C H A P T E R 6
Quadratic Equations and Inequalities

Section 6.1 Factoring and Extracting Square Roots

Solutions to Odd-Numbered Exercises

1. $x^2 - 12x + 35 = 0$

$(x - 5)(x - 7) = 0$

$x = 5 \qquad x = 7$

3. $x^2 + x - 72 = 0$

$(x + 9)(x - 8) = 0$

$x = -9 \qquad x = 8$

5. $x^2 + 4x = 45$

$x^2 + 4x - 45 = 0$

$(x + 9)(x - 5) = 0$

$x = -9 \qquad x = 5$

7. $x^2 - 12x + 36 = 0$

$(x - 6)(x - 6) = 0$

$x - 6 = 0 \qquad x - 6 = 0$

$x = 6 \qquad x = 6$

9. $9x^2 + 24x + 16 = 0$

$(3x + 4)(3x + 4) = 0$

$3x + 4 = 0 \qquad 3x + 4 = 0$

$3x = -4 \qquad 3x = -4$

$x = -\frac{4}{3} \qquad x = -\frac{4}{3}$

11. $4x^2 - 12x = 0$

$4x(x - 3) = 0$

$4x = 0 \qquad x - 3 = 0$

$x = 0 \qquad x = 3$

13. $u(u - 9) - 12(u - 9) = 0$

$(u - 9)(u - 12) = 0$

$u - 9 = 0 \qquad u - 12 = 0$

$u = 9 \qquad u = 12$

15. $3x(x - 6) - 5(x - 6) = 0$

$(x - 6)(3x - 5) = 0$

$x - 6 = 0 \qquad 3x - 5 = 0$

$x = 6 \qquad x = \frac{5}{3}$

17. $(y - 4)(y - 3) = 6$

$y^2 - 7y + 12 - 6 = 0$

$y^2 - 7y + 6 = 0$

$(y - 6)(y - 1) = 0$

$y - 6 = 0 \qquad y - 1 = 0$

$y = 6 \qquad y = 1$

19. $2x(3x + 2) = 5 - 6x^2$

$6x^2 + 4x = 5 - 6x^2$

$12x^2 + 4x - 5 = 0$

$(6x + 5)(2x - 1) = 0$

$6x + 5 = 0 \qquad 2x - 1 = 0$

$x = -\frac{5}{6} \qquad x = \frac{1}{2}$

21. $x^2 = 64$

$x = \pm\sqrt{64}$

$x = \pm 8$

23. $6x^2 = 54$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$

25. $25x^2 = 16$

$x^2 = \frac{16}{25}$

$x = \pm\sqrt{\frac{16}{25}}$

$x = \pm\frac{4}{5}$

27. $\frac{1}{2}y^2 = 32$

$y^2 = 64$

$y = \pm\sqrt{64}$

$y = \pm 8$

29. $4x^2 - 25 = 0$

$4x^2 = 25$

$x^2 = \frac{25}{4}$

$x = \pm\sqrt{\frac{25}{4}}$

$x = \pm\frac{5}{2}$

31. $4u^2 - 225 = 0$

$u^2 = \frac{225}{4}$

$u = \pm\sqrt{\frac{225}{4}}$

$u = \pm\frac{15}{2}$

33. $(x + 4)^2 = 169$

$x + 4 = \pm\sqrt{169}$

$x = -4 \pm 13$

$x = 9, -17$

35. $(x - 3)^2 = 0.25$

$x - 3 = \pm\sqrt{0.25}$

$x = 3 \pm 0.5$

$x = 3.5, 2.5$

37. $(x-2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

$x = 2 \pm \sqrt{7}$

39. $(2x+1)^2 = 50$

$2x + 1 = \pm\sqrt{50}$

$2x = -1 \pm 5\sqrt{2}$

$x = \dfrac{-1 \pm 5\sqrt{2}}{2}$

41. $(4x-3)^2 - 98 = 0$

$(4x-3)^2 = 98$

$4x - 3 = \pm\sqrt{98}$

$4x = 3 \pm 7\sqrt{2}$

$x = \dfrac{3 \pm 7\sqrt{2}}{4}$

43. $z^2 = -36$

$z = \pm\sqrt{-36}$

