

Section 5.3 Multiplying and Dividing Radical Expressions

1. $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$
 $= \sqrt{16}$
 $= 4$
3. $\sqrt{3} \cdot \sqrt{6} = \sqrt{3 \cdot 6}$
 $= \sqrt{18}$
 $= \sqrt{9 \cdot 2}$
 $= 3\sqrt{2}$
5. $\sqrt[3]{12} \cdot \sqrt[3]{6} = \sqrt[3]{12 \cdot 6}$
 $= \sqrt[3]{8 \cdot 9}$
 $= 2\sqrt[3]{9}$
7. $\sqrt[4]{8} \cdot \sqrt[4]{6} = \sqrt[4]{8 \cdot 6} = \sqrt[4]{2^4 \cdot 3}$
 $= 2\sqrt[4]{3}$
9. $\sqrt{5}(2 - \sqrt{3}) = 2\sqrt{5} - \sqrt{5}\sqrt{3} = 2\sqrt{5} - \sqrt{15}$
11. $\sqrt{2}(\sqrt{20} + 8) = \sqrt{2}\sqrt{20} + 8\sqrt{2}$
 $= \sqrt{40} + 8\sqrt{2}$
 $= 2\sqrt{10} + 8\sqrt{2}$
13. $\sqrt{6}(\sqrt{12} - \sqrt{3}) = \sqrt{6}\sqrt{12} - \sqrt{6}\sqrt{3}$
 $= \sqrt{72} - \sqrt{18}$
 $= \sqrt{36 \cdot 2} - \sqrt{9 \cdot 2}$
 $= 6\sqrt{2} - 3\sqrt{2}$
 $= 3\sqrt{2}$
15. $\sqrt{2}(\sqrt{18} - \sqrt{10}) = \sqrt{2}\sqrt{18} - \sqrt{2}\sqrt{10}$
 $= \sqrt{36} - \sqrt{20}$
 $= 6 - \sqrt{4 \cdot 5}$
 $= 6 - 2\sqrt{5}$
17. $\sqrt{y}(\sqrt{y} + 4) = (\sqrt{y})^2 + 4\sqrt{y}$
 $= y + 4\sqrt{y}$
19. $\sqrt{a}(4 - \sqrt{a}) = \sqrt{a} \cdot 4 - \sqrt{a}\sqrt{a}$
 $= 4\sqrt{a} - a$
21. $\sqrt[3]{4}(\sqrt[3]{2} - 7) = \sqrt[3]{4}\sqrt[3]{2} - 7\sqrt[3]{4}$
 $= \sqrt[3]{8} - 7\sqrt[3]{4}$
 $= 2 - 7\sqrt[3]{4}$
23. $(\sqrt{3} + 2)(\sqrt{3} - 2) = (\sqrt{3})^2 - 2^2$
 $= 3 - 4$
 $= -1$
25. $(\sqrt{5} + 3)(\sqrt{3} - 5) = \sqrt{15} - 5\sqrt{5} + 3\sqrt{3} - 15$
27. $(\sqrt{20} + 2)^2 = (\sqrt{20})^2 + 2 \cdot \sqrt{20} \cdot 2 + 2^2$
 $= 20 + 4\sqrt{20} + 4$
 $= 24 + 4\sqrt{4 \cdot 5}$
 $= 24 + 8\sqrt{5}$
29. $(\sqrt[3]{6} - 3)(\sqrt[3]{4} + 3) = \sqrt[3]{6}\sqrt[3]{4} + 3\sqrt[3]{6} - 3\sqrt[3]{4} - 9$
 $= \sqrt[3]{24} + 3\sqrt[3]{6} - 3\sqrt[3]{4} - 9$
 $= \sqrt[3]{8 \cdot 3} + 3\sqrt[3]{6} - 3\sqrt[3]{4} - 9$
 $= 2\sqrt[3]{3} + 3\sqrt[3]{6} - 3\sqrt[3]{4} - 9$
31. $(10 + \sqrt{2x})^2 = 10^2 + 2 \cdot 10 \cdot \sqrt{2x} + (\sqrt{2x})^2$
 $= 100 + 20\sqrt{2x} + 2x$
33. $(9\sqrt{x} + 2)(5\sqrt{x} - 3) = (9\sqrt{x})(5\sqrt{x}) - 27\sqrt{x} + 10\sqrt{x} - 6$
 $= 45x - 17\sqrt{x} - 6$
35. $(3\sqrt{x} - 5)(3\sqrt{x} + 5) = (3\sqrt{x})^2 - 5^2$
 $= 9x - 25$
37. $(\sqrt[3]{2x} + 5)^2 = (\sqrt[3]{2x})^2 + 2 \cdot 5\sqrt[3]{2x} + 5^2$
 $= \sqrt[3]{(2x)^2} + 10\sqrt[3]{2x} + 25$
 $= \sqrt[3]{4x^2} + 10\sqrt[3]{2x} + 25$

