

135. $892 = (26.6 + t)^2$

$$\sqrt{892} = 26.6 + t$$

$$\sqrt{892} - 26.6 = t$$

$$3 \approx t$$

Year 1993

137. (a) $h_0 = 100$ feet $v_0 = 0$ feet/sec $h = 0$

$$0 = 16t^2 + 0 \cdot t + 100$$

$$16t^2 = 100$$

$$t^2 = 6.25$$

$$t = \sqrt{6.25}$$

$$t = 2.5 \text{ seconds}$$

Extracting the roots method was used because the quadratic equation did not have a linear term.

(b) $h_0 = 100$ feet $v_0 = 32$ feet/sec $h = 100$ feet

$$100 = -16t^2 + 32t + 100$$

$$0 = -16t^2 + 32t$$

$$0 = -16t(t - 2)$$

$$-16t = 0 \quad t - 2 = 0$$

$$t = 0 \text{ seconds} \quad t = 2 \text{ seconds}$$

Factoring method was used because the quadratic equation did not have a constant term.

139. Factoring and the Zero-Factor Property allow you to solve a quadratic equation by converting it into two linear equations that you already know how to solve.

143. To solve an equation of quadratic form, determine an algebraic expression u such that substitution yields the quadratic equation $au^2 + bu + c = 0$. Solve this quadratic equation for u and then, through back-substitution, find the solution of the original equation.141. False. The solutions are $x = 5$ and $x = -5$.

Section 6.2 Completing the Square

1. $x^2 + 8x + 16 \quad \left[16 = \left(\frac{8}{2}\right)^2\right]$

3. $y^2 - 20y + 100 \quad \left[100 = \left(-\frac{20}{2}\right)^2\right]$

5. $x^2 - 16x + 64 \quad \left[64 = \left(-\frac{16}{2}\right)^2\right]$

7. $t^2 + 5t + \frac{25}{4} \quad \left[\frac{25}{4} = \left(\frac{5}{2}\right)^2\right]$

9. $x^2 - 9x + \frac{81}{4} \quad \left[\frac{81}{4} = \left(-\frac{9}{2}\right)^2\right]$

11. $a^2 - \frac{1}{3}a + \frac{1}{36} \quad \left[\frac{1}{36} = \left[-\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\right]^2\right]$

13. $y^2 - \frac{3}{5}y + \frac{9}{100} \quad \left[\frac{9}{100} = \left[-\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\right]^2\right]$

15. $r^2 - 0.4r + 0.04 \quad \left[0.04 = \left(-\frac{0.4}{2}\right)^2\right]$

17. (a) $x^2 - 20x + 100 = 100$

$$(x - 10)^2 = 100$$

$$x - 10 = \pm 10$$

$$x = 10 \pm 10$$

$$x = 20, 0$$

(b) $x^2 - 20x = 0$

$$x(x - 20) = 0$$

$$x = 0 \quad x = 20$$

19. (a) $x^2 + 6x + 9 = 0 + 9$

$$(x + 3)^2 = 9$$

$$x + 3 = \pm 3$$

$$x = -3 \pm 3$$

$$x = -6, 0$$

(b) $x^2 + 6x = 0$

$$x(x + 6) = 0$$

$$x = 0 \quad x + 6 = 0$$

$$x = -6, 0$$

21. (a) $y^2 - 5y = 0$

$$y^2 - 5y + \frac{25}{4} = \frac{25}{4}$$

$$\left(y - \frac{5}{2}\right)^2 = \frac{25}{4}$$

$$y - \frac{5}{2} = \pm\frac{5}{2}$$

$$y = \frac{5}{2} \pm \frac{5}{2}$$

$$= 0, 5$$

(b) $y^2 - 5y = 0$

$$y(y - 5) = 0$$

$$y = 0 \quad y - 5 = 0$$

$$y = 5$$

25. (a) $x^2 + 2x + 1 = 24 + 1$

$$(x + 1)^2 = 25$$

$$x + 1 = \pm 5$$

$$x = -1 \pm 5$$

$$x = 4, -6$$

(b) $x^2 + 2x - 24 = 0$

$$(x + 6)(x - 4) = 0$$

$$x = -6 \quad x = 4$$

29. (a) $x^2 - 3x + \frac{9}{4} = 18 + \frac{9}{4}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{81}{4}$$

