CHAPTER 5
Radicals and Complex Numbers

Section 5.1 Radicals and Rational Exponents

Solutions to Odd-Numbered Exercises

1. $\sqrt{64} = 8$ because $8 \cdot 8 = 64$

3. $-\sqrt{49} = -7$ because $7 \cdot 7 = 49$

5. $\sqrt[3]{-8} = -2$ because $-2 \cdot -2 \cdot -2 = -8$

7. $\sqrt{-1}$ is not a real number because no real number multiplied by itself yields -1.

9. Because $7^2 = 49$, 7 is a square root of 49.

11. Because $4.2^3 = 74.088$, 4.2 is a cube root of 74.088.

13. Because $45^2 = 2025$, 45 is called the square root of 2025.

15. $\sqrt{8^2} = |8| = 8$
(index is even)

17. $\sqrt{(-10)^2} = |-10| = 10$
(index is even)

19. $\sqrt{-9^2}$ = not a real number
(even root of a negative number)

21. $-\sqrt{\left(\frac{2}{3}\right)^2} = -\frac{2}{3}$
(index is even)

23. $\sqrt{-\left(\frac{3}{10}\right)^2}$ = not a real number
(even root of a negative number)

25. $\left(\sqrt{5}\right)^2 = 5$
(inverse property of powers and roots)

27. $-\left(\sqrt{23}\right)^2 = -23$
(inverse property of powers and roots)

29. $\sqrt[3]{(5)^3} = 5$
(index is odd)

31. $\sqrt[3]{10^3} = 10$
(index is odd)

33. $-\sqrt[3]{(-6)^3} = 6$
(index is odd)

35. $\sqrt[3]{\left(-\frac{1}{4}\right)^3} = -\frac{1}{4}$
(index is odd)

37. $\left(\sqrt[3]{11}\right)^3 = 11$
(inverse property of powers and roots)

39. $\left(-\sqrt[3]{24}\right)^3 = -24$
(inverse property of powers and roots)

41. $\sqrt[4]{3^4} = 3$
(inverse property of powers and roots)

43. $-\sqrt[4]{-5^4}$ = not a real number
(even root of a negative number)

45. $\sqrt{6}$ is not rational because 6 is not a perfect square.

47. $\sqrt{900}$ is rational because $30 \cdot 30 = 900$, a perfect square.

49.

Radical Form	*Rational Exponent Form*
$\sqrt{16} = 4$	$16^{1/2} = 4$

51.

Radical Form	*Rational Exponent Form*
$\sqrt[3]{27^2} = 9$	$27^{2/3} = 9$

53.

Radical Form	*Rational Exponent Form*
$\sqrt[4]{256^3} = 64$	$256^{3/4} = 64$

55. $25^{1/2} = \sqrt{25} = 5$
Root is 2. Power is 1.

57. $-36^{1/2} = -\sqrt{36} = -6$
Root is 2. Power is 1.

59. $-(16)^{3/4} = -\left(\sqrt[4]{16}\right)^3 = -8$
Root is 4. Power is 3.

61. $32^{-2/5} = \dfrac{1}{\left(\sqrt[5]{32}\right)^2} = \dfrac{1}{2^2} = \dfrac{1}{4}$

Root is 5. Power is 2.

63. $(-27)^{-2/3} = \dfrac{1}{(-27)^{2/3}} = \dfrac{1}{\left(\sqrt[3]{-27}\right)^2}$
$= \dfrac{1}{9}$

Root is 3. Power is 2.

65. $\left(\frac{8}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Root is 3. Power is 2.

67. $\left(\frac{121}{9}\right)^{-1/2} = \left(\frac{9}{121}\right)^{1/2} = \sqrt{\frac{9}{121}} = \frac{3}{11}$

Root is 2. Power is 1.

69. $(3^3)^{2/3} = 3^{3\cdot 2/3} = 3^2 = 9$

Root is 3. Power is 2.

71. $-(4^4)^{3/4} = -4^{4\cdot 3/4} = -4^3 = -64$

Root is 4. Power is 3.

73. $\left(\dfrac{1}{5^3}\right)^{-2/3} = (5^3)^{2/3} = 5^{3\cdot 2/3} = 5^2 = 25$

Root is 3. Power is 2.

75. $\sqrt{t} = t^{1/2}$

Root is 2. Power is 1.

77. $x\sqrt[4]{x^3} = x \cdot x^{3/4} = x^{1+3/4} = x^{7/4}$

Root is 4. Power is 3.

79. $u^2\sqrt[3]{u} = u^2 \cdot u^{1/3} = u^{2+1/3} = u^{7/3}$

Root is 3. Power is 1.