$$\begin{aligned}
 39. (\sqrt[3]{y} + 2)(\sqrt[3]{y^2} - 5) &= \sqrt[3]{y} \cdot \sqrt[3]{y^2} - 5\sqrt[3]{y} + 2\sqrt[3]{y^2} - 10 \\
 &= \sqrt[3]{y^3} - 5\sqrt[3]{y} + 2\sqrt[3]{y^2} - 10 \\
 &= y - 5\sqrt[3]{y} + 2\sqrt[3]{y^2} - 10
 \end{aligned}$$

$$\begin{aligned}
 41. (\sqrt[3]{t} + 1)(\sqrt[3]{t^2} + 4\sqrt[3]{t} - 3) &= \sqrt[3]{t}\sqrt[3]{t^2} + \sqrt[3]{t} \cdot 4\sqrt[3]{t} - 3\sqrt[3]{t} + \sqrt[3]{t^2} + 4\sqrt[3]{t} - 3 \\
 &= \sqrt[3]{t^3} + 4\sqrt[3]{t^2} - 3\sqrt[3]{t} + \sqrt[3]{t^2} + 4\sqrt[3]{t} - 3 \\
 &= t + 5\sqrt[3]{t^2} + \sqrt[3]{t} - 3
 \end{aligned}$$

$$43. 5x\sqrt{3} + 15\sqrt{3} = 5\sqrt{3}(x + 3)$$

$$\begin{aligned}
 45. 4\sqrt{12} - 2x\sqrt{27} &= 4\sqrt{4 \cdot 3} - 2x\sqrt{9 \cdot 3} \\
 &= 8\sqrt{3} - 6x\sqrt{3} \\
 &= 2\sqrt{3}(4 - 3x)
 \end{aligned}$$

$$\begin{aligned}
 47. 6u^2 + \sqrt{18u^3} &= 6u^2 + \sqrt{9 \cdot 2u^2 \cdot u} \\
 &= 6u^2 + 3u\sqrt{2u} \\
 &= 3u(2u + \sqrt{2u})
 \end{aligned}$$

$$\begin{aligned}
 49. 2 + \sqrt{5}, \text{ conjugate} &= 2 - \sqrt{5} \\
 \text{product} &= (2 + \sqrt{5})(2 - \sqrt{5}) \\
 &= 2^2 - (\sqrt{5})^2 \\
 &= 4 - 5 = -1
 \end{aligned}$$

$$\begin{aligned}
 51. \sqrt{11} - \sqrt{3}, \text{ conjugate} &= \sqrt{11} + \sqrt{3} \\
 \text{product} &= (\sqrt{11} - \sqrt{3})(\sqrt{11} + \sqrt{3}) \\
 &= (\sqrt{11})^2 - (\sqrt{3})^2 \\
 &= 11 - 3 = 8
 \end{aligned}$$

$$\begin{aligned}
 53. \sqrt{15} + 3, \\
 \text{conjugate} &= \sqrt{15} - 3 \\
 \text{product} &= (\sqrt{15} + 3)(\sqrt{15} - 3) \\
 &= \sqrt{15} \cdot \sqrt{15} - 3\sqrt{15} + 3\sqrt{15} - 9 \\
 &= 15 - 9 = 6
 \end{aligned}$$

$$\begin{aligned}
 55. \sqrt{x} - 3, \text{ conjugate} &= \sqrt{x} + 3 \\
 \text{product} &= (\sqrt{x} - 3)(\sqrt{x} + 3) \\
 &= (\sqrt{x})^2 - 3^2 \\
 &= x - 9
 \end{aligned}$$