$$x - \frac{3}{2} = \pm\frac{9}{2}$$

$$x = \frac{3}{2} \pm \frac{9}{2}$$

$$x = \frac{12}{2}, -\frac{6}{2}$$

$$x = 6, -3$$

(b) $x^2 - 3x - 18 = 0$

$$(x - 6)(x + 3) = 0$$

$$x = 6 \quad x = -3$$

23. (a) $t^2 - 8t + 16 = -7 + 16$

$$(t - 4)^2 = 9$$

$$t - 4 = \pm 3$$

$$t = 4 \pm 3$$

$$t = 7, 1$$

(b) $t^2 - 8t + 7 = 0$

$$(t - 7)(t - 1) = 0$$

$$t = 7 \quad t = 1$$

27. (a) $x^2 + 7x + \frac{49}{4} = -12 + \frac{49}{4}$

$$\left(x + \frac{7}{2}\right)^2 = \frac{1}{4}$$

$$x + \frac{7}{2} = \pm\frac{1}{2}$$

$$x = -\frac{7}{2} \pm \frac{1}{2}$$

$$x = -\frac{6}{2}, -\frac{8}{2}$$

$$x = -3, -4$$

(b) $x^2 + 7x + 12 = 0$

$$(x + 4)(x + 3) = 0$$

$$x = -4 \quad x = -3$$

31. (a) $2x^2 - 14x + 12 = 0$

$$x^2 - 7x + 6 = 0$$

$$x^2 - 7x = -6$$

$$x^2 - 7x + \frac{49}{4} = -6 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = -\frac{24}{4} + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{25}{4}$$

$$x - \frac{7}{2} = \pm\frac{5}{2}$$

$$x = \frac{7}{2} \pm \frac{5}{2}$$

$$x = \frac{12}{2}, \frac{2}{2}$$

$$x = 6, 1$$

(b) $2x^2 - 14x + 12 = 0$

$$x^2 - 7x + 6 = 0$$

$$(x - 6)(x - 1) = 0$$

$$x = 6 \quad x = 1$$

33. (a) $4x^2 + 4x - 15 = 0$

$$x^2 + x - \frac{15}{4} = 0$$

$$x^2 + x = \frac{15}{4}$$

$$x^2 + x + \frac{1}{4} = \frac{15}{4} + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{16}{4}$$