81. $s\sqrt[4]{s^5} = s^4 \cdot s^{5/2} = s^{4+5/2} = s^{13/2}$

Root is 2. Power is 5.

83. $\dfrac{\sqrt{x}}{\sqrt{x^3}} = \dfrac{x^{1/2}}{x^{3/2}} = x^{1/2-3/2} = x^{-1} = \dfrac{1}{x}$

85. $\dfrac{\sqrt[4]{t}}{\sqrt{t^5}} = \dfrac{t^{1/4}}{t^{5/2}} = t^{1/4-5/2}$
$= t^{1/4-10/4}$
$= t^{-9/4}$
$= \dfrac{1}{t^{9/4}}$

87. $\sqrt[3]{x^2} \cdot \sqrt[3]{x^7} = x^{2/3} \cdot x^{7/3} = x^{2/3+7/3} = x^{9/3}$
$= x^3$

89. $\sqrt[4]{y^3} \cdot \sqrt[3]{y} = y^{3/4} \cdot y^{1/3} = y^{3/4+1/3}$
$= y^{9/12+4/12} = y^{13/12}$

91. $\sqrt[4]{x^3y} = (x^3y)^{1/4}$
$= x^{3/4}y^{1/4}$

93. $z^2\sqrt{y^5z^4} = z^2 \cdot (y^5z^4)^{1/2} = z^2y^{5/2}z^2$
$= z^{2+2}y^{5/2} = z^4y^{5/2}$

95. $3^{1/4} \cdot 3^{3/4} = 3^{1/4+3/4}$
$= 3^{4/4}$
$= 3^1$

97. $(2^{1/2})^{2/3} = 2^{1/2\cdot 2/3}$
$= 2^{1/3}$
$= \sqrt[3]{2}$

99. $\dfrac{2^{1/5}}{2^{6/5}} = 2^{1/5-6/5}$
$= 2^{-5/5}$
$= 2^{-1} = \dfrac{1}{2}$

101. $(c^{3/2})^{1/3} = c^{3/2\cdot 1/3}$
$= c^{1/2}$
$= \sqrt{c}$

103. $\dfrac{18y^{4/3}z^{-1/3}}{24y^{-2/3}z} = \dfrac{6 \cdot 3y^{4/3-(-2/3)}z^{-1/3-1}}{6 \cdot 4}$

$= \dfrac{3y^{6/3}z^{-4/3}}{4}$

$= \dfrac{3y^2}{4z^{4/3}}$

105. $(3x^{-1/3}y^{3/4})^2 = 3^2x^{-2/3}y^{3/2}$

$= \dfrac{9y^{3/2}}{x^{2/3}}$

107. $\left(\dfrac{x^{1/4}}{x^{1/6}}\right)^3 = (x^{1/4-1/6})^3$

$= (x^{3/12-2/12})^3$

$= (x^{1/12})^3$

$= x^{3/12}$

$= x^{1/4}$

109. $\sqrt{\sqrt[4]{y}} = (y^{1/4})^{1/2}$

$= y^{1/4 \cdot 1/2}$

$= y^{1/8}$

$= \sqrt[8]{y}$

111. $\sqrt[4]{\sqrt{x^3}} = \sqrt[4]{x^{3/2}}$

$= (x^{3/2})^{1/4}$

$= x^{3/2 \cdot 1/4}$

$= x^{3/8}$

113. $\dfrac{(x+y)^{3/4}}{\sqrt[4]{x+y}} = \dfrac{(x+y)^{3/4}}{(x+y)^{1/4}}$

$= (x+y)^{3/4-1/4}$

$= (x+y)^{2/4}$

$= (x+y)^{1/2}$

$= \sqrt{x+y}$

115. $\dfrac{(3u-2v)^{2/3}}{\sqrt{(3u-2v)^3}} = \dfrac{(3u-2v)^{2/3}}{(3u-2v)^{3/2}} = (3u-2v)^{2/3-3/2}$