$z = \pm 6i$

45. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

47. $9u^2 + 17 = 0$

$9u^2 = -17$

$u = \pm\sqrt{-\dfrac{17}{9}}$

$= \pm i\dfrac{\sqrt{17}}{3}$

49. $(t-3)^2 = -25$

$t - 3 = \pm\sqrt{-25}$

$t = 3 \pm 5i$

51. $(3z+4)^2 + 144 = 0$

$(3z+4)^2 = -144$

$3z + 4 = \pm\sqrt{-144}$

$3z + 4 = \pm 12i$

$3z = -4 \pm 12i$

$z = \dfrac{-4 \pm 12i}{3}$

$z = -\dfrac{4}{3} \pm 4i$

53. $(2x+3)^2 = -54$

$2x + 3 = \pm\sqrt{-54}$

$2x = -3 \pm 3i\sqrt{6}$

$x = -\dfrac{3}{2} \pm \dfrac{3i\sqrt{6}}{2}$

55. $9(x+6)^2 = -121$

$(x+6)^2 = \dfrac{-121}{9}$

$x + 6 = \pm\sqrt{\dfrac{-121}{9}}$

$x = -6 \pm \dfrac{11}{3}i$

57. $(x-1)^2 = -27$

$x - 1 = \pm\sqrt{-27}$

$x = 1 \pm 3i\sqrt{3}$

59. $(x+1)^2 + 0.04 = 0$

$(x+1)^2 = -0.04$

$x + 1 = \pm\sqrt{-0.04}$

$x = -1 \pm 0.2i$

61. $\left(c - \dfrac{2}{3}\right)^2 + \dfrac{1}{9} = 0$

$\left(c - \dfrac{2}{3}\right)^2 = -\dfrac{1}{9}$

$c - \dfrac{2}{3} = \pm\sqrt{-\dfrac{1}{9}}$

$c = \dfrac{2}{3} \pm \dfrac{1}{3}i$

63. $\left(x + \dfrac{7}{3}\right)^2 = -\dfrac{38}{9}$

$x + \dfrac{7}{3} = \pm\sqrt{-\dfrac{38}{9}}$

$x = -\dfrac{7}{3} \pm \dfrac{i}{3}\sqrt{38}$

65. $2x^2 - 5x = 0$

$x(2x-5) = 0$

$x = 0 \qquad 2x - 5 = 0$

$x = \dfrac{5}{2}$

67. $2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$x = \dfrac{3}{2} \qquad x = -4$

69. $x^2 - 900 = 0$

$x^2 = 900$

$x = \pm 30$

71. $x^2 + 900 = 0$

$x^2 = -900$

$x = \pm\sqrt{-900}$

$x = \pm 30i$

73. $\frac{2}{3}x^2 = 6$

$\frac{3}{2} \cdot \frac{2}{3}x^2 = 6 \cdot \frac{3}{2}$

$x^2 = 9$

$x = \pm 3$

75. $(x - 5)^2 - 100 = 0$

$(x - 5)^2 = 100$

$x - 5 = \pm 10$

$x = 15, -5$

77. $(x - 5)^2 + 100 = 0$

let $u = (x - 5)$

$u^2 + 100 = 0$

$(u + 10i)(u - 10i) = 0$

$u + 10i = 0$ $\quad u - 10i = 0$

$u = -10i$ $\quad u = 10i$

$x - 5 = -10i$ $\quad x - 5 = 10i$

$x = 5 - 10i$ $\quad x = 5 + 10i$

79. $(x + 2)^2 + 18 = 0$

$(x + 2)^2 = -18$

$x + 2 = \pm\sqrt{-18}$

$x = -2 \pm 3i\sqrt{2}$

81. *Keystrokes:*

Y= X,T,θ x² − 9 GRAPH

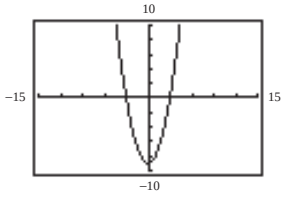
x-intercepts are -3 and 3.

$0 = x^2 - 9$

$= (x - 3)(x + 3)$

$x - 3 = 0$ $\quad x + 3 = 0$

$x = 3$ $\quad x = -3$



83. *Keystrokes:*

Y= X,T,θ x² − 2 X,T,θ − 15 GRAPH

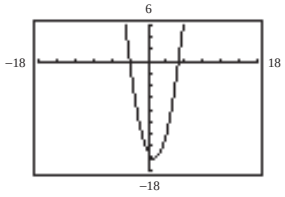
x-intercepts are -3 and 5.