$$x + \frac{1}{2} = \pm\sqrt{4}$$

$$x = -\frac{1}{2} \pm 2$$

$$x = \frac{3}{2}, -\frac{5}{2}$$

(b) $4x^2 + 4x - 15 = 0$

$$(2x - 3)(2x + 5) = 0$$

$$x = \frac{3}{2} \quad x = -\frac{5}{2}$$

35. $x^2 - 4x - 3 = 0$

$$x^2 - 4x + 4 = 3 + 4$$

$$(x - 2)^2 = 7$$

$$x - 2 = \pm\sqrt{7}$$

$$x = 2 \pm \sqrt{7}$$

$$x \approx 4.65, -0.65$$

37. $x^2 + 4x - 3 = 0$

$$x^2 + 4x + 4 = 3 + 4$$

$$(x + 2)^2 = 7$$

$$x + 2 = \pm\sqrt{7}$$

$$x = -2 \pm \sqrt{7}$$

$$x \approx 0.65, -4.65$$

39. $u^2 - 4u + 1 = 0$

$$u^2 - 4u + 4 = -1 + 4$$

$$(u - 2)^2 = 3$$

$$u - 2 = \pm\sqrt{3}$$

$$u = 2 \pm \sqrt{3}$$

$$u \approx 3.73, 0.27$$

41. $x^2 + 2x + 3 = 0$

$$x^2 + 2x + 1 = -3 + 1$$

$$(x + 1)^2 = -2$$

$$x + 1 = \pm\sqrt{-2}$$

$$x = -1 \pm i\sqrt{2}$$

$$x \approx -1 + 1.41i$$

$$x \approx -1 - 1.41i$$

43. $x^2 - 10x - 2 = 0$

$$x^2 - 10x + 25 = 2 + 25$$

$$(x - 5)^2 = 27$$

$$x - 5 = \pm\sqrt{27}$$

$$x = 5 \pm 3\sqrt{3}$$

$$x \approx 10.20, -0.20$$

45. $y^2 + 20y + 10 = 0$

$$y^2 + 20y + 100 = -10 + 100$$

$$(y + 10)^2 = 90$$

$$y + 10 = \pm\sqrt{90}$$

$$y = -10 \pm 3\sqrt{10}$$

$$y \approx -0.51, -19.49$$

47. $t^2 + 5t + 3 = 0$

$$t^2 + 5t + \frac{25}{4} = -3 + \frac{25}{4}$$

$$\left(t + \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$t + \frac{5}{2} = \pm\sqrt{\frac{13}{4}}$$

$$t = -\frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

$$t = \frac{-5 \pm \sqrt{13}}{2}$$

$$t \approx -0.70, -4.30$$

49. $v^2 + 3v - 2 = 0$

$$v^2 + 3v + \frac{9}{4} = 2 + \frac{9}{4}$$

$$\left(v + \frac{3}{2}\right)^2 = \frac{17}{4}$$

$$v + \frac{3}{2} = \pm\sqrt{\frac{17}{4}}$$

$$v = -\frac{3}{2} \pm \sqrt{\frac{17}{4}}$$

$$v = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

$$v = \frac{-3 \pm \sqrt{17}}{2}$$

$$v \approx 0.56, -3.56$$

51. $-x^2 + x - 1 = 0$

$$x^2 - x + 1 = 0$$

$$x^2 - x + \frac{1}{4} = -1 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$$

$$x - \frac{1}{2} = \pm \sqrt{-\frac{3}{4}}$$

$$x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

$$x \approx 0.5 + 0.87i$$

$$x \approx 0.5 - 0.87i$$

55. $x^2 - \frac{2}{3}x - 3 = 0$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = 3 + \frac{1}{9}$$

$$\left(x - \frac{1}{3}\right)^2 = \frac{28}{9}$$

$$x - \frac{1}{3} = \pm \sqrt{\frac{28}{9}}$$

$$x = \frac{1}{3} \pm \frac{2}{3}\sqrt{7}$$

$$x = \frac{1 \pm 2\sqrt{7}}{3}$$

$$x \approx 2.10, -1.43$$

59. $2x^2 + 8x + 3 = 0$

$$x^2 + 4x + 4 = -\frac{3}{2} + 4$$

$$(x + 2)^2 = \frac{5}{2}$$

$$x + 2 = \pm \sqrt{\frac{5}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = -2 \pm \frac{\sqrt{10}}{2}$$

$$x \approx -0.42, -3.58$$

53. $x^2 - 7x + 12 = 0$

$$x^2 - 7x + \frac{49}{4} = -12 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{-48}{4} + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{1}{4}$$