$= (3u-2v)^{4/6-9/6}$

$= (3u-2v)^{-5/6}$

$= \dfrac{1}{(3u-2v)^{5/6}}$

117. $\sqrt{73} \approx 8.5440$

Scientific: 73 $\boxed{\sqrt{\ }}$

Graphing: $\boxed{\sqrt{\ }}$ 73 ENTER

119. $315^{2/5} = \left(\sqrt[5]{315}\right)^2 \approx 9.9845$

Scientific: 315 $\boxed{y^x}$ (2 ÷ 5) =

Graphing: 315 ^ (2 ÷ 5) ENTER

121. $1698^{-3/4} \approx 0.0038$

Scientific: 1698 $\boxed{y^x}$ (3 ÷ 4 +/−) =

Graphing: 1698 ^ ((−) 3 ÷ 4) ENTER

123. $\sqrt[4]{342} \approx 4.3004$

$\sqrt[4]{342} = 342^{1/4}$

Scientific: 342 $\boxed{y^x}$ (1 ÷ 4) =

Graphing: 342 ^ (1 ÷ 4) ENTER

125. $\sqrt[3]{545^2} \approx 66.7213$

$\sqrt[3]{545^2} = 545^{2/3}$

Scientific: 545 $\boxed{y^x}$ (2 ÷ 3) =

Graphing: 545 ^ (2 ÷ 3) ENTER

127. $\dfrac{8-\sqrt{35}}{2} \approx 1.0420$

Scientific: (8 − 35 $\boxed{\sqrt{\ }}$) ÷ 2 =

Graphing: (8 − $\boxed{\sqrt{\ }}$ 35) ÷ 2 ENTER

129. $\dfrac{3+\sqrt{17}}{9} \approx 0.7915$

Scientific: (3 + 17 $\boxed{\sqrt{\ }}$) ÷ 9 =

Graphing: (3 + $\boxed{\sqrt{\ }}$ 17) ÷ 9 ENTER

131. $f(x) = 3\sqrt{x},\ x \geq 0,$

Domain $= [0, \infty)$

133. The domain of $g(x) = \dfrac{2}{\sqrt[4]{x}}$ is the set of all nonnegative real numbers or $(0, \infty)$.

135. $f(x) = \sqrt{-x}, \quad -x \geq 0$

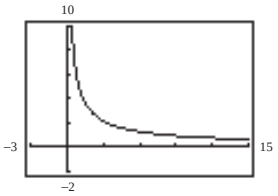
$x \leq 0$

Domain $= (-\infty, 0]$

137. *Keystrokes:*

Y= 5 ÷ 4 MATH 5 X,T,θ MATH 3 GRAPH

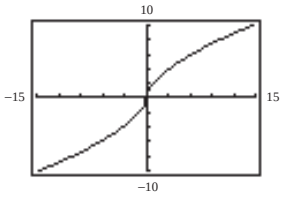
Domain is $(0, \infty)$ so graphing utility did complete the graph.



139. *Keystrokes:*

Y= 2 X,T,θ ^ (3 ÷ 5) GRAPH

Domain is $(-\infty, \infty)$ so graphing utility did complete the graph.



141. $x^{1/2}(2x - 3) = 2x^{3/2} - 3x^{1/2}$

143. $y^{-1/3}(y^{1/3} + 5y^{4/3}) = y^0 + 5y^{3/3}$

$= 1 + 5y$

145. $r = 1 - \left(\dfrac{25{,}000}{75{,}000}\right)^{1/8}$

$= 1 - \left(\dfrac{1}{3}\right)^{1/8}$

$\approx 0.128 \approx 12.8\%$

147. *Verbal Model:* $\boxed{\text{Area}} = \boxed{\text{side}} \cdot \boxed{\text{side}}$

Labels: Area $= 529$

side $= x$

Equation:

$529 = x \cdot x$

$529 = x^2$

$\sqrt{529} = x$

$23 = x$

23 feet $\times$ 23 feet

149. $d = \sqrt{l^2 + w^2 + h^2}$

$= \sqrt{9^2 + 5^2 + 2^2}$

$= \sqrt{81 + 25 + 4}$

$= \sqrt{110}$

≈ 10.49 cm

151. (a) $r = \sqrt{\dfrac{2v}{\pi l}}$

$r = \sqrt{\dfrac{2(35{,}350)}{\pi(100)}}$

$r \approx 15$ feet

(b) $h = \sqrt{r^2 - \left(\dfrac{a}{2}\right)^2}$

$h = \sqrt{15^2 - \left(\dfrac{a}{2}\right)^2}$

(c) $h = \sqrt{15^2 - \left(\dfrac{25}{2}\right)^2}$

$h = \sqrt{225 - 156.25}$

$h = \sqrt{68.75}$

$h \approx 8.29$ feet

(d) $a = 2\sqrt{r^2 - h^2}$

$a = 2\sqrt{15^2 - 8^2}$

$= 2\sqrt{225 - 64}$

$= 2\sqrt{161}$

≈ 25.38 feet

153. Given $\sqrt[n]{a}$, a is the radicand and n is the index.

155. No. $\sqrt{2}$ is an irrational number. Its decimal representation is a nonterminating, nonrepeating decimal.

157. (a) "Last digits:" 1 (Perfect square 81)
4 (Perfect square 64)
5 (Perfect square 25)
6 (Perfect square 36)
9 (Perfect square 49)
0 (Perfect square 100)

(b) Yes, 4,322,788,986 ends in a 6, but it is not a perfect square.