$0 = x^2 - 2x - 15$

$0 = (x - 5)(x + 3)$

$x - 5 = 0$ $\quad x + 3 = 0$

$x = 5$ $\quad x = -3$



85. *Keystrokes:*

Y= 4 − (X,T,θ − 3) x² GRAPH

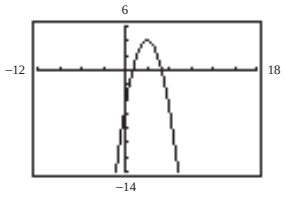
x-intercepts are 1 and 5.

$0 = 4 - (x - 3)^2$

$(x - 3)^2 = 4$

$x - 3 = \pm 2$

$x = 5, 1$



87. *Keystrokes:*

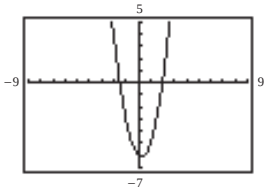
Y= 2 X,T,θ x² − X,T,θ − 6 GRAPH

x-intercepts are $-\frac{3}{2}$ and 2.

$0 = 2x^2 - x - 6$

$0 = (2x + 3)(x - 2)$

$x = -\frac{3}{2} \qquad x = 2$



89. *Keystrokes:*

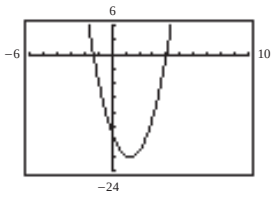
Y= 3 X,T,θ x² − 8 X,T,θ − 16 GRAPH

x-intercepts are $-\frac{4}{3}$ and 4.

$0 = 3x^2 - 8x - 16$

$0 = (3x + 4)(x - 14)$

$x = -\frac{4}{3} \qquad x = 4$



91. *Keystrokes:*

Y= X,T,θ x² + 7 GRAPH

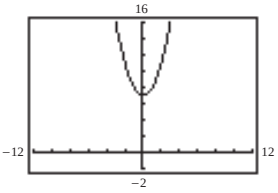
$0 = x^2 + 7$

$-7 = x^2$

$\pm\sqrt{-7} = x$

$\pm i\sqrt{7} = x$

The equation has complex roots.



93. *Keystrokes:*

Y= (X,T,θ − 1) x² + 1 GRAPH

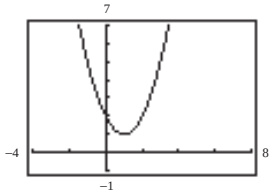
$0 = (x - 1)^2 + 1$

$-1 = (x - 1)^2$

$\pm i = x - 1$

$1 \pm i = x$

The equation has complex roots.



95. *Keystrokes:*

Y= (X,T,θ + 3) x² + 5 GRAPH

$0 = (x + 3)^2 + 5$

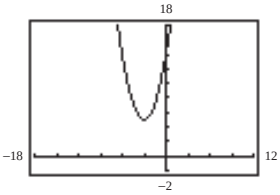
$-5 = (x + 3)^2$

$\pm\sqrt{-5} = x + 3$

$\pm\sqrt{5}i = x + 3$

$-3 \pm \sqrt{5}i = x$

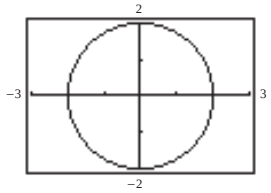
The equation has complex roots.



97. $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

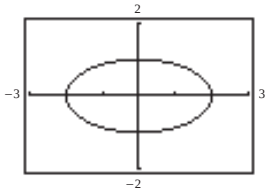


99. $x^2 + 4y^2 = 4$

$4y^2 = 4 - x^2$

$y^2 = \dfrac{4 - x^2}{4}$

$y = \pm\sqrt{\dfrac{4 - x^2}{4}} = \pm\dfrac{\sqrt{4 - x^2}}{2}$



101. $x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$(x - 2)(x + 2)(x - 1)(x + 1) = 0$