$$x - \frac{7}{2} = \pm \sqrt{\frac{1}{4}}$$

$$x = \frac{7}{2} \pm \frac{1}{2}$$

$$x = 4, 3$$

57. $v^2 + \frac{3}{4}v - 2 = 0$

$$v^2 + \frac{3}{4}v + \frac{9}{64} = 2 + \frac{9}{64}$$

$$\left(v + \frac{3}{8}\right)^2 = \frac{128}{64} + \frac{9}{64}$$

$$\left(v + \frac{3}{8}\right)^2 = \frac{137}{64}$$

$$v + \frac{3}{8} = \pm \sqrt{\frac{137}{64}}$$

$$v = -\frac{3}{8} \pm \frac{\sqrt{137}}{8}$$

$$v \approx 1.09, -1.84$$

61. $3x^2 + 9x + 5 = 0$

$$x^2 + 3x + \frac{9}{4} = -\frac{5}{3} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{-20 + 27}{12}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{7}{12}$$

$$x + \frac{3}{2} = \pm \sqrt{\frac{7}{12}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{21}}{6}$$

$$x = \frac{-9 \pm \sqrt{21}}{6}$$

$$x \approx -0.74, -2.26$$

63. $4y^2 + 4y - 9 = 0$

$$y^2 + y + \frac{1}{4} = \frac{9}{4} + \frac{1}{4}$$

$$\left(y + \frac{1}{2}\right)^2 = \frac{10}{4}$$

$$y + \frac{1}{2} = \pm \sqrt{\frac{10}{4}}$$

$$y = -\frac{1}{2} \pm \frac{\sqrt{10}}{2}$$

$$y = \frac{-1 \pm \sqrt{10}}{2}$$

$$y \approx 1.08, -2.08$$

65. $5x^2 - 3x + 10 = 0$

$$x^2 - \frac{3}{5}x = -2$$

$$x^2 - \frac{3}{5}x + \frac{9}{100} = -2 + \frac{9}{100}$$

$$\left(x - \frac{3}{10}\right)^2 = \frac{-200}{100} + \frac{9}{100}$$

$$\left(x - \frac{3}{10}\right)^2 = -\frac{191}{100}$$

$$x - \frac{3}{10} = \pm \sqrt{-\frac{191}{100}}$$

$$x = \frac{3}{10} \pm \frac{\sqrt{191}}{10}i$$

$$x \approx 0.30 + 1.38i, 0.30 - 1.38i$$

67. $x(x - 7) = 2$

$$x^2 - 7x + \frac{49}{4} = 2 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{8 + 49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{57}{4}$$

$$x - \frac{7}{2} = \pm \sqrt{\frac{57}{4}}$$

$$x = \frac{7}{2} \pm \frac{\sqrt{57}}{2}$$

$$x = \frac{7 \pm \sqrt{57}}{2}$$

$$x \approx 7.27, -0.27$$

69. $0.5t^2 + t + 2 = 0$

$$t^2 + 2t = -4$$

$$t^2 + 2t + 1 = -4 + 1$$

$$(t + 1)^2 = -3$$

$$t + 1 = \pm \sqrt{-3}$$

$$t + 1 = \pm \sqrt{3}i$$

$$t = -1 \pm \sqrt{3}i$$

$$t \approx -1 + 1.73i, -1 - 1.73i$$

71. $0.1x^2 + 0.2x + 0.5 = 0$

$$x^2 + 2x + 5 = 0$$

$$x^2 + 2x + 1 = -5 + 1$$

$$(x + 1)^2 = -4$$

$$x + 1 = \pm \sqrt{-4}$$

$$x = -1 \pm 2i$$

73. $\frac{x}{2} - \frac{1}{x} = 1$

$$2x\left(\frac{x}{2} - \frac{1}{x}\right) = (1)2x$$

$$x^2 - 2 = 2x$$

$$x^2 - 2x + 1 = 2 + 1$$

$$(x - 1)^2 = 3$$

$$x - 1 = \pm \sqrt{3}$$

$$x = 1 \pm \sqrt{3}$$

75.
$$\frac{x^2}{4} = \frac{x+1}{2}$$

$$2x^2 = 4x + 4$$

$$2x^2 - 4x - 4 = 0$$

$$x^2 - 2x - 2 = 0$$

$$x^2 - 2x + 1 = 2 + 1$$

$$(x-1)^2 = 3$$

$$x-1 = \pm\sqrt{3}$$

$$x = 1 \pm \sqrt{3}$$

79. Keystrokes:

$$\boxed{Y=}\boxed{(X,T,\theta)}\boxed{x^2}\boxed{+}\boxed{4}\boxed{(X,T,\theta)}\boxed{-}\boxed{1}\boxed{\text{GRAPH}}$$