Section 5.2 Simplifying Radical Expressions

1. $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{2^2 \cdot 5} = 2\sqrt{5}$

3. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$

5. $\sqrt{96} = \sqrt{16 \cdot 6} = \sqrt{4^2 \cdot 6} = 4\sqrt{6}$

7. $\sqrt{216} = \sqrt{36 \cdot 6} = \sqrt{6^2 \cdot 6} = 6\sqrt{6}$

9. $\sqrt{1183} = \sqrt{169 \cdot 7} = \sqrt{13^2 \cdot 7} = 13\sqrt{7}$

11. $\sqrt{0.04} = \sqrt{4 \cdot 0.01} = \sqrt{4}\sqrt{0.01} = 2 \cdot 0.1 = 0.2$

13.
$$\begin{aligned}\sqrt{0.0072} &= \sqrt{36 \cdot 2 \cdot 0.0001} \\ &= \sqrt{36} \cdot \sqrt{2} \cdot \sqrt{0.0001} \\ &= 6 \cdot 0.01\sqrt{2} \\ &= 0.06\sqrt{2}\end{aligned}$$

15.
$$\begin{aligned}\sqrt{2.42} &= \sqrt{121 \cdot 2 \cdot 0.01} \\ &= \sqrt{121} \cdot \sqrt{2} \cdot \sqrt{0.01} \\ &= 11 \cdot 0.1\sqrt{2} \\ &= 1.1\sqrt{2}\end{aligned}$$

17. $\sqrt{\dfrac{15}{4}} = \dfrac{\sqrt{15}}{2}$

19. $\sqrt{\dfrac{13}{25}} = \dfrac{\sqrt{13}}{5}$

21.
$$\begin{aligned}\sqrt{9x^5} &= \sqrt{3^2x^4 \cdot x} \\ &= 3 \cdot x^2 \cdot \sqrt{x} \\ &= 3x^2\sqrt{x}\end{aligned}$$

23. $\sqrt{48y^4} = \sqrt{16 \cdot 3 \cdot y^4} = 4y^2\sqrt{3}$

25. $\sqrt{117y^6} = \sqrt{9 \cdot 13 \cdot y^6} = 3|y^3|\sqrt{13}$

27. $\sqrt{120x^2y^3} = \sqrt{4 \cdot 30 \cdot x^2 \cdot y^2 \cdot y} = 2|x|y\sqrt{30y}$

29.
$$\begin{aligned}\sqrt{192a^5b^7} &= \sqrt{64 \cdot 3 \cdot a^4 \cdot a \cdot b^6 \cdot b} \\ &= 8a^2b^3\sqrt{3ab}\end{aligned}$$

31. $\sqrt[3]{48} = \sqrt[3]{16 \cdot 3} = \sqrt[3]{2^4 \cdot 3} = 2\sqrt[3]{3 \cdot 2} = 2\sqrt[3]{6}$

33.
$$\begin{aligned}\sqrt[3]{112} &= \sqrt[3]{8 \cdot 14} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{14} \\ &= 2\sqrt[3]{14}\end{aligned}$$

35.
$$\begin{aligned}\sqrt[3]{40x^5} &= \sqrt[3]{8 \cdot 5 \cdot x^3 \cdot x^2} \\ &= 2x\sqrt[3]{5x^2}\end{aligned}$$

37.
$$\begin{aligned}\sqrt[4]{324y^6} &= \sqrt[4]{81 \cdot 4 \cdot y^4 \cdot y^2} \\ &= 3|y|\sqrt[4]{4y^2} \\ &= 3|y|\sqrt[4]{2^2y^2} \\ &= 3|y|\sqrt{2y}\end{aligned}$$

39. $\sqrt[3]{x^4y^3} = \sqrt[3]{x^3 \cdot x \cdot y^3} = xy\sqrt[3]{x}$

41.
$$\begin{aligned}\sqrt[4]{3x^4y^2} &= \sqrt[4]{x^4} \cdot \sqrt[4]{3y^2} \\ &= |x|\sqrt[4]{3y^2}\end{aligned}$$

43. $\sqrt[5]{32x^5y^6} = \sqrt[5]{2^5 \cdot x^5 \cdot y^5 \cdot y} = 2xy\sqrt[5]{y}$