$$\begin{aligned}
 57. \sqrt{2u} - \sqrt{3}, \text{ conjugate} &= \sqrt{2u} + \sqrt{3} \\
 \text{product} &= (\sqrt{2u} - \sqrt{3})(\sqrt{2u} + \sqrt{3}) \\
 &= (\sqrt{2u})^2 - (\sqrt{3})^2 \\
 &= 2u - 3
 \end{aligned}$$

$$\begin{aligned}
 59. 2\sqrt{2} + \sqrt{4}, \text{ conjugate} &= 2\sqrt{2} - \sqrt{4} \\
 \text{product} &= (2\sqrt{2} + \sqrt{4})(2\sqrt{2} - \sqrt{4}) \\
 &= (2\sqrt{2})^2 - (\sqrt{4})^2 \\
 &= 4 \cdot 2 - 4 \\
 &= 8 - 4 = 4
 \end{aligned}$$

$$\begin{aligned}
 61. \sqrt{x} + \sqrt{y}, \text{ conjugate} &= \sqrt{x} - \sqrt{y} \\
 \text{product} &= (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) \\
 &= (\sqrt{x})^2 - (\sqrt{y})^2 \\
 &= x - y
 \end{aligned}$$

$$\begin{aligned}
 63. \frac{4 - 8\sqrt{x}}{12} &= \frac{4(1 - 2\sqrt{x})}{12} \\
 &= \frac{1 - 2\sqrt{x}}{3}
 \end{aligned}$$

$$\begin{aligned}
 65. \frac{-2y + \sqrt{12y^3}}{8y} &= \frac{-2y + 2y\sqrt{3y}}{8y} \\
 &= \frac{2y(-1 + \sqrt{3y})}{8y} \\
 &= \frac{-1 + \sqrt{3y}}{4}
 \end{aligned}$$

$$\begin{aligned} 67. \text{ (a) } f(2 - \sqrt{3}) &= (2 - \sqrt{3})^2 - 6(2 - \sqrt{3}) + 1 \\ &= 4 - 4\sqrt{3} + 3 - 12 + 6\sqrt{3} + 1 \\ &= 2\sqrt{3} - 4 \end{aligned}$$

$$\begin{aligned} \text{(b) } f(3 - 2\sqrt{2}) &= (3 - 2\sqrt{2})^2 - 6(3 - 2\sqrt{2}) + 1 \\ &= 9 - 12\sqrt{2} + 8 - 18 + 12\sqrt{2} + 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 69. \text{ (a) } f(1 + \sqrt{2}) &= (1 + \sqrt{2})^2 - 2(1 + \sqrt{2}) - 1 \\ &= 1 + 2\sqrt{2} + 2 - 2 - 2\sqrt{2} - 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{(b) } f(\sqrt{4}) &= (\sqrt{4})^2 - 2\sqrt{4} - 1 \\ &= 4 - 4 - 1 \\ &= -1 \end{aligned}$$

$$71. \frac{6}{\sqrt{2} - 2} = \frac{6}{\sqrt{2} - 2} \cdot \frac{\sqrt{2} + 2}{\sqrt{2} + 2} = \frac{6(\sqrt{2} + 2)}{(\sqrt{2})^2 - 2^2} = \frac{6(\sqrt{2} + 2)}{2 - 4} = \frac{6(\sqrt{2} + 2)}{-2} = -3(\sqrt{2} + 2)$$

$$73. \frac{7}{\sqrt{3} + 5} = \frac{7}{\sqrt{3} + 5} \cdot \frac{\sqrt{3} - 5}{\sqrt{3} - 5} = \frac{7(\sqrt{3} - 5)}{(\sqrt{3})^2 - 5^2} = \frac{7(\sqrt{3} - 5)}{3 - 25} = \frac{7(\sqrt{3} - 5)}{-22} = \frac{7(5 - \sqrt{3})}{22}$$

$$\begin{aligned} 75. \frac{3}{2\sqrt{10} - 5} &= \frac{3}{2\sqrt{10} - 5} \cdot \frac{2\sqrt{10} + 5}{2\sqrt{10} + 5} \\ &= \frac{3(2\sqrt{10} + 5)}{(2\sqrt{10})^2 - 5^2} \\ &= \frac{3(2\sqrt{10} + 5)}{40 - 25} \\ &= \frac{3(2\sqrt{10} + 5)}{15} \\ &= \frac{2\sqrt{10} + 5}{5} \end{aligned}$$