$$0 = x^2 + 4x - 1$$

$$1 = x^2 + 4x$$

$$1 + 4 = x^2 + 4x + 4$$

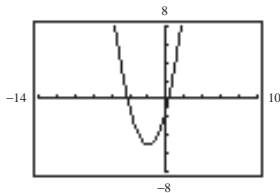
$$5 = (x+2)^2$$

$$\pm\sqrt{5} = x + 2$$

$$-2 \pm \sqrt{5} = x$$

$$x \approx .236$$

$$x \approx -4.236$$



77.
$$\sqrt{2x+1} = x-3$$

$$(\sqrt{2x+1})^2 = (x-3)^2$$

$$2x+1 = x^2 - 6x + 9$$

$$0 = x^2 - 8x + 8$$

$$+16 - 8 = x^2 - 8x + 16$$

$$8 = (x-4)^2$$

$$\pm\sqrt{8} = x - 4$$

$$4 \pm \sqrt{8} = x$$

$$4 \pm 2\sqrt{2} = x$$

81. Keystrokes:

$$\boxed{Y=}\boxed{(X,T,\theta)}\boxed{x^2}\boxed{-}\boxed{2}\boxed{(X,T,\theta)}\boxed{-}\boxed{5}\boxed{\text{GRAPH}}$$

$$0 = x^2 - 2x - 5$$

$$5 = x^2 - 2x$$

$$1 + 5 = x^2 - 2x + 1$$

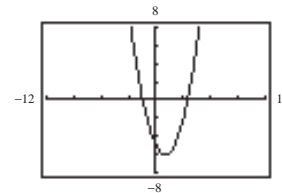
$$6 = (x-1)^2$$

$$\pm\sqrt{6} = x - 1$$

$$1 \pm \sqrt{6} = x$$

$$x \approx 3.449$$

$$x \approx -1.449$$



83. Keystrokes:

$$\boxed{Y=}\boxed{(1)}\boxed{\div}\boxed{3}\boxed{(X,T,\theta)}\boxed{x^2}\boxed{+}\boxed{2}\boxed{(X,T,\theta)}\boxed{-}\boxed{6}\boxed{\text{GRAPH}}$$

$$0 = \frac{1}{3}x^2 + 2x - 6$$

$$0 = x^2 + 6x - 18$$

$$18 = x^2 + 6x$$

$$9 + 18 = x^2 + 6x + 9$$

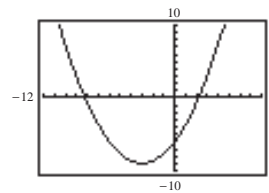
$$27 = (x+3)^2$$

$$\pm\sqrt{27} = x + 3$$

$$-3 \pm 3\sqrt{3} = x$$

$$x \approx 2.20$$

$$x \approx -8.20$$



85. Keystrokes:

 $\boxed{Y=}$ $\boxed{(-)}$ $\boxed{X,T,\theta}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{X,T,\theta}$ $\boxed{+}$ $\boxed{3}$ $\boxed{\text{GRAPH}}$

$$-x^2 - x + 3 = 0$$

$$x^2 + x - 3 = 0$$

$$x^2 + x = 3$$

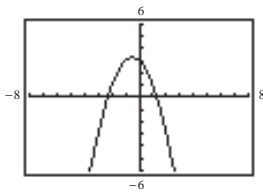
$$x^2 + x + \frac{1}{4} = 3 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{13}{4}$$

$$x + \frac{1}{2} = \pm \frac{\sqrt{13}}{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

$$x \approx 1.30, -2.30$$

89. Verbal model: $\boxed{\text{Area}} = \frac{1}{2} \cdot \boxed{\text{Base}} \cdot \boxed{\text{Height}}$ Labels: Base = x Height = $x + 2$