$x - 2 = 0 \quad x + 2 = 0 \quad x - 1 = 0 \quad x + 1 = 0$

$x = 2 \quad x = -2 \quad x = 1 \quad x = -1$

103. $x^4 - 5x^2 + 6 = 0$

$(x^2 - 3)(x^2 - 2) = 0$

$x^2 - 3 = 0 \quad x^2 - 2 = 0$

$x^2 = 3 \quad x^2 = 2$

$x = \pm\sqrt{3} \quad x = \pm\sqrt{2}$

105. $x^4 - 3x^2 - 4 = 0$

$(x^2 - 4)(x^2 + 1) = 0$

$x^2 - 4 = 0 \quad x^2 + 1 = 0$

$x^2 = 4 \quad x^2 = -1$

$x = \pm 2 \quad x = \pm i$

107. $(x^2 - 4)^2 + 2(x^2 - 4) - 3 = 0$

$[(x^2 - 4) + 3][(x^2 - 4) - 1] = 0$

$(x^2 - 1)(x^2 - 5) = 0$

$x^2 - 1 = 0 \quad x^2 - 5 = 0$

$x^2 = 1 \quad x^2 = 5$

$x = \pm 1 \quad x = \pm\sqrt{5}$

109. $x - 7\sqrt{x} + 10 = 0$

let $u = \sqrt{x}$

$\left(\sqrt{x}\right)^2 - 7\left(\sqrt{x}\right) + 10 = 0$

$u^2 - 7u + 10 = 0$

$(u - 5)(u - 2) = 0$

$u = 5 \quad u = 2$

$\sqrt{x} = 5 \quad \sqrt{x} = 2$

$x = 25 \quad x = 4$

Check: $25 - 7\sqrt{25} + 10 \stackrel{?}{=} 0$

$25 - 35 + 10 \stackrel{?}{=} 0$

$0 = 0$

Check: $4 - 7\sqrt{4} + 10 \stackrel{?}{=} 0$

$4 - 14 + 10 \stackrel{?}{=} 0$

$0 = 0$

111. $x^{2/3} - x^{1/3} - 6 = 0$

$(x^{1/3} - 3)(x^{1/3} + 2) = 0$

$x^{1/3} - 3 = 0 \quad x^{1/3} + 2 = 0$

$x^{1/3} = 3 \quad x^{1/3} = -2$

$x = 27 \quad x = -8$

113. $2x^{2/3} - 7x^{1/3} + 5 = 0$

let $u = x^{1/3}$

$2(x^{1/3})^2 - 7x^{1/3} + 5 = 0$

$2u^2 - 7u + 5 = 0$

$(2u - 5)(u - 1) = 0$

$2u - 5 = 0 \qquad u - 1 = 0$

$2u = 5 \qquad u = 1$

$u = \frac{5}{2} \qquad x^{1/3} = 1$

$x^{1/3} = \frac{5}{2} \qquad (x^{1/3}) = 1^3$

$(x^{1/3}) = \left(\frac{5}{2}\right)^3 \qquad x = 1$

$x = \frac{125}{8}$

115. $x^{2/5} - 3x^{1/5} + 2 = 0$

$(x^{1/5} - 2)(x^{1/5} - 1) = 0$

$x^{1/5} = 2 \qquad x^{1/5} = 1$

$x = 2^5 \qquad x = 1^5$

$x = 32 \qquad x = 1$

117. $2x^{2/5} - 7x^{1/5} + 3 = 0$

$(2x^{1/5} - 1)(x^{1/5} - 3) = 0$

$x^{1/5} = \frac{1}{2} \qquad x^{1/5} = 3$

$x = \left(\frac{1}{2}\right)^5 \qquad x = 3^5$

$x = \frac{1}{32} \qquad x = 243$

119. $\frac{1}{x^2} - \frac{3}{x} + 2 = 0$

$1 - 3x + 2x^2 = 0$

$2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

$2x - 1 = 0 \qquad x - 1 = 0$

$x = \frac{1}{2} \qquad x = 1$

121. $(x - 5)(x - (-2)) = 0$

$(x - 5)(x + 2) = 0$

$x^2 - 5x + 2x - 10 = 0$

$x^2 - 3x - 10 = 0$

123. $\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$

$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$

$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$

$x^2 - 2x + 1 - 2 = 0$

$x^2 - 2x - 1 = 0$

125. $(x - 5i)(x - (-5i)) = 0$

$(x - 5i)(x + 5i) = 0$

$x^2 - 25i^2 = 0$

$x^2 + 25 = 0$

127. $0 = -16t^2 + 256$

$16t^2 = 256$

$t^2 = 16$

$t = 4$ seconds

129. $0 = -16t^2 + 128$