$$\begin{aligned} 77. \frac{2}{\sqrt{6} + \sqrt{2}} &= \frac{2}{\sqrt{6} + \sqrt{2}} \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{2(\sqrt{6} - \sqrt{2})}{6 - 2} \\ &= \frac{2(\sqrt{6} - \sqrt{2})}{4} \\ &= \frac{\sqrt{6} - \sqrt{2}}{2} \end{aligned}$$

$$79. \frac{9}{\sqrt{3} - \sqrt{7}} = \frac{9}{\sqrt{3} - \sqrt{7}} \cdot \frac{\sqrt{3} + \sqrt{7}}{\sqrt{3} + \sqrt{7}} = \frac{9(\sqrt{3} + \sqrt{7})}{(\sqrt{3})^2 - (\sqrt{7})^2} = \frac{9(\sqrt{3} + \sqrt{7})}{3 - 7} = \frac{9(\sqrt{3} + \sqrt{7})}{-4} = \frac{-9(\sqrt{3} + \sqrt{7})}{4}$$

$$81. (\sqrt{7} + 2) \div (\sqrt{7} - 2) = \frac{\sqrt{7} + 2}{\sqrt{7} - 2} \cdot \frac{\sqrt{7} + 2}{\sqrt{7} + 2} = \frac{(\sqrt{7})^2 + 2\sqrt{7} + 2\sqrt{7} + 4}{(\sqrt{7})^2 - 2^2} = \frac{7 + 4\sqrt{7} + 4}{7 - 4} = \frac{11 + 4\sqrt{7}}{3}$$

$$83. (\sqrt{x} - 5) \div (2\sqrt{x} - 1) = \frac{\sqrt{x} - 5}{2\sqrt{x} - 1} \cdot \frac{2\sqrt{x} + 1}{2\sqrt{x} + 1} = \frac{2x + \sqrt{x} - 10\sqrt{x} - 5}{(2\sqrt{x})^2 - 1^2} = \frac{2x - 9\sqrt{x} - 5}{4x - 1}$$

$$\begin{aligned} 85. \frac{3x}{\sqrt{15} - \sqrt{3}} &= \frac{3x}{\sqrt{15} - \sqrt{3}} \cdot \frac{\sqrt{15} + \sqrt{3}}{\sqrt{15} + \sqrt{3}} = \frac{3x(\sqrt{15} + \sqrt{3})}{(\sqrt{15})^2 - (\sqrt{3})^2} = \frac{3x(\sqrt{15} + \sqrt{3})}{15 - 3} \\ &= \frac{3x(\sqrt{15} + \sqrt{3})}{12} = \frac{x\sqrt{15} + x\sqrt{3}}{4} \end{aligned}$$

$$87. \frac{2t^2}{\sqrt{5} - \sqrt{t}} = \frac{2t^2}{\sqrt{5} - \sqrt{t}} \cdot \frac{\sqrt{5} + \sqrt{t}}{\sqrt{5} + \sqrt{t}} = \frac{2t^2(\sqrt{5} + \sqrt{t})}{(\sqrt{5})^2 - (\sqrt{t})^2} = \frac{2t^2(\sqrt{5} + \sqrt{t})}{5 - t} = \frac{2t^2(\sqrt{5} + \sqrt{t})}{5 - t}$$

$$89. \frac{8a}{\sqrt{3a} + \sqrt{a}} = \frac{8a}{\sqrt{3a} + \sqrt{a}} \cdot \frac{\sqrt{3a} - \sqrt{a}}{\sqrt{3a} - \sqrt{a}} = \frac{8a(\sqrt{3a} - \sqrt{a})}{(\sqrt{3a})^2 - (\sqrt{a})^2} = \frac{8a(\sqrt{3a} - \sqrt{a})}{3a - a} = \frac{8a(\sqrt{3a} - \sqrt{a})}{2a} = 4(\sqrt{3a} - \sqrt{a})$$

