac{\left(\dfrac{110}{x}\right)^2}{600}\right]$

$$20.39 = 5x + \frac{121}{6x}$$

$$122.34x = 30x^2 + 121$$

$$0 = 30x^2 - 122.34x + 121$$

$$x = \frac{-(-122.34) \pm \sqrt{(-122.34)^2 - 4(30)(121)}}{2(30)}$$

$$x = \frac{122.34 \pm \sqrt{477.0756}}{60}$$

$$x \approx 2.39, 1.67$$

$$v = \frac{110}{2.39} \approx 46 \text{ mi/hr}$$

or

$$v = \frac{110}{1.67} \approx 65 \text{ mi/hr}$$

61. (a) $a + b = 20$ $\qquad A = \pi ab$

$b = 20 - a$ $\qquad A = \pi a(20 - a)$

(b)

a	4	7	10	13	16
A	201.1	285.9	314.2	285.9	201.1

$A = \pi(4)(20 - 4)$
$= \pi(4)(16)$
$= 64\pi$
≈ 201.1

$A = \pi(7)(20 - 7)$
$= \pi(7)(13)$
$= 91\pi$
≈ 285.9

$A = \pi(10)(20 - 10)$
$= \pi(10)(10)$
$= 100\pi$
≈ 314.2

$A = \pi(13)(20 - 13)$
$= \pi(13)(7)$
$= 91\pi$
≈ 285.9

$A = \pi(16)(20 - 16)$
$= \pi(16)(4)$
$= 64\pi$
≈ 201.1

(c) $300 = \pi a(20 - a)$

$0 = 20\pi a - \pi a^2 - 300$

$0 = \pi a^2 - 20\pi a + 300$

$$a = \frac{-(-20\pi) \pm \sqrt{(-20\pi)^2 - 4(\pi)(300)}}{2(\pi)}$$

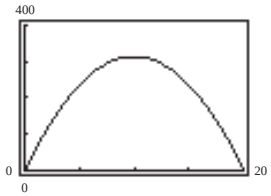
$$a = \frac{20\pi \pm \sqrt{177.9305761}}{2\pi}$$

$a \approx 12.1, 7.9$

(d) $A = \pi a(20 - a)$

Keystrokes:

Y= π X,T,θ (20 − X,T,θ) GRAPH



63. Guidelines for solving word problems:

(a) Write a verbal model that will describe what you need to know.

(b) Assign labels to each part of the verbal model—numbers to the known quantities and letters to the variable quantities.

(c) Use the labels to write an algebraic model based on the verbal model.

(d) Solve the resulting algebraic equation and check your solution.

65. Unit Analysis

$$\frac{9 \text{ dollars}}{\cancel{\text{hour}}} \cdot (20 \cancel{\text{hours}}) = 180 \text{ dollars}$$

67. An example of a quadratic equation that has only one repeated solution is $(x + 4)^2 = 0$. Any equation of the form $(x - c)^2 = 0$, where c is a constant will have only one repeated solution.

Section 6.5 Quadratic and Rational Inequalities

1. $x(2x - 5) = 0$

$x = 0 \quad 2x - 5 = 0$

$x = \frac{5}{2}$

Critical numbers $= 0, \frac{5}{2}$

3. $4x^2 - 81 = 0$

$x^2 = \frac{81}{4}$

$x = \pm\frac{9}{2}$

Critical numbers: $\frac{9}{2}, -\frac{9}{2}$

5. $x(x + 3) - 5(x + 3) = 0$

$(x - 5)(x + 3) = 0$

$x = 5 \quad x = -3$

Critical numbers: $5, -3$

7. $x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3 \quad x = 1$

Critical numbers $= 3, 1$

9. $4x^2 - 20x + 25 = 0$

$(2x - 5)^2 = 0$

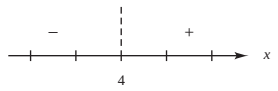
$2x - 5 = 0$

$x = \frac{5}{2}$

Critical number: $\frac{5}{2}$

11. Negative: $(-\infty, 4)$

Positive: $(4, \infty)$



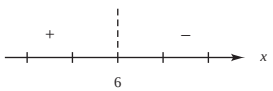
Choose a test value from each interval.

$(-\infty, 4) \Rightarrow x = 0 \Rightarrow 0 - 4 = -4 < 0$

$(4, \infty) \Rightarrow x = 5 \Rightarrow 5 - 4 = 1 > 0$

13. Negative: $(6, \infty)$

Positive: $(-\infty, 6)$



Choose a test value from each interval.

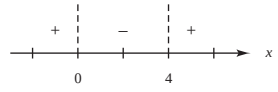
$(-\infty, 6) \Rightarrow x = 0 \Rightarrow 3 - \frac{1}{2}(0) = 3 > 0$

$(6, \infty) \Rightarrow x = 8 \Rightarrow 3 - \frac{1}{2}(8) = -1 < 0$

15. Positive: $(-\infty, 0)$

Negative: $(0, 4)$

Positive: $(4, \infty)$



Choose a test value from each interval.

$(-\infty, 0) \Rightarrow x = -1 \Rightarrow 2(-1)(-1 - 4) = 10 > 0$

$(0, 4) \Rightarrow x = 1 \Rightarrow 2(1)(1 - 4) = -6 < 0$

$(4, \infty) \Rightarrow x = 5 \Rightarrow 2(5)(5 - 4) = 10 > 0$