$16t^2 = 128$

$t^2 = 8$

$t = \pm\sqrt{8}$

$t = \pm 2\sqrt{2}$

$t = 2\sqrt{2} \approx 2.828$ seconds

131. $0 = 144 + 128 - 16^2$

$0 = -16t^2 + 128t + 144$

$0 = -16(t^2 - 8t - 9)$

$0 = -16(t - 9)(t + 1)$

$t - 9 = 0 \qquad t + 1 = 0$

$t = 9$ seconds ~~$t = -1$~~

133. $1685.40 = 1500(1 + r)^2$

$1.1236 = (1 + r)^2$

$1.06 = 1 + r$

$.06 = r$

$6\% = r$

135. $892 = (26.6 + t)^2$

$\sqrt{892} = 26.6 + t$

$\sqrt{892} - 26.6 = t$

$3 \approx t$

Year 1993

137. (a) $h_0 = 100$ feet $v_0 = 0$ feet/sec $h = 0$

$0 = 16t^2 + 0 \cdot t + 100$

$16t^2 = 100$

$t^2 = 6.25$

$t = \sqrt{6.25}$

$t = 2.5$ seconds

Extracting the roots method was used because the quadratic equation did not have a linear term.

(b) $h_0 = 100$ feet $v_0 = 32$ feet/sec $h = 100$ feet

$100 = -16t^2 + 32t + 100$

$0 = -16t^2 + 32t$

$0 = -16t(t - 2)$

$-16t = 0$ $t - 2 = 0$

$t = 0$ seconds $t = 2$ seconds

Factoring method was used because the quadratic equation did not have a constant term.

139. Factoring and the Zero-Factor Property allow you to solve a quadratic equation by converting it into two linear equations that you already know how to solve.

143. To solve an equation of quadratic form, determine an algebraic expression u such that substitution yields the quadratic equation $au^2 + bu + c = 0$. Solve this quadratic equation for u and then, through back-substitution, find the solution of the original equation.

141. False. The solutions are $x = 5$ and $x = -5$.

Section 6.2 Completing the Square

1. $x^2 + 8x + 16$ $\left[16 = \left(\frac{8}{2}\right)^2\right]$

3. $y^2 - 20y + 100$ $\left[100 = \left(-\frac{20}{2}\right)^2\right]$

5. $x^2 - 16x + 64$ $\left[64 = \left(-\frac{16}{2}\right)^2\right]$

7. $t^2 + 5t + \frac{25}{4}$ $\left[\frac{25}{4} = \left(\frac{5}{2}\right)^2\right]$

9. $x^2 - 9x + \frac{81}{4}$ $\left[\frac{81}{4} = \left(-\frac{9}{2}\right)^2\right]$

11. $a^2 - \frac{1}{3}a + \frac{1}{36}$ $\left[\frac{1}{36} = \left[\left(-\frac{1}{3}\right)\left(\frac{1}{2}\right)^2\right]\right]$

13. $y^2 - \frac{3}{5}y + \frac{9}{100}$ $\left[\frac{9}{100} = \left[\left(-\frac{3}{5}\right)\left(\frac{1}{2}\right)^2\right]\right]$

15. $r^2 - 0.4r + 0.04$ $\left[0.04 = \left(-\frac{0.4}{2}\right)^2\right]$

17. (a) $x^2 - 20x + 100 = 100$

$(x - 10)^2 = 100$

$x - 10 = \pm 10$

$x = 10 \pm 10$

$x = 20, 0$

(b) $x^2 - 20x = 0$

$x(x - 20) = 0$

$x = 0$ $x = 20$

19. (a) $x^2 + 6x + 9 = 0 + 9$

$(x + 3)^2 = 9$

$x + 3 = \pm 3$

$x = -3 \pm 3$

$x = -6, 0$

(b) $x^2 + 6x = 0$

$x(x + 6) = 0$

$x = 0$ $x + 6 = 0$

$x = -6, 0$