Equation: $12 = \frac{1}{2}x(x + 2)$

$24 = x^2 + 2x$

$0 = x^2 + 2x - 24$

$0 = (x + 6)(x - 4)$

$x = -6$ $x = 4$ cm base

not a solution $x + 2 = 6$ cm height

87. (a) Area of square = $x \cdot x = x^2$

Area of vertical rectangle = $4 \cdot x = 4x$

Area of horizontal rectangle = $4 \cdot x = 4x$

Total area = $x^2 + 4x + 4x = x^2 + 8x$

(b) Area of small square = $4 \cdot 4 = 16$

Total area = $x^2 + 8x + 16$

(c) $(x + 4)(x + 4) = x^2 + 8x + 16$ 91. Verbal model: $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$ Labels: Length = x

Width = $\frac{200 - 4x}{3}$

Equation: $1400 = 2 \left[x \cdot \left(\frac{200 - 4x}{3} \right) \right]$

$1400 = 2 \left[\frac{200}{3}x - \frac{4x^2}{3} \right]$

$1400 = \frac{400x}{3} - \frac{8x^2}{3}$

$4200 = 400x - 8x^2$

$8x^2 - 400x + 4200 = 0$

$x^2 - 50x + 525 = 0$

$(x - 35)(x - 15) = 0$

$x - 35 = 0$

$x - 15 = 0$

$x = 35$ ft.

$x = 15$ ft.

$\frac{200 - 4x}{3} = 20$ ft.

$\frac{200 - 4x}{3} = 46\frac{2}{3}$ ft.

93. Verbal model: $\boxed{\text{side 1}}^2 + \boxed{\text{side 2}}^2 = \boxed{\text{Hypotenuse}}^2$

Labels: side 1 = x
side 2 = $400 - x$

Equation: $x^2 + (400 - x)^2 = 300^2$
 $x^2 + 160,000 - 2(400)x + x^2 = 90,000$
 $x^2 + 160,000 - 800x + x^2 = 90,000$
 $2x^2 - 800x + 70,000 = 0$
 $x^2 - 400x + 35,000 = 0$
 $x^2 - 400x = -35,000$
 $x^2 - 400x + 40,000 = -35,000 + 40,000$
 $(x - 200)^2 = 5000$
 $x - 200 = \pm\sqrt{5000}$
 $x - 200 = \pm 50\sqrt{2}$
 $x = 200 \pm 50\sqrt{2}$
 $x = 200 \pm 50\sqrt{2}$ meters
 $x \approx 270.71$ meters and 129.29 meters

95. Equation: $12,000 = x(100 - \frac{1}{10}x)$
 $12,000 = 100x - \frac{1}{10}x^2$
 $120,000 = 1000x - x^2$
 $x^2 - 1000x + 120,000 = 0$
 $x^2 - 1000x = -120,000$
 $x^2 - 1000x + 250,000 = -120,000 + 250,000$
 $(x - 500)^2 = 130,000$
 $x - 500 = \pm\sqrt{130,000}$
 $x - 500 = \pm 100\sqrt{13}$
 $x = 500 \pm 100\sqrt{13}$
 $x \approx 860.56, 139.44$

Thus, 139 or 861 units must be sold.

97. $\frac{25}{4}$. Divide the coefficient of the first-degree term by 2, and square the result to obtain $(\frac{5}{2})^2 = \frac{25}{4}$.

99. Yes. $x^2 + 1 = 0$

101. True. Given the solutions $x = r_1$ and $x = r_2$, the quadratic equation can be written as $(x - r_1)(x - r_2) = 0$.