$$\begin{aligned}
 91. \frac{3(x-4)}{x^2-\sqrt{x}} &= \frac{3(x-4)}{x^2-\sqrt{x}} \cdot \frac{x^2+\sqrt{x}}{x^2+\sqrt{x}} = \frac{3(x-4)(x^2+\sqrt{x})}{(x^2)^2-(\sqrt{x})^2} = \frac{3(x-4)(x^2+\sqrt{x})}{x^4-x} \\
 &= \frac{3(x-4)(x^2+\sqrt{x})}{x(x^3-1)} = \frac{3(x-4)(x^2+\sqrt{x})}{x(x-1)(x^2+x+1)}
 \end{aligned}$$

$$\begin{aligned}
 93. \frac{\sqrt{u+v}}{\sqrt{u-v}-\sqrt{u}} &= \frac{\sqrt{u+v}}{\sqrt{u-v}-\sqrt{u}} \cdot \frac{\sqrt{u-v}+\sqrt{u}}{\sqrt{u-v}+\sqrt{u}} = \frac{\sqrt{u+v}(\sqrt{u-v}+\sqrt{u})}{u-v-u} \\
 &= \frac{\sqrt{u+v}(\sqrt{u-v}+\sqrt{u})}{-v} = \frac{-\sqrt{u+v}(\sqrt{u-v}+\sqrt{u})}{v}
 \end{aligned}$$

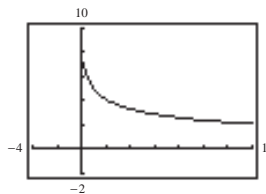
95. Keystrokes:

$$y_1 \text{ [Y=] } 10 \text{ [÷] [C] [✓] [X,T,θ] [+] 1 [)] [ENTER]$$

$$y_2 \text{ [(] } 10 \text{ [C] [✓] [X,T,θ] [-] 1 [)] [÷] [C] [X,T,θ] [-] 1 [)] [GRAPH]$$

$y_1 = y_2$, except at $x = 1$

$$\frac{10}{\sqrt{x}+1} = \frac{10}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{10(\sqrt{x}-1)}{x-1}, x \neq 1$$



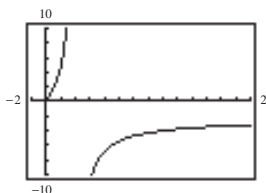
97. Keystrokes:

$$y_1 \text{ [Y=] } 2 \text{ [✓] [X,T,θ] [÷] [C] } 2 \text{ [-] [✓] [X,T,θ] [)] [ENTER]$$

$$y_2 \text{ [(] } 2 \text{ [✓] [X,T,θ] [+] [X,T,θ] [)] [÷] [C] } 4 \text{ [-] [X,T,θ] [)] [GRAPH]$$

$y_1 = y_2$

$$\begin{aligned}
 \frac{2\sqrt{x}}{2-\sqrt{x}} &= \frac{2\sqrt{x}}{2-\sqrt{x}} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} \\
 &= \frac{2\sqrt{x}(2+\sqrt{x})}{2^2-(\sqrt{x})^2} \\
 &= \frac{2\sqrt{x}(2+\sqrt{x})}{4-x} \\
 &= \frac{2(2\sqrt{x}+x)}{4-x}
 \end{aligned}$$



$$99. \frac{\sqrt{2}}{7} = \frac{\sqrt{2}}{7} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{7\sqrt{2}}$$

$$\begin{aligned}
 101. \frac{\sqrt{7}+\sqrt{3}}{5} &= \frac{\sqrt{7}+\sqrt{3}}{5} \cdot \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} \\
 &= \frac{(\sqrt{7})^2-(\sqrt{3})^2}{5(\sqrt{7}-\sqrt{3})} \\
 &= \frac{7-3}{5(\sqrt{7}-\sqrt{3})} \\
 &= \frac{4}{5(\sqrt{7}-\sqrt{3})}
 \end{aligned}$$

103. Area = $h \cdot w$

$$\begin{aligned}
 &= \sqrt{24^2 - (8\sqrt{3})^2} \cdot 8\sqrt{3} \\
 &= \sqrt{576 - 192} \cdot 8\sqrt{3} \\
 &= \sqrt{384} \cdot 8\sqrt{3} \\
 &= 8\sqrt{1152} \\
 &= 8\sqrt{2^7 \cdot 3^2} \\
 &= 8 \cdot 2^3 \cdot 3\sqrt{2} \\
 &= 192\sqrt{2} \text{ square inches}
 \end{aligned}$$

$$105. \frac{500k}{\frac{1}{\sqrt{k^2+1}} + \frac{k^2}{\sqrt{k^2+1}}} = \frac{500k}{\frac{1+k^2}{\sqrt{k^2+1}}} = \frac{500k\sqrt{k^2+1}}{1+k^2}$$

$$107. \sqrt{3}(1 - \sqrt{6}) = \sqrt{3} - \sqrt{3} \cdot \sqrt{6}$$

Distributive Property

$$= \sqrt{3} - \sqrt{9 \cdot 2}$$

Multiplication Property of Radicals

$$= \sqrt{3} - 3\sqrt{2}$$

Simplify radicals.

$$109. (3 - \sqrt{2})(3 + \sqrt{2}) = 9 - 2 = 7$$

Multiplying the number by its conjugate yields the difference of two squares. Squaring a square root eliminates the radical.

Mid-Chapter Quiz for Chapter 5

$$1. \sqrt{225} = 15 \text{ because } 15 \cdot 15 = 225$$

$$2. \sqrt[4]{\frac{81}{16}} = \frac{3}{2} \text{ because } \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{81}{16}$$

$$3. 64^{1/2} = \sqrt{64} = 8 \text{ because } 8 \cdot 8 = 64$$

$$4. (-27)^{2/3} = \sqrt[3]{(-27)^2} = (\sqrt[3]{-27})^2 = (-3)^2 = 9$$

$$5. \sqrt{27x^2} = \sqrt{9 \cdot 3 \cdot x^2} = 3|x|\sqrt{3}$$

$$6. \sqrt[4]{81x^6} = \sqrt[4]{81 \cdot x^4 \cdot x^2} = 3|x|\sqrt[4]{x^2} = 3|x|\sqrt{x}$$

$$7. \sqrt{\frac{4u^3}{9}} = \frac{\sqrt{4 \cdot u^2 \cdot u}}{\sqrt{9}} = \frac{2|u|\sqrt{u}}{3}$$

$$8. \sqrt[3]{\frac{16}{u^6}} = \frac{\sqrt[3]{16}}{\sqrt[3]{u^6}} = \frac{\sqrt[3]{16}}{u^2} = \frac{\sqrt[3]{8 \cdot 2}}{u^2} = \frac{2\sqrt[3]{2}}{u^2}$$

$$9. \sqrt{200y} - 3\sqrt{8y} = \sqrt{100 \cdot 2y} - 3\sqrt{4 \cdot 2y} \\ = 10\sqrt{2y} - 6\sqrt{2y} \\ = 4\sqrt{2y}$$

$$10. 6x\sqrt[3]{5x^2} + 2\sqrt[3]{40x^4} = 6x\sqrt[3]{5x^2} + 2\sqrt[3]{8 \cdot 5 \cdot x^3 \cdot x} \\ = 6x\sqrt[3]{5x^2} + 4x\sqrt[3]{5x}$$

$$11. \sqrt{8}(3 + \sqrt{32}) = 3\sqrt{8} + \sqrt{256} \\ = 3\sqrt{4 \cdot 2} + \sqrt{2^8} \\ = 6\sqrt{2} + 2^4 \\ = 6\sqrt{2} + 16$$

$$12. (\sqrt{50} - 4)\sqrt{2} = \sqrt{100} - 4\sqrt{2} \\ = \sqrt{10^2} - 4\sqrt{2} \\ = 10 - 4\sqrt{2}$$

$$13. (\sqrt{6} + 3)(4\sqrt{6} - 7) = \sqrt{6} \cdot 4\sqrt{6} - 7\sqrt{6} + 12\sqrt{6} - 21 \\ = 24 + 5\sqrt{6} - 21 \\ = 3 + 5\sqrt{6}$$

$$14. (9 + 2\sqrt{3})(2 + 7\sqrt{3}) = 18 + 63\sqrt{3} + 4\sqrt{3} + 2\sqrt{3} \cdot 7\sqrt{3} \\ = 18 + 67\sqrt{3} + 14(3) \\ = 18 + 67\sqrt{3} + 42 \\ = 60 + 67\sqrt{3}$$